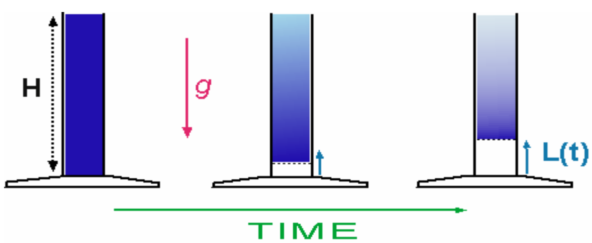
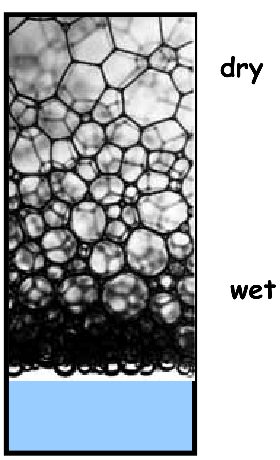
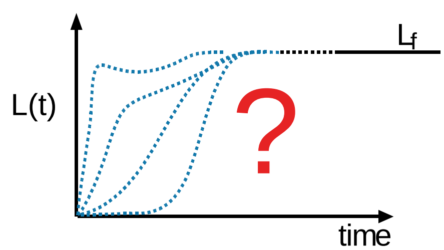
# FOAM DRAINAGE – part 1

Drainage ?




*what sets the drainage speed ?*

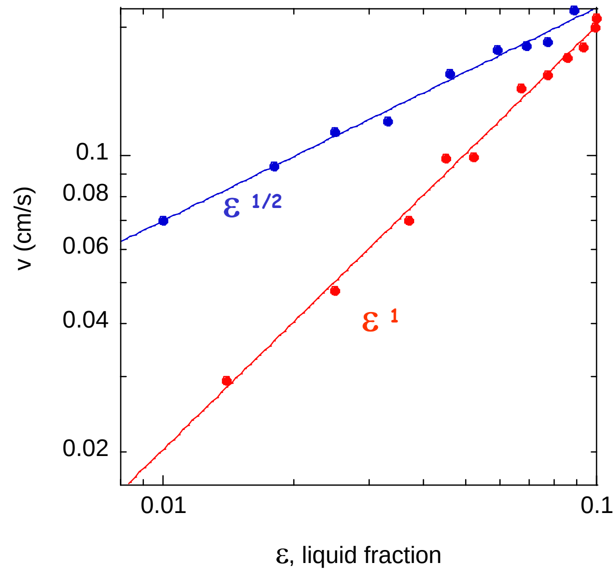
*important parameters ?*

same bubble size…

same
bulk viscosity…

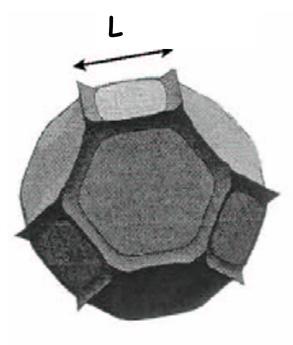
BUT

2 different
surfactants

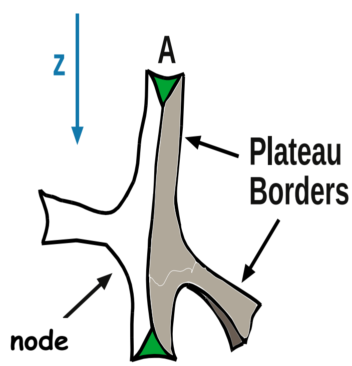



→ $V = a\, \varepsilon^b$     what sets a and b ?

## Geometry and liquid content (1)

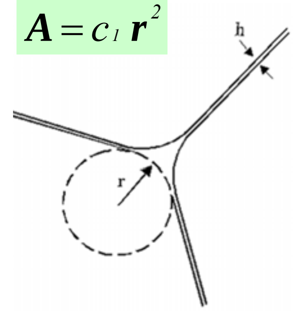



$D = 2.7\, L$




A : Area of a Plateau border

$A = c_1\, r^2$




$c_1 = \sqrt{3} - \pi/2 = 0.161$

Volume of a Plateau border   $V_{PB} \sim r^2\, L$

Volume of a node   $V_{node} \sim r^3$   $V_{node} = c_n\, r^3$   With $c_n = 0.3$

