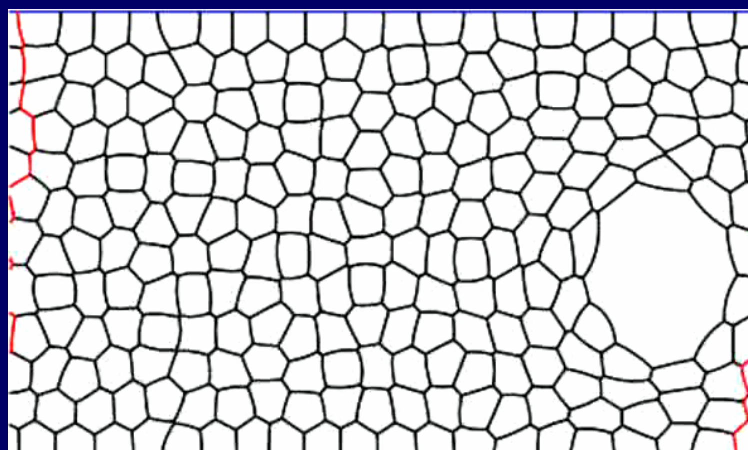
Prifysgol Cymru
Aberystwyth
1872 The University of Wales

Simon Cox

Simulations of foam rheology



Principles and Goals

- Use knowledge of foam structure …

… to predict response when experiments are not easy

… to isolate causes of certain effects

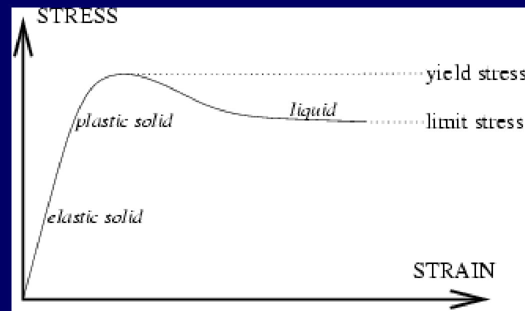
…to save time in designing new experiments

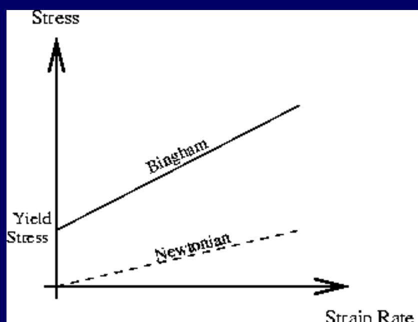
… to use parameters that experiment cannot reach

…

Foam is a Complex Fluid

- *Elastic solids at low strain*
- *Behave as plastic solids as strain increases*
- *Liquid-like at very high strain*



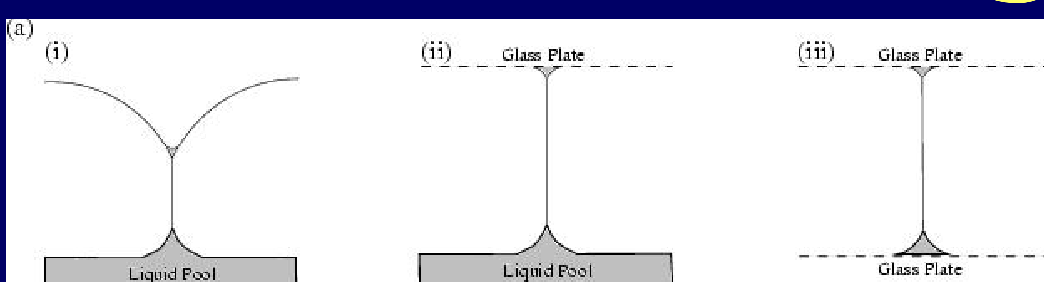


Shear thinning fluid with a yield stress suggests a Bingham or Herschel Bulkley model.

