Rutherford Scattering

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November 15, 2010

Abstract

A thin piece of gold foil is bombarded with alpha particles using an americium-241 source at several angles. The resulting scattering of the alpha particles is examined by finding the count rate at the angles. It was found that most of the particles were unscattered while a minority were scattered. It is predicted by Rutherford's model that the count rate is proportional to cosec^4 of the angle and this relationship was verified. This disproves the plum-pudding model and supports Rutherford's theory.

⋆ Introduction

The idea of the atom has been around for millenia yet little could be said about it. By the end of the 19th Century, it became apparent that atoms were a useful tool that could be used to model phenomena in physics and chemistry. J. J. Thomson proposed the plum pudding model in 1904 where the atom is a mass of positive charge with small negatively charged electrons embedded in it like plums in a plum pudding.

In 1909, Ernest Rutherford directed the famous experiment where a thin sheet of gold foil was bombarded with alpha particles. The plum pudding model predicted that the alpha particles would be scattered by the gold atoms by small angles. However, it was found that most of the alpha particles passed straight through the gold foil with little scattering and a very small amount of the alpha particles were scattered by some large angle and some were even backscattered.

The only explanation Rutherford had was that the plum pudding model could not be right. He proposed his own model where the atom is mostly empty space with a dense concentration of positive charge at the centre orbited by electrons. With this model, Rutherford derived the differential scattering cross section which corresponded with the experiment.

Differential Scattering Cross Section

Let dN be the number of particles scattered per unit time through angles between θ and $\theta + d\theta$ where θ is the scattering angle. Let n be the number of particles passing in unit time through unit area of the alpha-particle beam cross section. Then the effective scattering cross section $d\sigma$ is defined as,

$$d\sigma = \frac{dN}{n} \tag{1}$$

The impact parameter, ρ, is a function of the scattering angle. Therefore, the particles that are scattered through θ and $\theta + d\theta$ lie between $\rho(\theta)$ and $\rho(\theta) + \rho(d\theta)$. The number of particles that are passing are the particles that pass through the annulus of thickness $d\rho$ and radius ρ so $dN = 2\pi\rho\ d\rho\ .\ n = dA\ .\ n$.

$$\Rightarrow d\sigma = 2\pi\rho\ d\rho$$

In order to find the dependence of $d\sigma$ on the angle of scattering, rewrite the equation to get,

$$d\sigma = 2\pi\rho(\theta)\left|\frac{d\rho(\theta)}{d\theta}\right|d\theta \tag{2}$$

The modulus is because the derivative may be negative. The solid angle element(which is the surface area subtended by a surface onto a sphere), $d\Omega$, between cones with vertical angles θ and $\theta + d\theta$ is $d\Omega = 2\pi \sin\theta \; d\theta$. This gives,

$$d\sigma = \frac{\rho(\theta)}{\sin\theta}\left|\frac{d\rho}{d\theta}\right|d\Omega \tag{3}$$

$\frac{d\sigma}{d\Omega}$ is referred to as the differential scattering cross-section. Rutherford derived from his model,

$$\rho^2 = (\frac{\alpha^2}{m^2 v_\infty^4})\cot^2\frac{1}{2}\theta$$

where α is a constant related to the Coulomb field, m is the mass of the alpha particle and v_∞ is the velocity of the alpha particle at infinity. Substituting this into Equation (3) gives,

$$d\sigma = (\frac{\alpha}{2mv_\infty^2})^2 d\Omega \; \mathrm{cosec}^4\frac{\theta}{2}$$

Recalling that dN is defined as the number of particles scattered per unit time, this is simply equal to the count rate R. So $d\sigma = \frac{R}{n}$ giving,

$$R \propto \mathrm{cosec}^4\frac{\theta}{2}$$

$$\Rightarrow \log R \propto \log \mathrm{cosec}^4\frac{\theta}{2} \tag{4}$$

$$\Rightarrow \log R \propto 4\log \mathrm{cosec}\frac{\theta}{2} \tag{5}$$

ie, $\log R = \log k\mathrm{cosec}^4\frac{\theta}{2} = 1\log\mathrm{cosec}^4\frac{\theta}{2} + \log k$ and a similar result for equation (5).

