

1S1 Tutorial Sheet 1¹

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Useful facts:

- **Domain and range:** The *domain* of a function $f(x)$ is the values of x is defined for, the *range* is the set of all values $f(x)$ takes when x takes all values in the domain. If no domain is given explicitly, the *natural domain* is the set of all values of x for which $f(x)$ is defined and real.
- **Inverse:** A function $f(x)$ has an *inverse* if it is strictly monotonically increasing or strictly monotonically decreasing. This means that it is either going up, $f(x_1) > f(x_2)$ whenever $x_1 > x_2$, or it is going down, $f(x_1) < f(x_2)$ whenever $x_1 > x_2$. If you draw a horizontal line anywhere on the graph of an invertible function, it will only cut it once.
- **Limit:** (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a but not equal to a then we write

$$\lim_{x \rightarrow a} f(x) = L \quad (1)$$

- **One-sided Limit:** (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and greater than a then we write

$$\lim_{x \rightarrow a+} f(x) = L \quad (2)$$

If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and less than a then we write

$$\lim_{x \rightarrow a-} f(x) = L \quad (3)$$

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (2) What is the natural domain of

$$f(x) = \sqrt{(x-1)(3-x)} \quad (4)$$

2. (2) We define the function

$$f(x) = x^2 \quad (5)$$

on the domain $x \geq 0$. Is this function invertible? If so, what is its inverse?

3. Consider the piecewise function

$$f(x) = \begin{cases} x & x < 2 \\ 3 & x = 2 \\ 1 & 2 < x \leq 3 \\ 4 - x & x > 3 \end{cases} \quad (6)$$

- (a) (2) Graph the function.

- (b) (2) Calculate

$$\lim_{x \rightarrow 2-} f(x), \quad \lim_{x \rightarrow 2+} f(x), \quad f(2), \quad \lim_{x \rightarrow 3} f(x) \quad (7)$$

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. What is the natural domain of

$$f(x) = \sqrt{-x} \quad (8)$$

2. Graph and invert

$$f(x) = \begin{cases} x^2 & x > 0 \\ x & x < 0 \end{cases} \quad (9)$$

3. Find

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} \quad (10)$$

4. Graph the function

$$f(x) = \frac{|x|}{x} \quad (11)$$

What is its domain? What are the one-sided limits at $x = 0$?

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