The Geiger Counter

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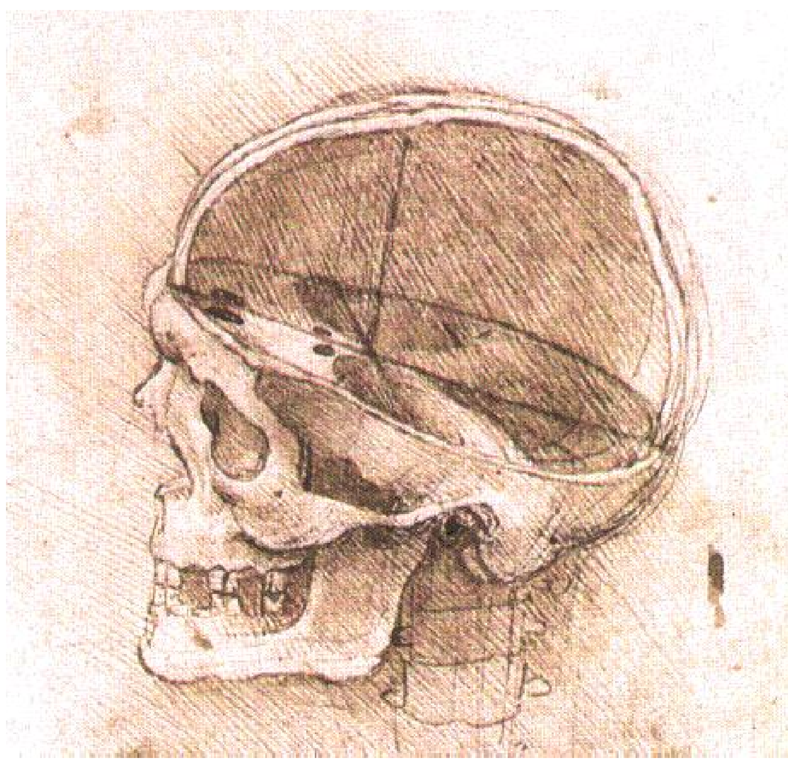

Abstract

The half life of indium-116 was found using a Geiger counter. The half life was found to be 3300 ± 50s. The dead time of the Geiger counter was then found using 2 carbon-14 sources. It was found that the dead time was $(1.07 \pm 0.06) \times 10^{-3}$s. It was found that the dead time contributes significantly to the inaccuracy in measuring high activity samples.

⋆ Introduction

In radioactive decay, it is impossible to determine when a single atom will decay. However, it is found that in a sample, the rate of decay is directly proportional to the number of radioactive particles, N.

$$\frac{dN}{dt} \propto -N$$

$$\Rightarrow \frac{dN}{dt} = -\lambda N$$

λ is known as the decay constant. The minus sign is because the sample is decreasing. The solution is,

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

$$\Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$\Rightarrow \ln(\frac{dN}{dt}) = -\lambda t + c \tag{2}$$

where N_0 is the initial number of particles. We can also define a quantity known as the half life, $t_{\frac{1}{2}}$, which is the time it takes for the sample to decay to half the original amount. The half life can be derived as follows,

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$\ln 2 = \lambda t_{\frac{1}{2}}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \tag{3}$$

The half life is a characteristic property of a radioactive isotope.

One way of measuring the rate of decay of a sample is with a Geiger Muller tube. It is essentially, a tube filled with an inert gas with a potential difference. Radiation will ionise the gas and a current will form which will be read as a count. One of the drawbacks with the tube is the so called dead time, τ. This is a period of time where after ionisation, the tube is unable to register another count. In very active samples, this is a severe problem.

To find the dead time, two sources of activity m_1 and m_2 are recorded seperately. Then the two sources combined are measured together to find the acitvity m. Let n_1, n_2 and n be the respective true count rates. The system will not measure for $m\tau$ of the time. Thus $nm\tau$ is lost due to the dead time. Hence, the true count is,

$$n = m + nm\tau$$

$$\Rightarrow n = \frac{m}{1 - m\tau}$$

Subbing this into $n = n_1 + n_2$,

$$\frac{m}{1 - m\tau} = \frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau}$$

Rearranging gives the quadratic,

$$\tau^2 - \frac{2}{m}\tau + \frac{m_1 + m_2 - m}{mm_1m_2} = 0$$

The solution is,

$$\tau = \frac{\frac{2}{m} \pm \sqrt{\frac{4}{m^2} - 4(\frac{m_1+m_2-m}{mm_1m_2})}}{2} \tag{4}$$

⋆ Experimental Method

The Geiger counter is plugged into the counter unit and the amplifier and bias supply unit. The background radiation was measured for 2 minutes so that it could taken away from all subsequent measurements.

An indium-116 source is placed in the Geiger counter and one minute counts are taken every five minutes for 2 hours. Graphs of the count rate versus time and ln count rate versus time were plotted to determine the half life.

Two carbon-14 sources are then placed in the Geiger counter with one covered and the count rate is found. The other source is then only covered and the procedure is repeated. The procedure is then repeated with both sources exposed.

