1S1 Tutorial Sheet 2[1]

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Useful facts:

- **Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a but not equal to a then we write

$$\lim_{x\to a} f(x) = L \tag{1}$$

- **One-sided Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and greater than a then we write

$$\lim_{x\to a+} f(x) = L \tag{2}$$

If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and less than a then we write

$$\lim_{x\to a-} f(x) = L \tag{3}$$

- **Infinite Limit**: (Informal definition). If the value of $f(x)$ can be made as large as we like by taking values of x sufficiently close to a but not equal to a then we write

$$\lim_{x\to a} f(x) = \infty \tag{4}$$

There are also minus infinity and one-sided versions of this, for example If the value of $f(x)$ can be made as large a negative number as we like by taking values of x sufficiently close to a and greater than a then we write

$$\lim_{x\to a+} f(x) = -\infty \tag{5}$$

- **Limit at Infinity**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking sufficiently large values of x then we write

$$\lim_{x\to \infty} f(x) = L \tag{6}$$

- **Limit at Infinity of Rational Functions**: Divide above and below by the highest power in the denominator.

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- **Moving around square roots**: For example

$$1+\sqrt{5x^2+2} = \frac{1+\sqrt{5x^2+2}}{1}\frac{1-\sqrt{5x^2+2}}{1-\sqrt{5x^2+2}} = \frac{1-(5x^2+2)}{1-\sqrt{5x^2+2}} = -\frac{1+5x^2}{1-\sqrt{5x^2+2}} \quad (7)$$

For working out limits like $\lim_{x\to\infty}(1+\sqrt{5x+2})$ it is a good idea to move the square root to the bottom;

$$\lim_{x\to\infty}(1+\sqrt{5x^2+2}) = -\lim_{x\to\infty}\left(\frac{1+5x^2}{1-\sqrt{5x^2+2}}\right) = -\lim_{x\to\infty}\left(\frac{1/x+5x}{1/x-\sqrt{5+2/x^2}}\right) = -\infty \quad (8)$$

where in the last term I have divided above and below by $|x|$ and used the fact that $|x| = x$ for x positive. Also $\lim_{x\to\infty} x = \infty$.

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (2) Evaluate the limits

$$\lim_{x\to 2}\frac{x^2-x-2}{x-2}, \qquad \lim_{x\to 4}\frac{x^2-8x+16}{x-4} \quad (9)$$

2. (3) Evaluate the limits

$$\lim_{x\to 2}\frac{x-2}{x^2-x-2}, \qquad \lim_{x\to 4-}\frac{x-4}{x^2-8x+16} \qquad \lim_{x\to 4+}\frac{x-4}{x^2-8x+16} \quad (10)$$

3. (2) Evaluate the limits

$$\lim_{x\to\infty}\frac{x^2-2}{x^2-x-2}, \qquad \lim_{x\to\infty}\frac{x^3-4}{x^2-8x+16} \quad (11)$$

4. (1) Evaluate the limit

$$\lim_{x\to\infty}(\sqrt{x^4+2x^2}-x^2) \quad (12)$$

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. Evaluate the limits

$$\lim_{x\to\infty}\frac{1+\sqrt{x^2+1}}{1+x}, \quad \lim_{x\to 3}\frac{x^2-x-6}{x-3}, \quad \lim_{x\to\infty}\frac{3x^2-4}{x^2-x+16}, \quad \lim_{x\to-\infty}\frac{x-4}{x^2-8x+16} \quad (13)$$

2. Evaluate the limits

$$\lim_{x\to\infty}(\sqrt{x^4+3x^2}-x^2), \qquad \lim_{x\to\infty}(\sqrt{x^6+2x^4+5x^3}-x^3-x), \quad \lim_{x\to 0}\frac{x(x-1)}{x^2+x} \quad (14)$$