Gamma-ray Spectroscopy

Louis Gregg

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1 Abstract

The the scintillator and photomultiplier was used and calibrated in this experiment using the radioactive source Cs^{137}.The position of the total energy peak was shown to vary with V^n and n was calculated to be 7.46. The Maestro software provided was used to obtain the γ-ray spectra of a number of radioactive sources were obtained; Cs^{137},Co^{60}, Na^{22} and KCl. A number of features were identified in the spectra including the Compton continuum, the 511KeV peak associated with pair annihilation and the total energy peaks of the respective sources. The efficiency of the spectrometer was estimated to be 14.6% by comparing the count obtained for the 511KeV peak and it's corresponding sum peak. The activity of the KCl source was calculated experimentally to be 8.369, which did not agree with our theoretically estimated value of 168.

2 Introduction

2.1 The Apparatus

The apparatus used in this experiment to detect γ rays has three main components; a scintillator, a photomultiplier(PM) and a multichannel analyzer(MCA). γ-rays produced by radioactive dicay can undergo photoelectric absorption. In this process, the γ-ray interacts with an electron in the shell of an atom. The γ-ray disappears and the energy of the photoelectron created is given by relation:

$$E_e = E_\gamma - E_b$$

where E_e is the photoelectron's energy, E_γ is the energy of the incident radiation and E_b is the energy with which the electron was initially bound to the atom.
A characteristic x-ray will also be generated in this process when the hole left by the photoelectron is filled by a higher energy electron. However, this x-ray photon is likely to undergo photoelectric absorption itself so that the total energy of both photoelectron is E_γ. Hence the corresponding peak recorded in the spectrometer is called the total energy peak, H_0.
Electron emitted in this way are then incident on the scintillator which in this experiment consists of a sodium iodide (NaI) crystal containing a small amount

of thallium as an activator element.
The Co^{60} source emits two γ-rays of different energies as part of its decay process. It is possible that both rays undergo photoelectric absorption producing two photoelectrons. For the occurrence of this case a peak is observed in the spectrum which corresponds to the detection of both of these photoelectron simultaneously, the sum peak. Its energy corresponds to $E_1 + E_2$, the sum of the energies of the two γ-rays.

2.2 Compton scattering

Compton scatter is the name given to the phenomenon in which a γ-ray photon is absorbed by an electron, assumed to be at rest.
Due to the conservation of momentum:

$$E_\gamma + E_e = E_{\gamma'} + E_{e'}$$

where E_γ and E_e are the initial energies of the γ-ray and electron, and $E_{\gamma'}$ and $E_{e'}$ are their resultant energies after interacting.
Similarly, by conservation of momentum:

$$P_\gamma = P_{\gamma'} + P_{e'}$$

The initial and resultant photon energies are $E_\gamma = hf$ and $E_{\gamma'} = hf'$.
The initial energy of the electron(assumed to have no kinetic energy) $= E_e = m_e c^2$, where$h$ is Planck's constant.
After scattering it is possible that the electron may be accelerated to a significant fraction of the speed of light and hence it's total energy should be given using the relativistic energy-momentum relation:

$$E_{e'} = \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}$$

The Expression for the conservation of momentum is now:

$$hf + m_e c^2 = hf' + \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}$$

Re-arranging to give the momentum of the scattered electron:

$$p_{e'}^2 c^2 = (hf - hf' + m_e c^2)^2 - m_e^2 c^4$$

This is the first equation for $P_{e'}^2 c^2$.
Solving the conservation of momentum equation for the scattered electron gives,

$$P_{e'} = P_\gamma - P_{\gamma'}$$

By making use of the scalar product with the previous relation:

$$\begin{aligned} P_{e'}^2 &= P_{e'} \cdot P_{e'} \\ &= (P_\gamma - P_{\gamma'}) \cdot (P_\gamma - P_{\gamma'}) \\ &= p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'} \cos\theta \end{aligned}$$

By multiplying across by c^2 and using the relations $E_\gamma = hf = p_\gamma c$ and $p = \hbar k$, the equation above can be re-written: This is the second equation for $P_{e'}^2 c^2$.