$$\tau = \tau_0 + K\dot{\gamma}^n$$

"Two-dimensional" Experiments

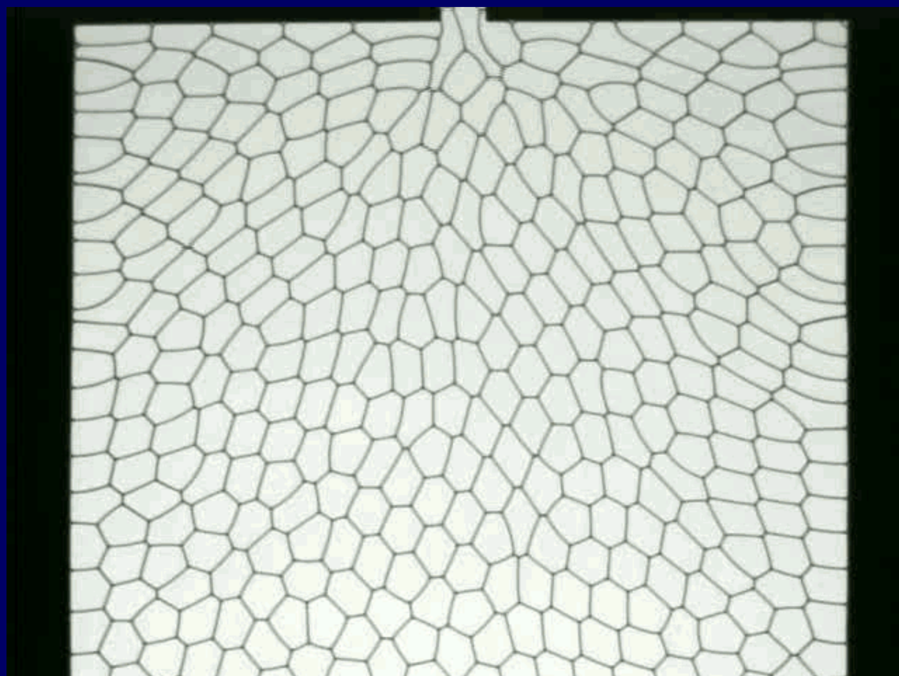
Easily observable



Lawrence Bragg Cyril Stanley Smith

Plateau & Laplace-Young → in equilibrium, each film is a circular arc; they meet three-fold at 120°.

2D contraction flow

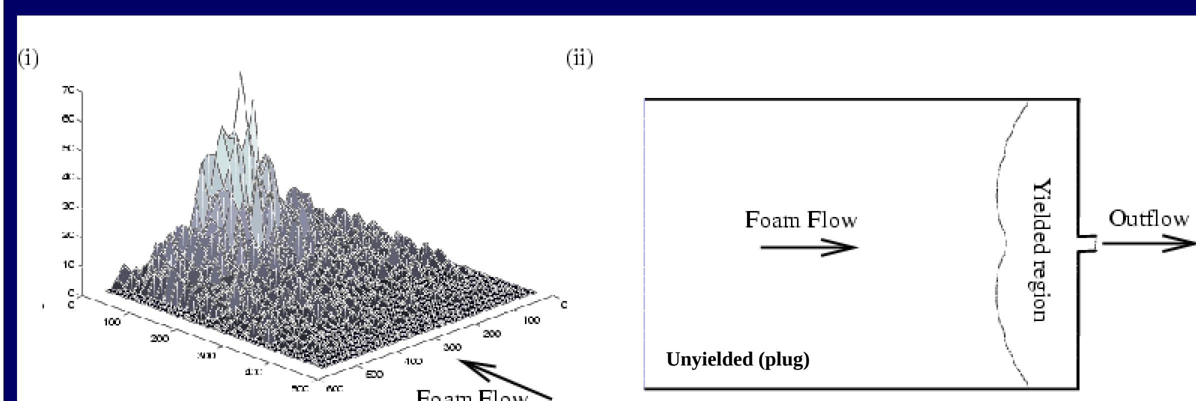


J.A. Glazier (Indiana)

Continuum approach

Experimental statistics for density of T1s

CFD (Fluent) calculation of Bingham fluid with slip boundary conditions



Streamlines, pressure drop, …

Bubble-scale modelling

How do parameters such as liquid viscosity, bubble size and area dispersity affect yield stress, consistency (plastic viscosity)?

Kraynik Ann. Rev. Fl. Mech. (1988)

Unlike many yield-stress fluids, foams have a **well-defined local structure**.

Exploit this **bubble-scale structure** (Plateau's Laws) to predict and model the rheological response of foams, and develop better continuum models / constitutive equations.

Explore both 2D and 3D situations.

Quasi-static simulations

Time-scale of equilibration & T_1s is faster than shear-rate and gas diffusion.

Foam passes through a sequence of equilibrium configurations.