Volume of a film   $V_{film} \sim h L^2$

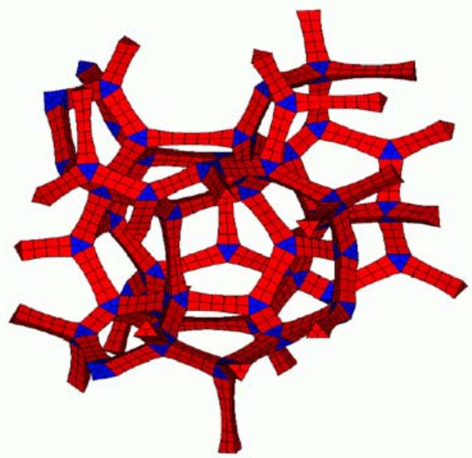
## Geometry and liquid content (2)

**Putting all together, and normalizing by the bubble gas volume ~ $L^3$ :**

$$\varepsilon = 0.17\,(r/L)^2 + 0.2\,(r/L)^3 + 1.2\,h/L$$

$c_2 = 0.17$ $c_3 = 0.2$

**Usual assumptions:** $h << r << L$



$$\varepsilon = 0.17\,(r/L)^2$$

$$r \approx 0.9\,D\sqrt{\varepsilon}$$

**Drainage : like a porous material problem, but with a variable pore size, r(ε)…**

## Liquid velocity : scaling concepts

**Porous media :** $\boldsymbol{G} = -\nabla p + \rho \boldsymbol{g} = \mu \boldsymbol{v} / k(\varepsilon)$

**in the steady state, constant liquid fraction :** $\nabla p = 0$

**Power balance:**

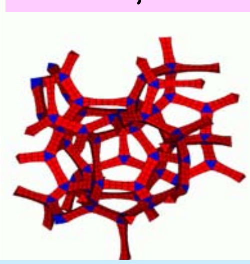
$$\rho g\,(r^2 L) v \sim \frac{\mu v^2}{r^2}(r^2 L)$$

**dissipation in the PBs**

$$v = K_c \frac{\rho g\, r^2}{\mu} = K_c \frac{\rho g\, L^2}{\mu}\varepsilon$$

**Permeability :** $K_c$

**PB resistance :** $R_c = 1/(3c_2 K_c) \approx 2/K_c$



$$\rho g\,(r^2 L) v \sim \frac{\mu v^2}{r^2}(r^3)$$

**dissipation in the nodes**

$$v = K_n \frac{\rho g\, r L}{\mu} = K_n \frac{\rho g\, L^2}{\mu}\sqrt{\varepsilon}$$

**Permeability :** $K_n$

**Node resistance :** $R_n = 2c_1/(3c_2^{1/2} c_n K_n) \approx 0.9/K_n$

*(Koehler et al, Langmuir, 2000)*

## Experimental evidences, but further needs…

Experimental evidence of the different regimes :

$$v = K_c \frac{\rho g\, r^2}{\mu} = K_c \frac{\rho g\, L^2}{\mu}\, \varepsilon$$

$$v = K_n \frac{\rho g\, r L}{\mu} = K_n \frac{\rho g\, L^2}{\mu} \sqrt{\varepsilon}$$

- **which experimental conditions set the drainage regime ?**

  It depends on the bubble size

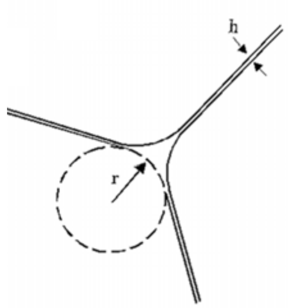
  It depends on the nature of the surfactant

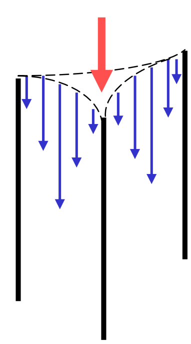
- **which values for the Kc (or Rc) and Kn (or Rn) ?**

  Within one regime, they are actually not constant values !

## Coupling bulk and surface flow…

First, considering the flow inside a Plateau border (PB) :



zero speed at the film junction + surface flow

→ SURFACE *SHEAR* FLOW

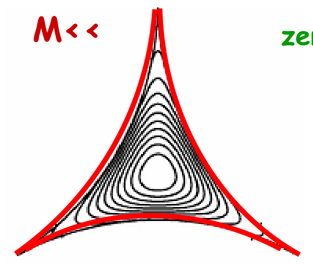
M : interfacial « mobility » parameter *(Leonard and Lemlich, 1965)*