$$P_{e'}^2 c^2 = (hf)^2 + (hf')^2 - 2h^2 ff' \cos\theta$$

Now, equating the two equations for the momentum of the scattered electron produces the equations:

$$(hf + m_e c^2 - hf)^2 = (hf)^2 + (hf')^2 - 2h^2 ff' \cos\theta)$$

and subsequently:

$$2hfm_e c^2 - 2hf' m_e c^2 = 2h^2 ff'(1 - \cos\theta)$$

The difference of the two photon energies is given by:

$$hf - hf' = \frac{hfhf'(1 - \cos\theta)}{m_e c^2}$$

and by re-arranging, the energy of the scattered photon is given by:

$$hf' = \frac{hf}{1 + \frac{hf(1-\cos\theta)}{m_e c^2}}$$

This is the energy of a photon scattered through an angle θ. Therefore $\cos\theta$ lies within the range $-1 \leq \cos\theta \leq 1$. Hence the scattered photon has a minimum energy of:

$$hf' = \frac{hf}{1 + \frac{hf}{m_e c^2}}$$

Also it should be noted that for the case of the the incident photon transferring all of it's energy to the electron($\theta = 180$, the electron's energy is given by:

$$E_e = E_\gamma - E_{\gamma'} = \frac{2E^2}{m_e c^2 + 2E}$$

This maximum for the energy of a scattered electron gives rise to a sudden drop in the energy of detected electrons in the MCA, known as the Compton edge.

2.3 Experiment 1

Experiment 1 aims to calibrate the PM voltage(V) and observe it's effect on the total energy peak's position. It will be be shown that $H_0 \propto V^n$. Also, the dependence of the resolution of the device on the PM voltage will be evaluated.

2.4 Experiment 2

In this experiment the spectra of spectra of Cs^{137}, Co^{60} and Na^{22} will be scrutinized and characteristics identified.
The Cs^{137} source decays with a long half life by beta emission to $^{137}_{56}$Ba. This Barium atom is at an excited state. 85% of de-excitations occur by emission of a characteristic and distinct γ-ray of energy 662KeV. The remaining de-excitations occur by a series of characteristic Barium x-rays. Both of these paths should be represented by a peak in the radiation spectrum obtained.
The $^{60}_{27}$Co source decay begins with the emission of a beta particle. This produces the nickel isotope $^{60}_{28}$Ni in an excited state. The de-excitation consists of two consecutive γ emissions, first at an energy of 1173KeV and second at 1332KeV to the nucleus' ground state. These separate emissions will appear as two peaks of equal intensity in the radiation spectrum.
The $^{22}_{11}$Na nucleus decay has a branching ratio of 90% β+(positron) emission and 10% electron capture. Both of these decays produce an excited-state $^{22}_{10}$Ne isotope. The Ne isotope then returns to the a stable state by means of a γ-ray of energy 1274KeV or by characteristic Ne X-ray emissions.
It is important to note that the strongest peak in the $^{22}_{11}$Na spectrum will occur at 511KeV and is distinct from the decay process within the nucleus. It results from the pair-annihilation of the emitted positron with an electron,This causes the emission of two γ-rays, each having same energy(511KeV) and opposite momentum of one another.

3 Experimental Method

3.1 Experiment 1

The Cs^{137} source was placed in the spectrometer. This source emits 662 KeV γ-rays. Withing the Maestro software which recorded the radiation spectrum, the target voltage was set to 600V. The fine gain was set to 0.55. The spectrum was recorded for a photomultiplier(PM) voltage of 600V.
Features including the Barium x-ray peak, the Compton continuum, the Compton edge and the 622 keV photopeak were noted. Then a series of Cs^{137} spectra were recorded for a range of PM voltages. From each of these spectra, the energy of the peak(H_0), as given in position along the x-axis(units:channel number or KeV) and the FWHM of the peak were recorded. These values are tabulated in the results section below.

