Your Titles

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1 Cosmology

The full Friedmann Equation is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \tag{1}$$

The scale factor can be related to redshift in all types of universe via the relation

$$a(t) = \frac{1}{1+z}, \tag{2}$$

which is nice.

If we consider the pressure exerted by radiation, as we often do when evaluating the evolution of a radiation dominated Universe (capital U!!), we can write

$$P = \frac{\rho_{rad} c^2}{3}. \tag{3}$$

The fluid equation governs the evolution of the density of matter in the Universe with time. It can be obtained by application of the Thermodynamic relation

$$dE = PdV = TdS, \tag{4}$$

for an expanding sphere of "comoving radius". I don't know what that term means. Fuck it. Anyway, assume it's a reversible expansion ($dS = 0$, isothermal?), obtain the energy enclosed by $E = mc^2$ for a volume of density ρ and rearranging before substitution into the TD equation above, you dig?

The fluid relation is then given by

$$\dot{\rho} + 3\left(\rho + \frac{P}{c^2}\right)\frac{\dot{a}}{a} = 0. \tag{5}$$

What a friendly fella!

The acceleration equation is obtained through combination of the other two relations. The acceleration equation is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right). \tag{6}$$

Expand on how to obtain and use this. I don't think he's asked a question involving this yet.

But, who knows? We don't rely on it heavily as I recall.

The critical density is the density required to ensure that the Universe is flat and that $k = 0$. We've observed that $k \approx 0$ to many decimal places, probably as driven there through our friend, the inflationary mechanism. Anyway, the density necessary to ensure this is

$$\rho_C(t) = \frac{3H^2}{8\pi G}. \tag{7}$$

This condition is currently met at the present time, by our observations.

The density fraction (correct name?) is the ratio of the density of the universe to ρ_C.

$$\Omega = \frac{\rho}{\rho_C} = \Omega_{Mat} + \Omega_{Rad} + \Omega_\Lambda = \frac{\rho_{Mat} + \rho_{rad} + \rho_\Lambda}{\rho_C}. \tag{8}$$

Note that IF Λ is constant, we can write for a GENERAL curvature (k)

$$\Omega + \Omega_\Lambda - 1 = \frac{kc^2}{a^2H^2}. \tag{9}$$

For the case of flat curvature (i.e. $k \approx 0$), this equation becomes

$$\Omega_{Mat,Rad} + \Omega_\Lambda = 1. \tag{10}$$

We consider Universes with a constant vacuum energy density (because, fuck knows what else it's doing; ain't nobody got time for that) and so the density of dark energy is also a constant, given by

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}. \tag{11}$$

2 ISM

The equation of radiative transfer is our friend and it can be written as

$$I_\nu = I_\nu(0)e^{-\tau_{\nu r}} + \int_0^{\tau_{\nu r}} \frac{j_\nu}{\kappa_\nu} e^{-\tau_\nu} d\tau_\nu. \tag{12}$$

The source function is defined as

$$S_\nu = \frac{j_\nu}{\kappa_{nu}}. \tag{13}$$

j_ν is the emissivity of the medium at a given frequency and κ_ν is the absorptivity at a given frequency. We define the optical depth as along a line BEGINNING at the observer (i.e. @ observer $\tau_\nu \equiv 0$).

κ is positive under this convention; I don't know why, but I ain't about to fuck with the status quo.

At the far side of the cloud or medium, $\tau_\nu \equiv \tau_{\nu r}$.

For a medium/gas/region that is described as uniform in temperature or as being in thermal equilibrium, we can assert that the source function becomes a blackbody;

$$S_\nu \equiv \frac{j_\nu}{\kappa_{nu}} \rightarrow B_\nu(T). \tag{14}$$

This can also be applied for a medium in LTE (check this?).

Also the optical depth is formally defined as

$$d\tau_\nu = \kappa_{nu} ds. \tag{15}$$

where ds is an section of pathlength along the Line of Sight (LOS).

If we integrate though an ENTIRE cloud of uniform density and absorptivity (would need to be in TE to ensure this), then our TOTAL optical depth is $\tau_{\nu r} \equiv \kappa_\nu \times L$, where L is the total length of the path.

The Planck function, in its frequency form, is defined as

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \mathrm{W\ sr^{-1} m^{-2} Hz^{-1}}. \tag{16}$$

The Planck function in its wavelength form is

$$B_\lambda(T) = \frac{2hc^2}{\Lambda^5} \frac{1}{e^{\frac{hc}{\Lambda kT}} - 1} \mathrm{W\ sr^{-1} m^{-3}}. \tag{17}$$

At low frequencies (e.g. far IR and radio), the Planck function becomes the Raleigh-Jeans Law. This conversion is obtained through the approximation

$$e^x \approx 1 + x \text{ for } x << 1. \tag{18}$$

At very high frequencies, the Wien approximation applies.

What is this again?? It's jolly good fun, that's what it is!