Expansion of the Universe

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1 Abstract

The Hubble constant was calculated using least-square fitting, for data taken by Edwin Hubble and for modern data of distant supernovae. The origional Hubble-data yielded a H_0 of $488 Kms^{-1}Mpc^{-1}$. The Modern data yielded a H_0 of $58.6 Kms^{-1}Mpc^{-1}$. The age of the universe was estimated from these values to be $2.444 * 10^9 years$ (Hubble data) and $1.6686 * 10^{10} years$ (modern data). The disparity between Hubble data and modern data was concluded to be do the limited data set available to Hubble at the time of his origional calculations.

2 Introduction

The aim of the experiment is to calculate the Hubble constant, H_0, by the method of least square fitting using Hubble's original data and modern data obtained from distant supernovae, respectively. Edwin Hubble concluded that the distance an object is from the observer, d, is proportional to the recessional velocity of the object in space, v. He equated these two factors linearly according to Hubble's Law

$$v = H_0 * d$$

where H_0 is defined as the Hubble constant. A table of Hubble's original values of the velocities and the distances of some nearby galaxies are provided. A first-guess estimate of H_0 and constant c will be made such that the line will roughly fit the values provided. A range of values around these initial estimates of H_0 and c will be tested for the data using the method of least squares. A value of S will be calculated for each value of H_0 and c , where S is given by the following summation:

$$S = \sum_{i=1}^{N} (y_i - y_i^{model})^2$$

In this summation, N is the number of data points provided, y_i are the velocities corresponding to respective values of distance,d, and y_i^{model} are values of velocity calculated using modelled values of H_0 and c for the same values of distance. This calculation of S will be completed for all modelled values of H_0 and c. The values of H_0 and c for which S is a minimum will have the "least squares". Theses values will fit the data provided to a linear equation. This process will be carried out for two sets of data.

The first set consists of objects at relatively small distances ($0.5 \rightarrow 2$ Mpc) and velocities ($650 \rightarrow 1800$ Kms^{-1}). The second data set is modern data for distant supernovae at much higher velocities. In this data set, the redshifts of the objects, the ditance modulus and an error associated with the distance modulus are provided. The velocity of an object can be calculated from the redshift using the following relation:

$$v = c * \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

In the equation above, c is the speed of light ($2.998 * 10^8$) and z is the redshift of the object (due to the net recessional velocity). The distance modulus,μ, is the difference between the apparent magnitute of an object and it's absolute magnitude. The distance of an object from the observer is then given from the relation:

$$d = 10^{\frac{\mu}{5}+1}$$

The uncertainty in distance can be obtained from the error in modulus through the relation $\delta d = 0.2 * \ln 10 * 10^{\frac{\mu}{5}+1} \delta\mu$, where δd is the distance error and $\delta\mu$ is the error in distance modulus.

3 Experimental Procedure

Hubble's origional data was read into IDL and the velocity and ditstance values were plotted (Figure 1).
A short script was used to obtain initial estimates of H_0 and c.

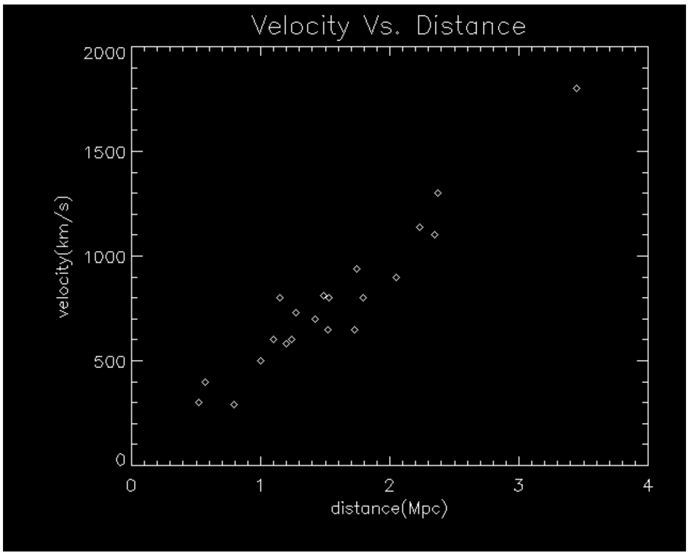

Figure 1: Hubble Data

The initial estimate for H_0 was $511.945 Kms^{-1}Mpc^{-1}$ and the initial estimate for c was $33.7841 Kms^{-1}$. A range of values in the vacinity of these initial estimates were then tested using the method of least squares. The step size between values tested was $2Kms^{-1}Mpc^{-1}$ and $1Kms^{-1}$, for H_0 and c respectively.A shaded surface image of the values of S obtained for modelled H_0 and c was produced (figure 2).

