**1S1 Tutorial Sheet 4**[1]

1 December 2006

**Useful facts:**

- **Constant**: If $f = c$ a constant $f' = 0$.
- **x to the n**: If $f = x^n$ then $f' = nx^{n-1}$.
- **Trigonometric functions**

$$\begin{aligned} \frac{d}{dx}\cos x &= -\sin x \\ \frac{d}{dx}\sin x &= \cos x \end{aligned} \tag{1}$$

- **Product rule**:

$$\frac{d}{dx}fg = f\frac{df}{dx} + g\frac{dg}{dx} \tag{2}$$

- **Quotient rule**:

$$\frac{d}{dx}\frac{f}{g} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2} \tag{3}$$

- **Chain rule**: If $f(u(x))$ then

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx} \tag{4}$$

- **Stationary points**: If $f'(x) > 0$ a function is **increasing** at $x$, if it is negative it is **decreasing**, if $f'(x) = 0$ then $x$ is a critical point. If $f''(x) > 0$ at a critical point it is a **minimum**, if $f''(x) < 0$ at a critical point it is a **maximum**, if $f''(x) = 0$ it is undecided, it could be a **point of inflection**, a point joining a **concave up** $f''(x) > 0$ interval from a **concave down**, $f''(x) < 0$, interval.

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**Questions**

The numbers in brackets give the numbers of marks available for the question.

1. (4) Work out $f'$ for

$$f(x) = x^2 \sin 3x, \qquad \sin x^2, \qquad f(x) = \frac{\cos x}{\sin^2 x}, \qquad f(x) = x \cos \sqrt{x} \tag{5}$$

2. (2) Find and classify the stationary points

$$f = 2x^4 - 4x^2 + 6 \tag{6}$$

3. (1) Show $f(x) = \tan x$ has a point of inflection at $x = 0$.

4. (1) Classify the stationary points of $3x^4 - 4x^4$

**Extra Questions**

The questions are extra; you don't need to do them in the tutorial class.

1. Differentiate

$$\frac{1}{\sqrt{1+x}}, \quad \sin\cos x, \quad \sin\frac{1}{1+x^2}, \quad \sqrt{\sin x}, \quad \tan 2x, \quad \sin\frac{1}{x} \tag{7}$$

2. Classify the stationary points for

$$f(x) = x^3+3x^2-9x+1, \qquad f(x) = x^{2/3}, \qquad f(x) = x^{1/3}(x+4), \qquad 2x^3-6x+7 \tag{8}$$

3. Find and classify the stationary points of $x^3 - 3x^2 + 3x - 1$. You may have to look at the sign of $f'(x)$ to do this.