Fresnel Biprism

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Abstract

The aim of the experiment is to determine the wavelength of sodium light using a fresnel biprism as an interferometer. The wavelength was found to be 628 ± 243nm which corresponds to the accepted value of 589.3nm within experimental error.

⋆ Introduction

When two coherent light sources meet, an interference pattern is formed. In Young's Double Slit Experiment, a light source is shone through a narrow slit undergoing diffraction and creating a point source. This is then shone through two narrow slits separated by a distance d such that two point sources are created. When the light is observed on a screen from a distance D away from the double slits, the resulting inteference pattern is a series of light and dark fringes. It can be shown from the geometry that the distance between the fringes, s, is given as $s = \frac{\lambda D}{d}$ where λ is the wavelength of the light souce.

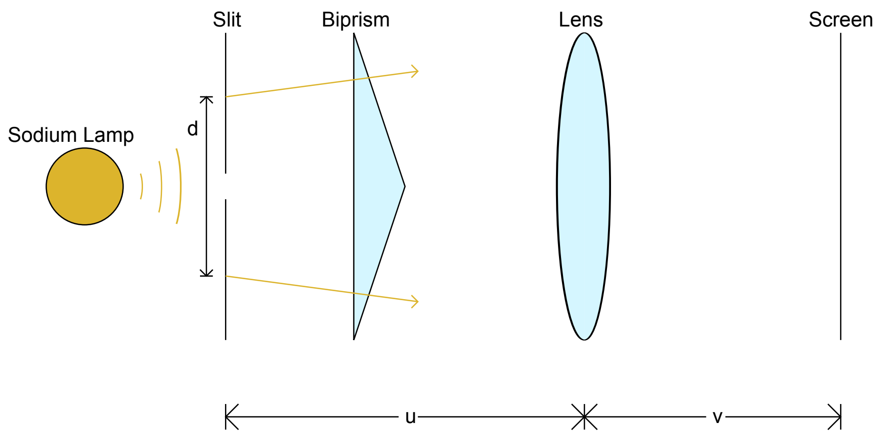

Figure 1: The Fresnel Biprism

The theory for the Fresnel Biprism is similar to double-slit experiment. A point source is shone through the fresnel biprism which consists of two prisms joint at their bases to form an isoceles triangle. The resulting refraction causes the light to appear as if it is coming from two point sources and a similar interference pattern is created. The two point sources are often referred to as the virtual sources. In a way, this makes it better than Young's experiment since when using the double slits, it is not possible to actually create ideal point sources.

The formula $s = \frac{\lambda D}{d}$ still holds. However, d cannot be directly measured as the sources are virtual. Instead, it is necessary to put a convex lens in front of the biprism to form real images of the virtual slits. There are two positions where the lens will form images and this can be found using,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

where u is the object distance, v is the image distance and f is the focal length. Since $u+v = D$,

$$\begin{aligned} \frac{1}{u} + \frac{1}{D-u} &= \frac{1}{f} \\ (D-u)(f) + uf &= (u)(D-u) \\ u^2 - Du + Df &= 0 \\ \Rightarrow u_{1,2} &= \frac{D \pm \sqrt{D^2 - 4Df}}{2} \end{aligned} \tag{1}$$

Solving for v gives,

$$v_{1,2} = \frac{D \mp \sqrt{D^2 - 4Df}}{2} \tag{2}$$

Let d_1 and d_2 be the image sizes. By definition of magnification, $\frac{d_1}{d} = -\frac{v_1}{u_1} = M_1$ and $\frac{d_2}{d} = -\frac{v_2}{u_2} = M_2$. However, note that $u_1 = v_2$ and $v_1 = u_2$ from the previous equations. So $\frac{v_1}{u_1}\frac{v_2}{u_2} = 1 = M_1 M_2$. This gives,

$$\begin{aligned} \frac{d_1}{d}\frac{d_2}{d} &= 1 \\ d^2 &= d_1 d_2 \\ d &= \sqrt{d_1 d_2} \end{aligned} \tag{3}$$

So the wavelength can be found by measuring d_1, d_2, D and s and using,

$$s = \frac{\lambda D}{\sqrt{d_1 d_2}} \tag{4}$$

⋆ Experimental Method

Set up the apparatus as shown in figure 1. The sodium lamp was turned on and allowed to warm up. The biprism is placed ≈ 20cm away from the slit. By eye, the biprism was rotated such that two closely spaced fringes are observed through the biprism. The eyepiece was calibrated in front of the biprism. The eyepiece is then moved to a distance D. Using the eyepiece, the distance between fringes, s, is measured by taking the distance, x, for n fringes and dividing by n. This was repeated several times for different bands of fringes to determine an average value of s. The convex lens is then placed in two positions such that the sharpest image of the virtual slits is formed on the eyepiece. d_1 and d_2 are then measured by finding the distance between the virtual slits on the eyepiece.

⋆ Results and Analysis

D	0.834 ± 0.001 m
d_1	0.007 ± 0.001 m
d_2	0.002 ± 0.001 m

s_1	2.5×10^{-4}m
s_2	1.25×10^{-4}m
s_3	1×10^{-4}m
s_4	0.83×10^{-4}m

d	3.74×10^{-3}m
s	1.4×10^{-4}m

Substituting into equation (4) gives $\lambda = 628$nm. To calculate the error,

$$\Delta(d_1 d_2) = d_1 d_2 \sqrt{(\frac{\Delta d_1}{d_1})^2 + (\frac{\Delta d_2}{d_2})^2} = 7.28 \times 10^{-6} \text{ m}^2$$

$$\Delta d = d\frac{\Delta(d_1 d_2)}{2 d_1 d_2} = 9.7 \times 10^{-4} \text{ m}$$

The error on s is found using the average error, $\Delta s = \frac{\sigma_s}{\sqrt{n}}$ where σ_s is the standard deviation.

$$\Delta s = \sqrt{\frac{\sum_{i=1}^{n}(s_i - \bar{s})^2}{n(n-1)}} = 0.4 \times 10^{-4} \text{ m}$$

$$\Rightarrow \Delta\lambda = \lambda\sqrt{(\frac{\Delta d}{d})^2 + (\frac{\Delta D}{D})^2 + (\frac{\Delta s}{s})^2} = 2.43 \times 10^{-7} \text{ m} = 243 \text{ nm}$$

⋆ Conclusion

The wavelength of sodium light was found to be 628 ± 243nm. This corresponds to the accepted value of 589.3nm within experimental error. The main sources of error were mostly human error. There was huge difficulty in accurately determining the distance between the fringes since the intensity diminished as the eyepiece was placed further away. The fringes could perhaps be measured more accurately if the slit at the light source could be adjusted more accurately and the power of the light source was increased. Placing the lens to find the sharpest point was also subject to human error. It was also noted that there was difficulty in getting the equipment to be exactly parallel to each other.

⋆ References

Eugene Hecht, *Optics*, Addison Wesley
Professor Jonathan Coleman, *2004 Physical Optics Part 4*, Trinity College Dublin