The Cornu Method

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Abstract

By using the Cornu method, Young's Modulus and Poisson's ratio for perspex could be found. The values were found to be 40 ± 10GPa and 0.397 ± 0.002 respectively. The experiment was a partial failure since Young's Modulus was not found to be within the accepted range of $2.7 - 3.5$GPa.

⋆ Introduction

Young's Modulus, Y, of a material is defined to be the ratio of the stress to the strain. The stress is the applied force per unit area and the strain is the ratio of the change in length to the original length. Essentially, it measures how much a material deforms when it is stretched.

Figure 1, shows masses, m, bending a perspex beam. At equillibrium, the moments balance each other and it can be shown that,

$$mgl = \frac{YAk^2}{R_1}$$

where g is the acceleration due to gravity, A is the cross-sectional area of the beam, k is the radius of gyration $(=\frac{b}{\sqrt{12}})$ and R_1 is the longitudinal radius of curvature of the beam. Rearranging, Young's Modulus can be expressed as,

$$Y = \frac{mglR_1}{Ak^2} \tag{1}$$

Poisson's Ratio, σ, is defined to be the ratio of the lateral strain to the longitudinal strain. When an object is stretched in the x-direction, it will compress in the y-direction and vice versa and Poisson's ratio measures this effect. Letting R_2 be the lateral radius of curvature, then

$$\sigma = \frac{R_1}{R_2} \tag{2}$$

R_1 and R_2 can be found using the Cornu method. Monochromatic light relected vertically from the lower surface of the glass plate and the top surface of the perspex beam will form fringes with constant distance, d, between each other. Due to the way the beam bends, it will be in the shape of a hyperbolic paraboloid. A hyperbolic paraboloid can be described by the equation,

$$z = \frac{x^2}{2R_1} - \frac{y^2}{2R_2} + c$$

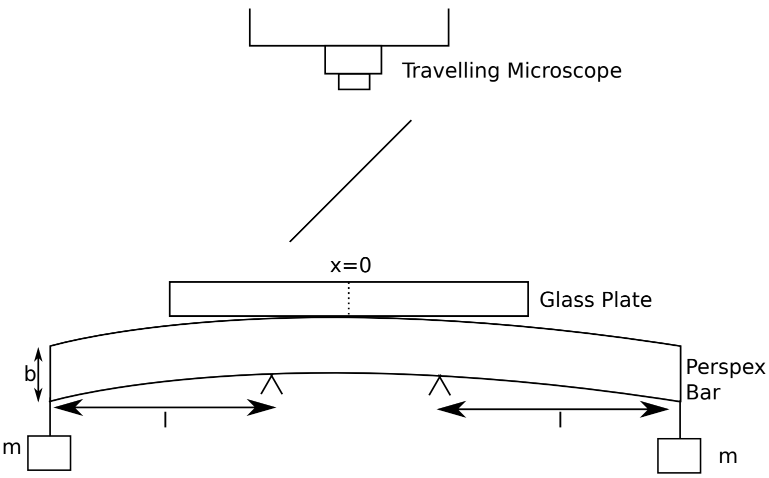

Figure 1: Diagram of Apparatus

Consider z as the distance between the beam and the glass plate. Let d_0 be the distance between the beam and plate at $(x, y) = (0, 0)$. Then,

$$\frac{x^2}{R_1} - \frac{y^2}{R_2} = 2(d - d_0) \tag{3}$$

The fringes will form at loci of points of constant d. From basic optics, constructive interference occurs for $d = \frac{N\lambda}{2}$. Thus, when $y = 0$, rearranging equation (3) gives

$$x^2 = R_1(N\lambda - 2d_0) \tag{4}$$

Similarly, along $x = 0$,

$$y^2 = -R_2(N\lambda - 2d_0) \tag{5}$$

Therefore, in order to find R_1 and R_2, we measure the distance between consecutive fringes along the x and y axis whilst taking the order.

Also, it can be shown that the fringes form two sets of hyperbolae with common asymptotes given by $x^2 R_2 = y^2 R_1$. Therefore,

$$\cot^2\theta = \frac{R_1}{R_2} = \sigma$$

⋆ Experimental Method

The apparatus is set up as shown in figure 1. The measurements for l, b, the width of the beam w, and thus A were found. A sodium lamp($\lambda = 589.3nm$) is turned on and shone horizontally on the 45 degree glass slide. The travelling microscope is centred at the hyperbola, ie when $(x, y) = (0, 0)$. x is measured as a function of N along the x-axis. Then y is measured as a function of N along the y-axis by rotating the bar 90 degrees.

By observing the asymptote, θ is estimated.

The glass plate is replaced with a convex lens and the pattern is observed.