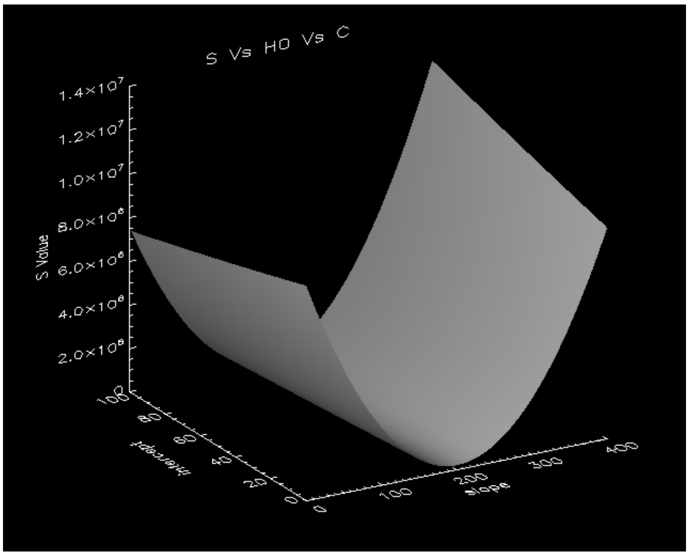

Figure 2: Variation of S with h_0 and c

The values of H_0 and c for which S were a minimum were used to overplot a best-fit line to the origional hubble data (figure 3).

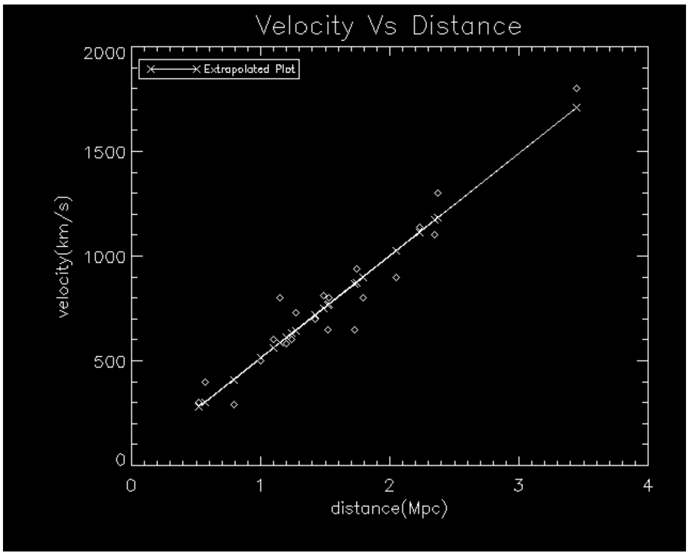

Figure 3: Overplot of least sqaure values

The age and size of the universe were calulated from the best fit value of H_0.
The distant supernovae data was then read into IDL and the distance, distance error and velocity values were calculated from the distance modulus, modulus error and redshift values. Velocity was plotted with respect to distance as previously(figure 4). Hubbles origional data was included in the plot to illustrate differences in scale.

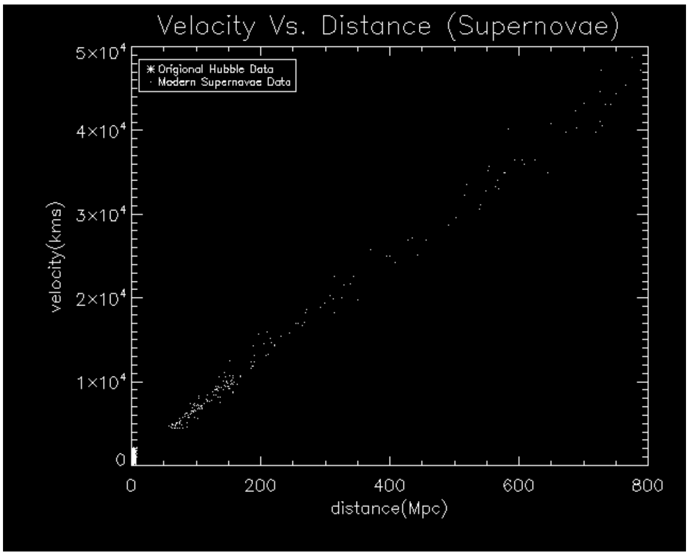

Figure 4: Hubble and Modern Data

A script obtained initial estimates of H_O and c to at $58.031 Kms^{-1}Mpc^{-1}$ and $1397.09 Kms^{-1}$. A range of values about this initial H_0 and c were tested with stepsizes of $0.1 Kms^{-1}Mpc^{-1}$ and $1 Kms^{-1}$. The magnitude of S was again plotted as a function of modelled values (figure 5).

