1S1 Tutorial Sheet 1[1]

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Useful facts:

- **Domain and range**: The *domain* of a function $f(x)$ is the values of x is is defined for, the *range* is the set of all values $f(x)$ takes when x takes all values in the domain. If no domain is given explicitely, the *natural domain* is the set of all values of x for which $f(x)$ is defined and real.

- **Inverse**: A function $f(x)$ has an *inverse* if it is strictly monotonically increasing or strictly monotonically decreasing. This means that it is either going up, $f(x_1) > f(x_2)$ whenever $x_1 > x_2$, or it is going down, $f(x_1) < f(x_2)$ whenever $x_1 > x_2$. If you draw a horizontal line anywhere on the graph of an invertible function, it will only cut it once.

- **Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a but not equal to a then we write

$$\lim_{x\to a} f(x) = L \tag{1}$$

- **One-sided Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and greater than a then we write

$$\lim_{x\to a+} f(x) = L \tag{2}$$

  If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and less than a then we write

$$\lim_{x\to a-} f(x) = L \tag{3}$$

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Questions

The numbers in brackets give the numbers of marks available for the question.

1. (2) What is the natural domain of
$$f(x) = \sqrt{(x-1)(3-x)} \tag{4}$$

2. (2) We define the function
$$f(x) = x^2 \tag{5}$$
on the domain $x \geq 0$. Is this function invertible? If so, what is its inverse?

3. Consider the piecewise function
$$f(x) = \begin{cases} x & x < 2 \\ 3 & x = 2 \\ 1 & 2 < x \leq 3 \\ 4 - x & x > 3 \end{cases} \tag{6}$$
   (a) (2) Graph the function.
   (b) (2) Calculate
$$\lim_{x \to 2-} f(x), \qquad \lim_{x \to 2+} f(x), \qquad f(2), \qquad \lim_{x \to 3} f(x) \tag{7}$$

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. What is the natural domain of
$$f(x) = \sqrt{-x} \tag{8}$$

2. Graph and invert
$$f(x) = \begin{cases} x^2 & x > 0 \\ x & x < 0 \end{cases} \tag{9}$$

3. Find
$$\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1} \tag{10}$$

4. Graph the function
$$f(x) = \frac{|x|}{x} \tag{11}$$
What is its domain? What are the one-sided limits at $x = 0$?