$$M = \frac{\mu\, r}{\mu_s} = 0.9 \frac{\mu\, D\sqrt{\varepsilon}}{\mu_s}$$

with $\mu_s$ = surface shear viscosity

## Plateau border resistance and interfacial mobility

**M<<**



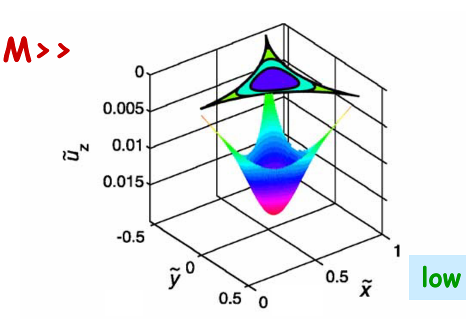
zero speed on the walls

high gradients

high resistance $R_c$

$$M = \frac{\mu\, r}{\mu_s} = 0.9\, \frac{\mu\, D\sqrt{\varepsilon}}{\mu_s}$$

**M>>**




only v=0 in the corners

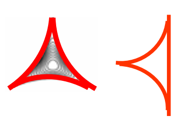
much less gradients

low resistance $R_c$

*(Koehler et al, JCIS, 2005)*

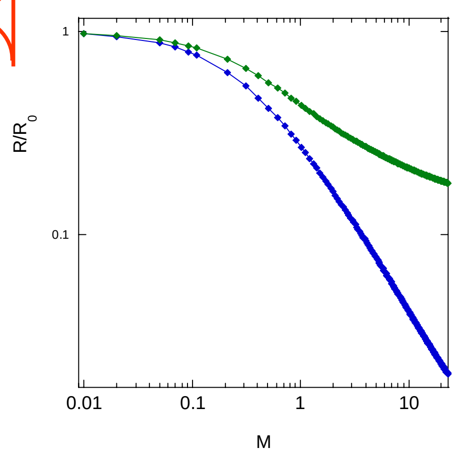
## Control parameter and interfacial mobility (3)

In red : v = 0



PB hydro.resistance, Rc, is thus a function of M :

M = 0 ⟶ $R_c^{\,0} = 300$

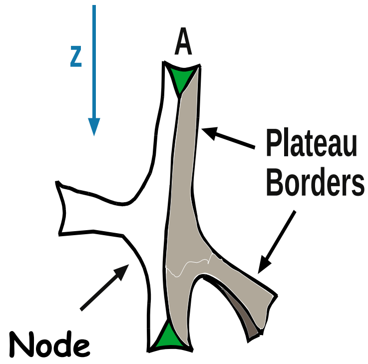



An explanation for the non-constant value of Rc (or Kc)

*numerical formula in : N'Guyen (JCIS, 2002)*

*Analytical formula in : Koehler , Hilgenfeldt, Stone (JCIS, 2005)*

… a simplified form valid for M ≤ 10 : $R_c = R_c^{\,0} / (1 + 2.4\, M)$




## interfacial mobility and node resistance

node hydrodynamic resistance, Rn,
must thus also be a function of M :

$$R_n = R_n^0 \; f(M) \; ?$$

(Cox et al, J.Phys. Cond. Matter, 2001)

### M <<1

a node in a Poiseuille flow, with rigid walls ?

- Node = correction to the PB length
- $R_n$ / $R_c$ ~ r / L
- As ε, or r, or M decreases : node resistance decreases and becomes smaller than $R_c$

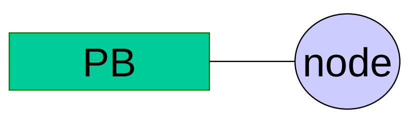
### M >> 1

a node with fluid interfaces ?

- $R_n$ must become large
- Simulations : $R_c$ ~ $R_n$

## All together…

A simple additive model of hydrodynamic resistances mounted in series :




$$v = \frac{\rho g L^2}{\mu} \varepsilon \left( \frac{1}{1/K_c(M) + \sqrt{\varepsilon}/K_n(M)} \right)$$

$$= \frac{\rho g L^2}{\mu} \varepsilon \frac{1}{(1/2)R_c(M) + (1/0.9)\sqrt{\varepsilon} R_n(M)})$$

One recovers the two extreme limits

In the intermediate range, this can also be written as : $v \sim \varepsilon^{\beta}$ , with $1/2 < \beta < 1$

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Experimentally :

Bubble diameter : from 150 μm to 8 mm

Bulk viscosity

Interfacial properties :

casein SDS/DOH SDS

→ factor 800 on the drainage speed

More than 3 orders of magnitude in M

## Low M, main dissipation in the Plateau borders :

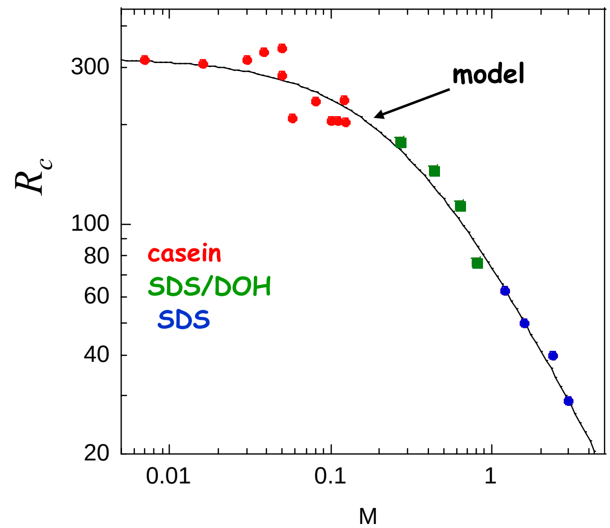
Expt. : ⟶ $v = \frac{1}{R_c} \frac{\rho g d^2}{\mu} \varepsilon^1$