3.2 Experiment 2

The PM voltage was returned to 600V and the spectra of Cs^{137}, $Co^{6}0$ and $Na^{2}2$ were recorded and scrutinized.
These spectra are included with the report.

3.3 Experiment 3

For experiment 3, the radioactive source used was KCl. This was a particularly low-activity source and was left in the apparatus to build a clear spectrum for two days.

4 Results and Analysis

4.1 Experiment 1

PM Voltage (V)	H_0	Peak Energy(KeV)	FWHM	R=FWHM/H_0
524	84.1	231.98	16.18	0.0697
549	120.83	333.28	23.58	0.0708
575	171.9	474.16	33.07	0.0697
599	240.41	663.21	46.61	0.0703
624	332.33	916.68	63.67	0.069
649	451.5	1245.4	57.34	0.046
675	608.72	1679.05	58.95	0.0351
699	813.14	2242.9	34.37	0.0153

Theoretically, $H_0 \propto V^n$. By plotting $ln(h_0)$ as a function of $ln(V)$, the plot was linear and was described by the following relation:

$$y = 7.46x - 42.23$$

Therefore n can be taken to be 7.46.
For A V OF 600v, this n value gives H_0 as follows:

$$ln(H_0) = 7.46 \times ln(V) - 42.23$$

$$H_0 = 242.5$$

This is within the range of the obtained value of 240.41 for 599V.

5 Experiment 2

The spectra were obtained for each of the sources provided. The total energy peaks were identified in each and they were labeled accordingly on the printouts. The sum peak was observed in the Co^{60} spectrum and the Compton continuum and Compton edge were observed in the spectra.

6 Experiment 3

The count rate for the 511 peak observed in the Na^{22} spectrum due to pair annihilation is given by the relation $Count1 = 3669 = 2A\epsilon$, where A is the

activity of the source and ϵ is the efficiency of the spectrometer at 511keV.
The count rate for the sum peak is $Count2 = 268 = A\epsilon^2$. Hence the efficiency of the detector is given by the relation:

$$\epsilon = \frac{2 \times 268}{3669} = 0.146 = 14.6\%$$

The KCL source used in this experiment has an abundance of the isotope k^{40} of 0.011%. 3.625g of KCl is provided. k^{40} has a half-life of 1.26×10^9 years. It has two main channels of decay, β^- (89%) and EC followed by the emission of a 1446keV γ-ray (11%). Only the γ-rays are detected in this spectrometer.
The mass of the isotope was calculated:

$$mass = 0.011 \times 3.625g = 0.03987g$$

The number isotope atoms was calculated:

$$N_0 = \frac{0.03987}{39.96 \times 1.66 \times 10^{-24}} = 6.01 \times 10^{20}$$

The specific activity of a sample is given by:

$$\begin{aligned} A &= N_0 \times \frac{ln(2)}{t_{1/2}} \\ &= 6.01 \times 10^{20} \times \frac{0.693}{1.26 \times 10^9 \times 364.25 \times 24 \times 3600} \\ &= 10503 \end{aligned}$$

Of the decays that occur, only 11% will emit γ radiation.
Also the previously calculated spectrometer efficiency value using the 511KeV peak/sum peak ratio to be 14.6% should be taken into account. Hence the observed rate could be estimated to be:

$$Rate = 10503 \times 0.11 \times 0.146 = 168$$

The total count rate for two days was 1.42481×10^6 The count rate over two days was calculated:

$$countrate = \frac{1.42481 \times 10^6}{170252.88seconds} = 8.369$$

7 Conclusion

The effect of the PM voltage on the observed spectrum was examined. The position of the total energy peak was shown to vary with V^n and n was calculated to be 7.46.
The Co^{60} sum peak was observed and both of the total energy peaks were observed.

The failure to obtain a good value for the rate of decay could be contributed to the loss of electrons from the device. Also the spectrometer sensitivity could have a dependence on voltage which meant the value used in estimating the count rate was inaccurate. Also,calculation of the estimated count rate made the assumption that the source had experienced 0 decays at the when it was added to the spectrometer, which was not the case.