⋆ Results and Analysis

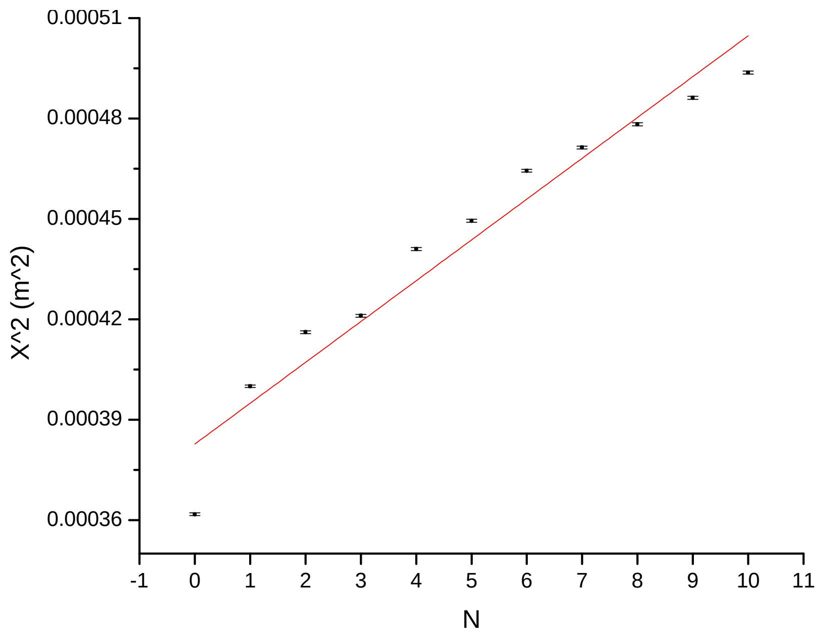

Figure 2: $y = ((1.220 \pm 0.004) \times 10^{-5})x + (3.827 \pm 0.002) \times 10^{-4}$

The slope of figure 2 is $(1.220 \pm 0.004) \times 10^{-5}$. Equation (4) gives $R_1 = 20.70 \pm 0.07$m. The slope of figure 4 is $(3.073 \pm 0.007) \times 10^{-5}$. This gives $R_2 = 52.1 \pm 0.1$m.

l was found to be 0.141 ± 0.001m, b was found to be 0.006 ± 0.001m, w was found to be 0.040 ± 0.001m. Thus, $A = (2.4 \pm 0.4) \times 10^{-4} m^2$. Using equation (1), $Y = 40 \pm 10$GPa where,

$$\Delta Y = Y \sqrt{(\frac{\Delta l}{l})^2 + (\frac{\Delta R_1}{R_1})^2 + \ldots}$$

Equation (2) gives $\sigma = 0.397 \pm 0.002$. θ was estimated to be $60\,\deg$. This gives $\sigma = 0.333\ldots$ which is reasonably close to the figure we produced.

When the glass plate was replaced with a convex lens, Newton's rings were observed. This is formed from the inteference of light reflecting from the bottom of the lens and the surface of the beam. Regions of equal distance from the centre will form the same interference pattern. Thus, the fringes appear as rings.

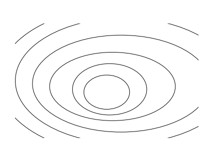

Figure 3: Newton's Rings

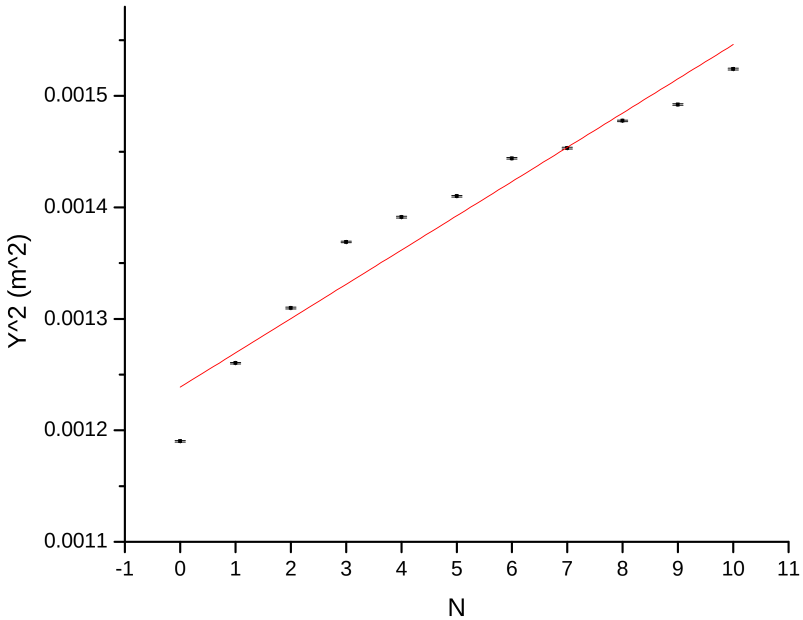

Figure 4: $y = ((3.073 \pm 0.007) \times 10^{-5})x + (1.240 \pm 0.004) \times 10^{-3}$

⋆ Conclusion

Young's Modulus for the perspex beam was found to be $Y = 40 \pm 10$GPa and Poisson's Ratio was found to be $\sigma = 0.397 \pm 0.002$. Accepted values for Young's Modulus is $2.7 - 3.5$GPa and Poisson's Ratio is 0.38. While our value for Poisson's Ratio was extremly close, Young's Modulus is out by an order of magnitude.

Sources of error include the potential error in reading the travelling microscope and the subjective nature of where each fringe is. However, it seems too much of a coincidence that we got the correct value for Poisson's Ratio and Young's Modulus is exactly out by a factor of 10. Despite our efforts to rigorously double-check the analysis, the source of the factor of 10 could not be found. We can only blame some form of human error as we were perhaps not in the correct frame of mind whilst performing this experiment due to the night before.