A Taylor series example[1]

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The Taylor series for $\sin x$ is given by

$$\sin x = \sum n \text{ odd}(-1)^{(n-1)/2}\frac{x^n}{n!} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + O(x^9) \tag{1}$$

This is worked out from the general formula for the Taylor series about $x = 0$

$$f(x) = \sum n = 0^\infty \frac{1}{n!} f^{(n)}(0) \tag{2}$$

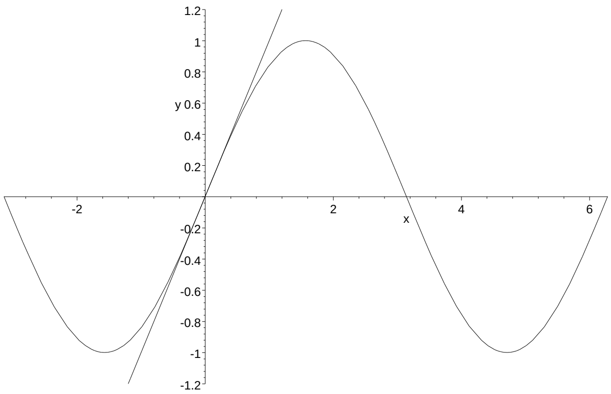
using $f(x) = \sin x$ so $f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$ and so on, with $\sin 0 = 0$ and $\cos 0 = 1$. Now, writing

$$f_N(x) = \sum_{n \text{ odd}, \ n \leq N} (-1)^{(n-1)/2}\frac{x^n}{n!} \tag{3}$$

so, for example,

$$f_5(x) = x - \frac{x^3}{6} + \frac{x^5}{120} \tag{4}$$

here are some graphs to show the Taylor series in action, the sine is in gray, the series in black. After a few terms it gets good near $x = 0$, but even at $O(x^{13})$ is isn't good beyond $x = Pi$. then



$f_1(x)$

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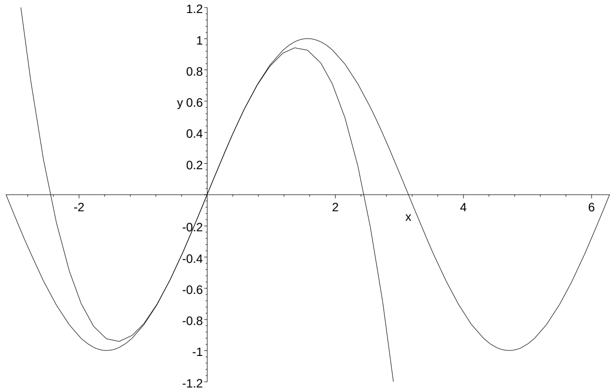
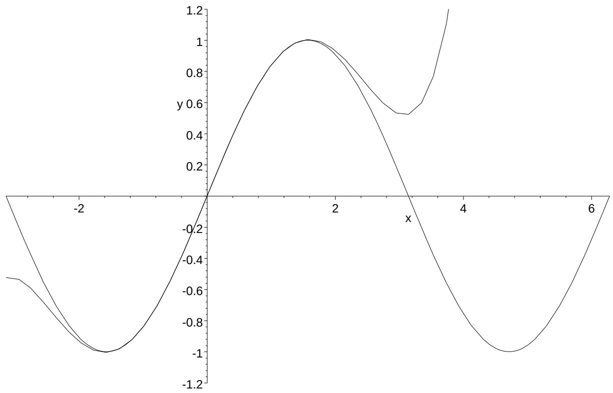
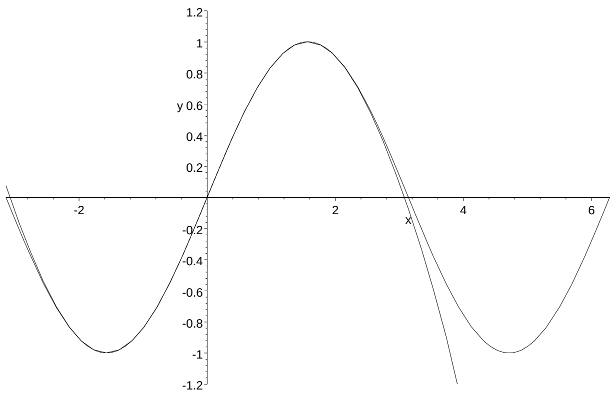


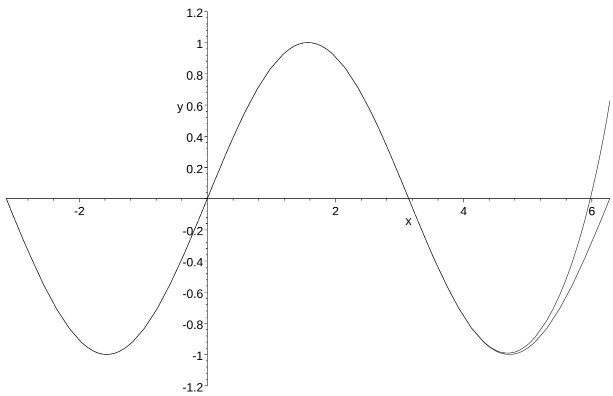
$f_3(x)$



$f_5(x)$



$f_7(x)$



$f_{13}(x)$