⋆ Experimental Method

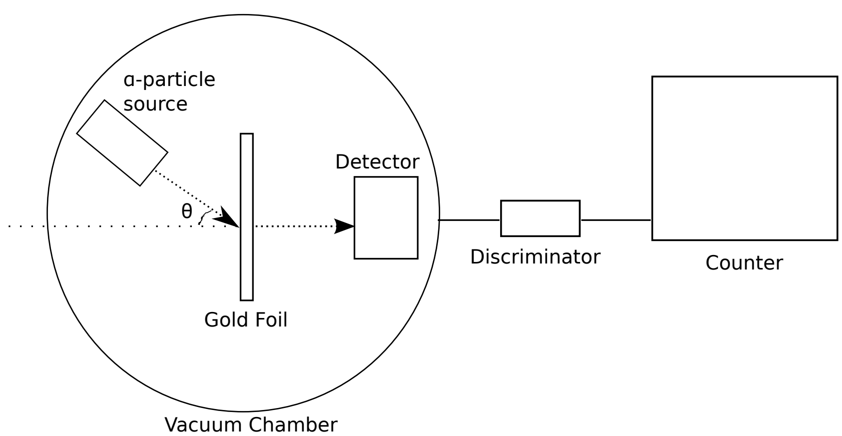

Figure 1: Apparatus of Experiment

The apparatus is set up as shown in figure 1. A discriminator is hooked up to filter out background noise. A 1mm collimating slit was used with the gold foil. The foil must be thin because if it is too thick, the alpha particles would not be able to penetrate the foil. It is placed in the chamber along with the alpha-particle source of 0.3 MBq(compared to Rutherford's experiment of 4GBq, this is about 10,000 times bigger). The pump is turned on until the pressure inside is ≈1mbar. This is done so that the alpha particles do not get scattered by air molecules due to the alpha particles low penetration power.

The detector detects counts of the alpha-particles, N, which are found for angles from 0° to ±50° for some period of time, t, to find the count rate. To speed up the process, a 5mm slit was replaced at ±30°. The count rate was measured at ±30° for both the 1mm and 5mm slit to find a conversion factor between the slits to be applied to counts found with the 5mm slit.

⋆ Results and Analysis

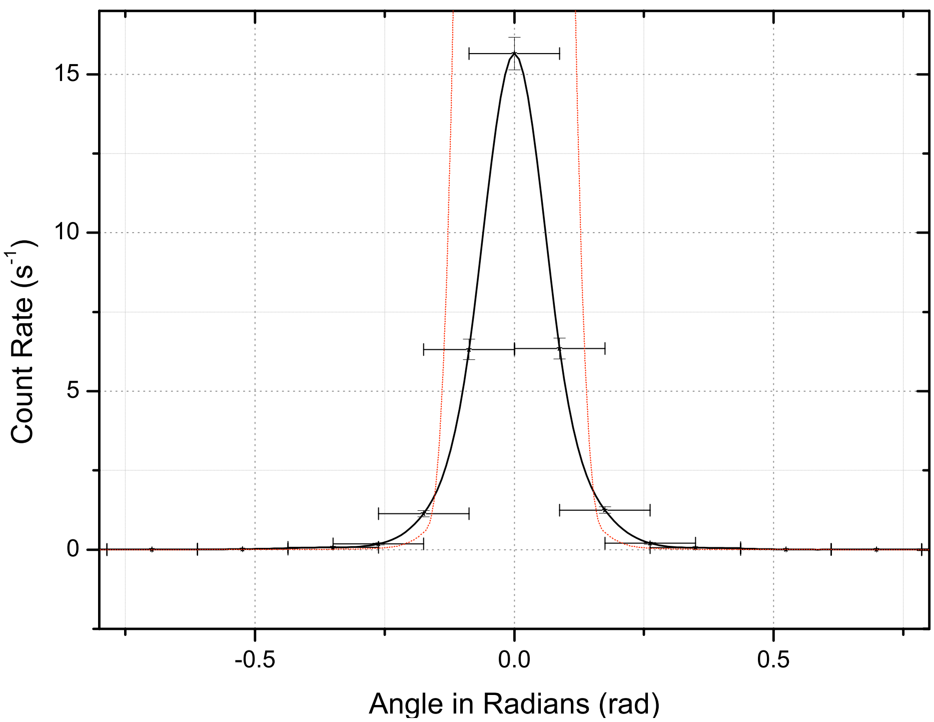

Figure 2: Graph of Count Rate vs Scattering Angle with cosec^4 normalised to the data

