An implicit differentiation example[1]

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Here is a quick example of implicit differentiation, say we want dy/dx when we know

$$x^2y^2 + x\sin y = 0 \tag{1}$$

We don't want to solve for y, so we just differentiation the equation

$$\frac{d}{dx}(x^2y^2 + x\sin y) = 0 \tag{2}$$

Now, by the product rule

$$\frac{d}{dx}x^2y^2 = 2xy^2 + x^2\frac{dy^2}{dx} \tag{3}$$

and, by the chain rule,

$$\frac{dy^2}{dx} = \frac{dy^2}{dy}\frac{dy}{dx} = 2y\frac{dy}{dx} \tag{4}$$

In the same way, using the product rule and the chain rule

$$\frac{d}{dx}x\sin y = \sin y + x\cos y\frac{dy}{dx} \tag{5}$$

Putting all of this together

$$\begin{aligned} \frac{d}{dx}(x^2y^2 + x\sin y) &= 2xy^2 + 2x^2y\frac{dy}{dx} + \sin y + x\cos y\frac{dy}{dx} && (6)\\ &= (2xy^2 + \sin y) + (2x^2y + x\cos y)\frac{dy}{dx} = 0 && (7) \end{aligned}$$

Solve for dy/dx gives

$$\frac{dy}{dx} = -\frac{2xy^2 + \sin y}{2x^2y + x\cos y} \tag{8}$$

Notice that the equation for dy/dx is linear in dy/dx and so is easy to solve, the equation for y was not.

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/231`