1S1 Tutorial Sheet 3[1]

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Useful facts:

- The **derivative**
$$f'(x) = \frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

- **Higher derivatives**:
$$\begin{aligned} f''(x) &= \frac{d^2 f}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right) \\ f'''(x) &= \frac{d^3 f}{dx^3} = \frac{d}{dx}\left(\frac{d^2 f}{dx^2}\right) \end{aligned} \tag{2}$$
and so on, a bracket number is used after f''', for example f'''' is written $f^{(4)}$.

- **Constant**: If $f = c$ a constant $f' = 0$.

- **x to the n**: If $f = x^n$ then $f' = nx^{n-1}$.

- **Linearity**:
$$\begin{aligned} \frac{d}{dx}(cf) &= c\frac{df}{dx} \\ \frac{d}{dx}(f+g) &= \frac{df}{dx} + \frac{dg}{dx} \end{aligned} \tag{3}$$

- **Product rule**:
$$\frac{d}{dx} fg = f\frac{df}{dx} + g\frac{dg}{dx} \tag{4}$$

- **Quotient rule**:
$$\frac{d}{dx}\frac{f}{g} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2} \tag{5}$$

- **Trigonometric functions**
$$\begin{aligned} \frac{d}{dx}\cos x &= -\sin x \\ \frac{d}{dx}\sin x &= \cos x \end{aligned} \tag{6}$$

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/1S1`

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (2) Work out f', f'' and keep going differentiating until you get zero.
$$f(x) = x^7 + 6x^4 + 4x - 1 \tag{7}$$

2. (2) Differentiate
$$f = \frac{1}{x^2} \tag{8}$$
and
$$f = x + \frac{1}{x} \tag{9}$$

3. (2) Use the product and quotient rules to differentiate
$$f = \sqrt{x}(x^2 + 1) \tag{10}$$
and
$$f = \frac{\sqrt{x}}{x^2 + 1} \tag{11}$$

4. (2) The tan is defined as
$$\tan x = \frac{\sin x}{\cos x} \tag{12}$$
and the secant as
$$\sec x = \frac{1}{\cos x} \tag{13}$$
Show using the quotient rule that
$$\frac{d}{dx} \tan x = \sec^2 x \tag{14}$$

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. Differentiate
$$\frac{1}{1+x}, \quad \frac{x^2 - x - 6}{x - 3}, \quad \frac{3x^2 - 4}{x^2 - x + 16}, \quad \frac{x^2 - 4}{\sqrt{x}}, \quad \sin x \cos x, \quad x \sin x \tag{15}$$

2. $f(x) = \sin^2 x + \cos^2 x$, we know by the Pythagorous theorem that $f(x) = 1$, but check that $f'(x) = 0$ from the original formula with sines and cosines. If $f'(x) = 0$ then $f(x)$ is a constant, check the value of the constant by working out $f(0)$. Is the answer consistent with the Pythagorous theorem?

3. Differentiate $\sec x$ using the quotient rule.