Figure 2 shows the count rate against the scattering angle. This clearly shows the relationship of R with $\text{cosec}^4\theta$. Although it is hard to tell from the graph, the points further away from 0 are small but nonzero as predicted by the cosec function. The relationship breaks down near 0 since cosec approaches infinity as it tends towards 0 and an infinite count rate is impossible.

However, using logarithmic plots will show the linear relationship between R and cosec^4 instantly by equations (4), (5). Figure 3 shows a linear relationship between log count rate and log cosec^4. The slope of the graph is 0.94 ± 0.01 while theory predicts it to be 1. Figure 4 shows a linear relationship between log count rate and log cosec. Note that there are less points than in the previous figure because cosec can return a negative number and logarithms are not defined. The slope of the graph is 3.24 ± 0.01 while theory predicts it to be 4. While these do not exactly agree with theory, note the huge error in the last 2 points which has skewed the line when fitting has been applied. The results are not too far from being right taking into account the experimental error.

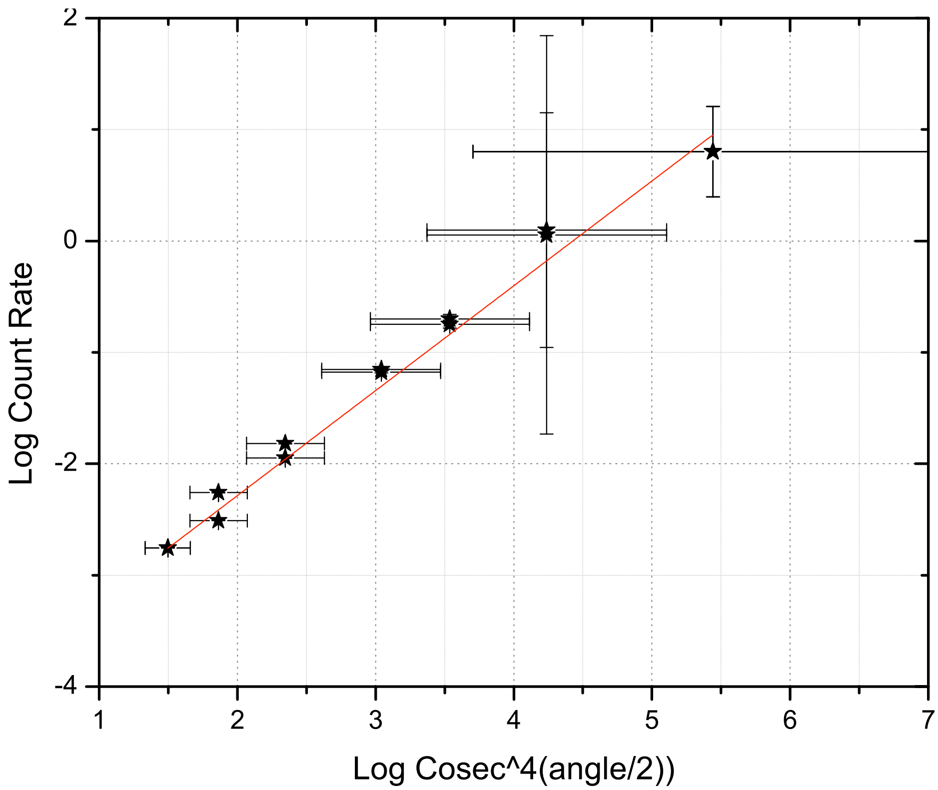

Figure 3: Graph of log Count Rate vs log $\mathrm{cosec}^4 \frac{\theta}{2}$ — $y = (0.94 \pm 0.01) - (4.16 \pm 0.01)$