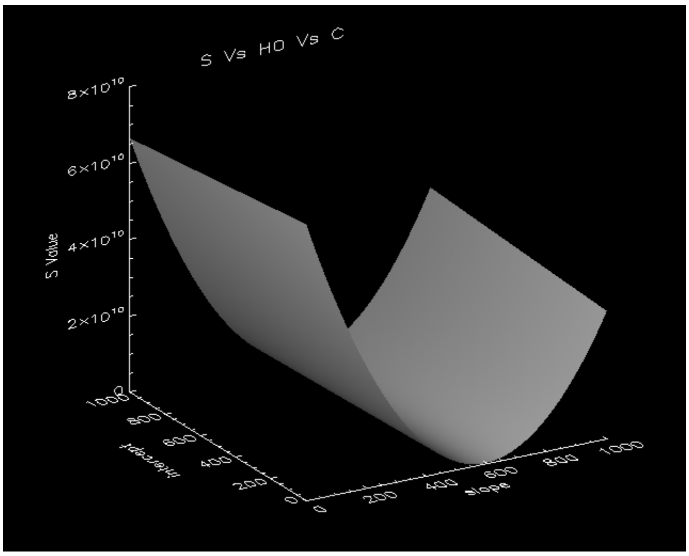

Figure 5: S with H_0 and c for Supernovae

Again, the optimum values of H_0 and c were used to overplot a best-fit line to the data (figure 6).

The distance modulus error was included for this plot.
Values for the Age and size of the universe were calulated from the H_0 values for modern data.

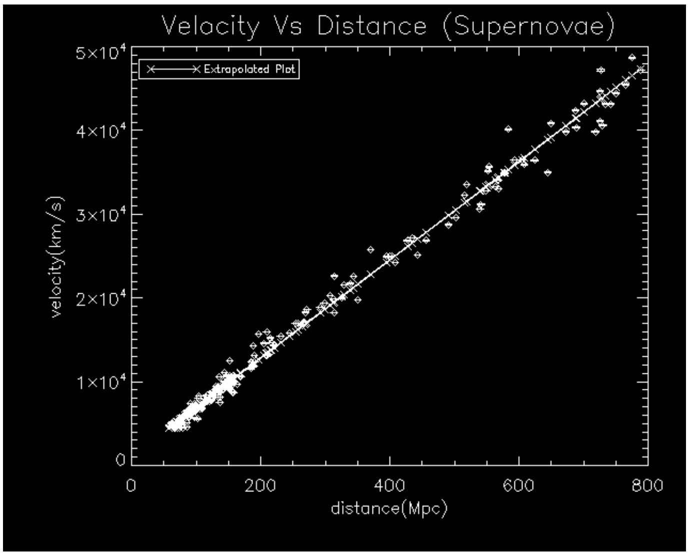

Figure 6: Overplot of least sqaure values (Supernovae)

4 Results and Discussion

The Hubble constant for Hubble's origional data was calculated to be $488Kms^{-1}Mpc^{-1}.c$ was calculated to be $25.00Kms^{-1}$. The age of the universe was then calculated from this value via the relation $T = \frac{1}{H_0}$, where T is the time elapsed since all objects were at the same point in space. An estimate of the age of the universe was calculated as follows:

$$\begin{aligned} T &= \frac{3.0856 * 10^{22} m}{488 * 1000 ms^{-1}} \\ &= 7.7142 * 10^{16} s \\ &= 2.444 * 10^{9} years \end{aligned}$$

Additionally a measureof the size of the universe(in the form of the total distance travelled by light to earth from the origin point of the unverse) can be estimated using the relation $D = T * c$, where D is the distance and c is the speed of light.

$$D = 7.7142 * 10^{16} * 2.998 * 10^{8} = 2.3127 * 1025m = 749.4951 Mpc$$

The Supernova data yielded a hubble constant of $58.6Kms^{-1}Mpc^{-1}$ and c value of $1117Kms^{-1}$. The Age of the universe was estimated.

$$
\begin{aligned}
T &= \frac{3.0856 * 10^{22} m}{58.6 * 1000 ms^{-1}} \\
&= 5.2657 * 10^{17} s \\
&= 1.6686 * 10^{10} years
\end{aligned}
$$

THe size fo the universe was estimated using this value of T, as with the Hubble data.

$$D = 5.2657 * 10^{17} * 2.998 * 10^{8} = 1.5786 * 1026m = 5116.0413Mpc$$

The values of H_0 obtained using Hubbles data and modern data were different by a factor of 10. This discrepancy could be contributed to Hubbles relatively small data set. The low sensentivity of measurement equipment available at the time restricted his observations to proportionally small and slow objects. This is made evident in figure 4 where both Hubbles data and the modern data used were plotted.
However it is important to note that the modern data used is also small section of a larger data set. Relativistic considerations must be taken into account for faster objects and therefore at larger scales the correlation between velocity and distance cannot be treated as linear.