⋆ Results and Analysis

The background radiation was found to be 99 counts for 2 minutes. This gives an error that is negligible so the background radiation is 0.825 counts/second. (The error in a measured count rate is the square root of the number)

Figure **??** shows the sample decaying exponentially as expected. An estimate of the half life can be made by observing that the initial count rate is 25 count/s and the count decays to 12.5 count/s in ≈3000s.

From figure **??**, equation (**??**) gives the value for the decay constant to be $(2.10 \pm 0.05) \times 10^{-4}s^{-1}$. This gives the half life of indium-116 to be 3300 ± 50s.

It was found that $m_1 = 620$ count/s, $m_2 = 534$ count/s and $m = 717$ count/s. As expected, due to the dead time, $m_1 + m_2 \neq m$. Substituting these into equation (**??**) gives, $\tau_1 = 1.71 \times 10^{-3}$s and $\tau_2 = 1.07 \times 10^{-3}$s.

τ_1 gives a negative value for the true count so it is not a meaningful solution. This means $\tau = 1.07 \times 10^{-3}$s. The error is quite complicated and was found from Mathematica. Thus, the dead time was found to be,

$$\tau = (1.07 \pm 0.06) \times 10^{-3}$$

Hence, the true values are $n_1 = 1840 \pm 150$ count/s, $n_2 = 1250 \pm 100$ count/s and $n = 3090 \pm 240$ count/s and $n_1 + n_2 = n$ within error.

⋆ Conclusion

The half life of indium-116 was found to be 3300 ± 50s which corresponds to the accepted value of 54.2 minutes.

The dead time of the Geiger counter was found to be $(1.07 \pm 0.06) \times 10^{-3}$. It was found that the measured activity of the carbon-14 was 720 ± 30 count/s when the actual count was 3090 ± 240 count/s. This is almost 4 times the amount showing how significant the inaccuracy is.

⋆ References

http://www.chemicalelements.com/elements/in.html

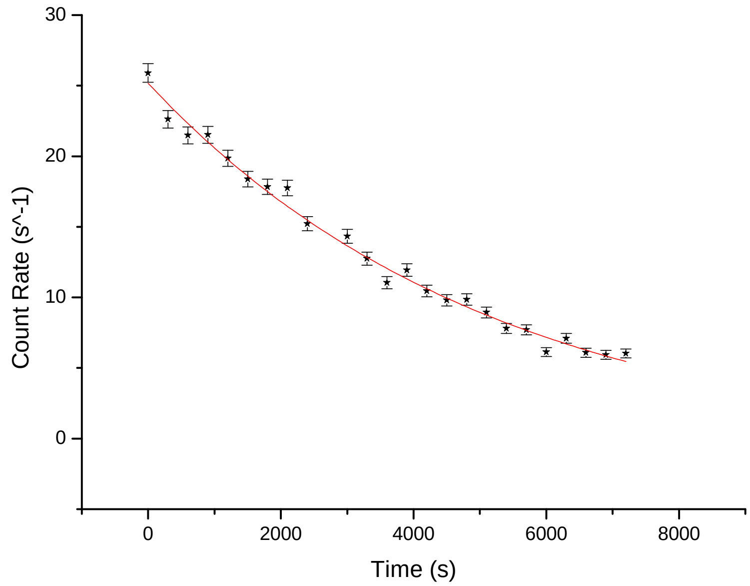

Figure 1: Graph of Count Rate versus Time

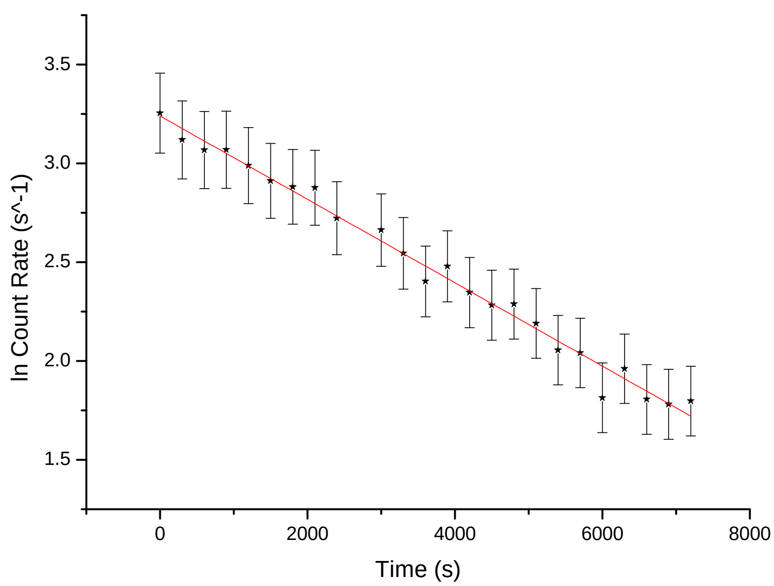

Figure 2: $y = -(2.10 \pm 0.05) \times 10^{-4}x + (3.24 \pm 0.02)$