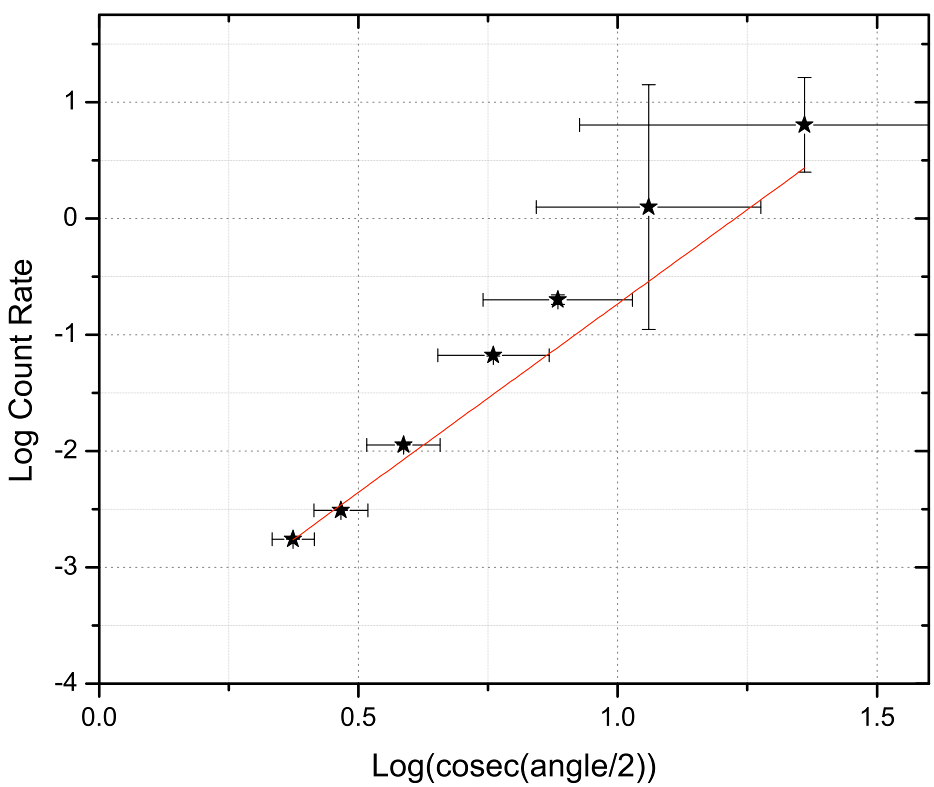

Figure 4: Graph of log Count Rate vs $\log\mathrm{cosec} \frac{\theta}{2}$ — $y = (3.24 \pm 0.01)x - (3.98 \pm 0.01)$

Errors

$$\Delta N = \sqrt{N}$$
$$\Delta R = R\sqrt{(\frac{\Delta N}{N})^2 + (\frac{\Delta t}{t})^2}$$
$$\Delta \log R = \frac{\Delta R}{R}$$

Using Wolfram Mathematica,

$$\Delta \log \mathrm{cosec}^4 \frac{\theta}{2} = 2\frac{\cot \frac{\theta}{2}}{\ln 10}\Delta\theta$$
$$\Delta \log \mathrm{cosec} \frac{\theta}{2} = 1/2\frac{\cot \frac{\theta}{2}}{\ln 10}\Delta\theta$$

⋆ Conclusion

Rutherford's theory was verified. It has been shown that there is a relationship between the count rate and the scattering angle and this can only be explained using Rutherford's model. Although the results did not exactly correspond to the predictions, this may have been due to time constraints. If more time was available, the count rates could be found more accurately reducing the error significantly. The error on the count rate can be seen quite clearly in figures 3 and 4.

When Rutherford first verified his theory, he was known to have said:

> "It was almost as if you fired a 15 inch shell into a piece of tissue paper and it came back and hit you."

Although we did not produce the most accurate results, we surely felt some of Rutherford's sentiments when performing the experiment.

⋆ References

L. D. Landau and E. M. Lifshitz; *Mechanics, Third Edition*; Elsevier Butterworth Heinemann