Model :

$$R_c = R_c^0 / (1 + 2.4\ M)$$

$$R_c^0 = 300 \qquad M = 0.9 \frac{\mu d \sqrt{\varepsilon}}{\mu_s}$$

Comparing data to the model :
a way to estimate $\mu_s$



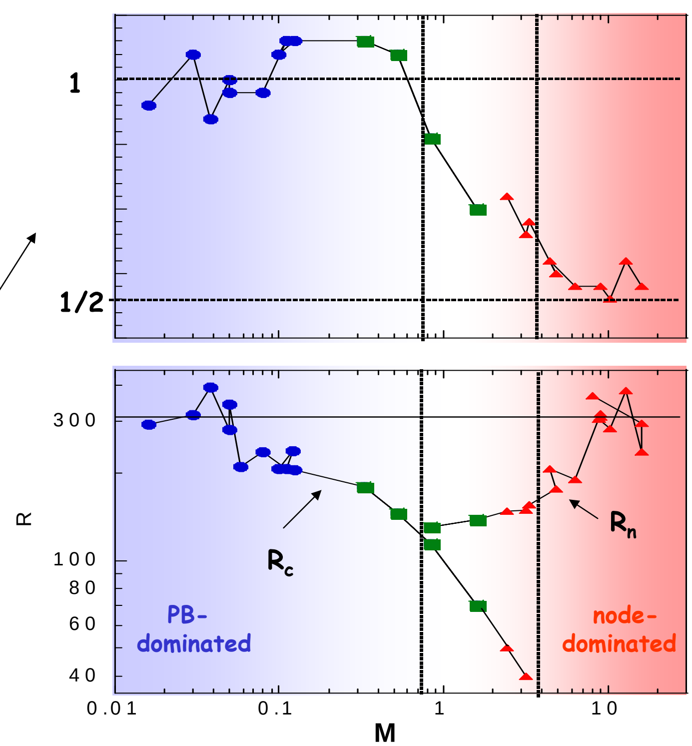

$\mu_s$ = $10^{-2}$ g/s — casein
$\mu_s$ = 1.8 $10^{-3}$ g/s — SDS / DOH
$\mu_s$ = 7 $10^{-5}$ g/s — SDS
⟶ $M$

very good values of $\mu_s$. in agreement with other studies. validates the assumptions.

## All the available data : a final simple picture...

casein
SDS/DOH
SDS

$$V = a\ \varepsilon^b$$




once plotted vs M, a simple crossover between 2 extreme regimes

in the crossover (M ≈ 1) : similar resistances

Only M is needed as a regime control parameter

...valid on more than 3 orders of magnitude of M !

## No significant transport in the thin films

simulations : films can be important only if : $h/L \geq 10^{-2}$ and $M >> 1$

*Koehler , Hilgenfeldt, Stone (JCIS, 2005)*

Experimentally :

**Large bubbles (D> a few mm)**

Large r : low $P_c$ : low $\Pi_d$ ( ~ 10 Pa)

High flow speeds

High M

→ Consistent with directly observed thickness : h > a few hundred of nm

→ But, still $V_{film} << V_{PB}$

**small bubbles (D < 1mm)**

Low r : high $P_c$ : high $\Pi_d$ ( > 500 Pa)

low flow speeds

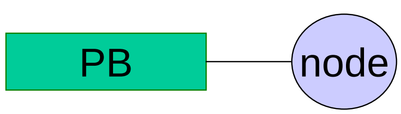
low M

Consistent with coarsening rate measurements : h = tens of nm

→ And, still $V_{film} << V_{PB}$

## What to remember ?

drainage speed results from a balance between viscous dissipation in Plateau borders and in nodes: each depending on the coupling between the bulk and the surface flows




$$M = \frac{\mu r}{\mu_s}$$

This coupling is described by M, which include the surface SHEAR viscosity.
M is also the control parameter which provide the drainage regime.

In the most simplified version :

$$V = (1/R)(\rho g/\mu)L^2 \varepsilon^b$$

Taking : <R> = 250 <b> between ½ and 1

→ Very reasonable value of v, and of T_drainage : H/V

*Next time : gas effect in drainage ??*