Idea is to satisfy Laplace-Young Law for each film at each step:

$$\gamma C = \Delta p$$

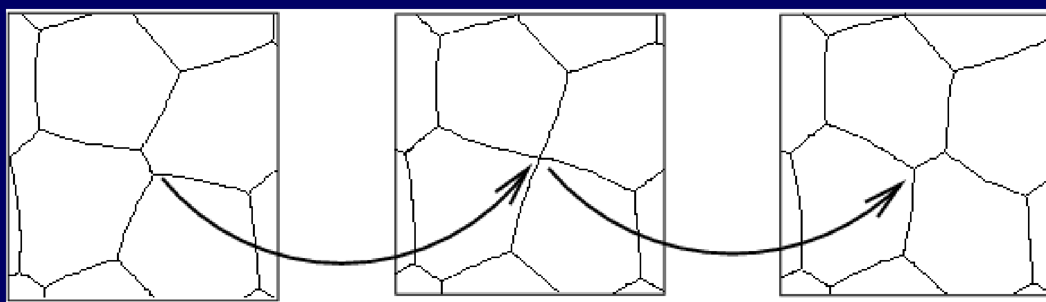
In practice, minimize total perimeter (based upon vertex positions and edge curvatures) subject to constraints:

$$E = \Sigma \gamma L_i + p_i(A_i - A_{i0}).$$

Make small increment in strain and re-converge to equilibrium.

Dissipation

Energy dissipated through neighbour-switching topological (T_1) changes:



Dissipation even in the zero shear-rate limit

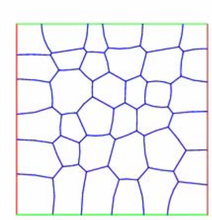
Software

Edges represented as circular arcs:

• Quasi-static, dry:

2D Froth, periodic, extensional shear (Kermode & Weaire, Aref & Herdtle)

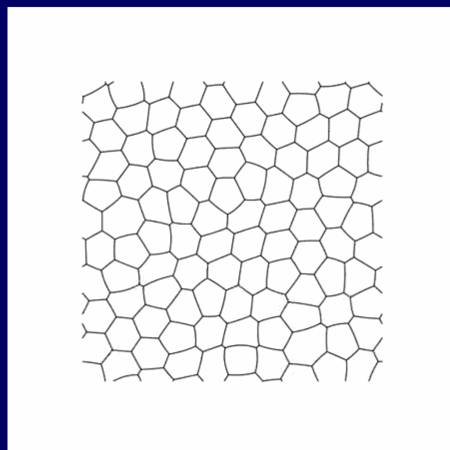
Surface Evolver, very flexible (Brakke)



Use cut-off length for T1s

Simple shear

Measure stresses, T1 field,
bubble velocities, …



Dry, periodic, quasi-static Surface Evolver

More software

Edges discretized, “almost” quasi-static:

• Potts model (Glazier, Graner)

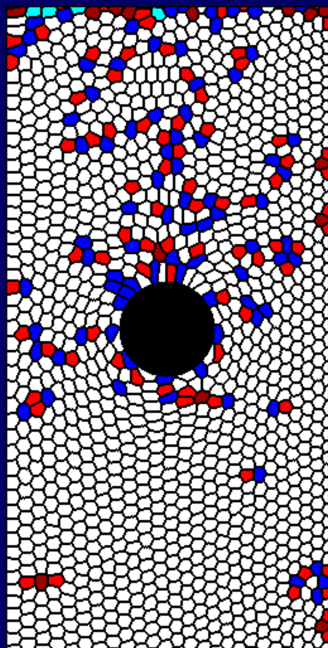
dry, many bubbles

• Lattice-Gas (Hutzler, Sun)

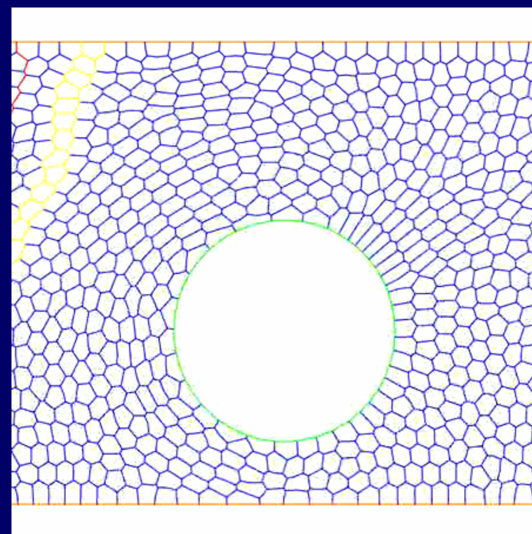
wet/dry

Use pixels to define T1 cut-off length

Stokes experiment in 2D

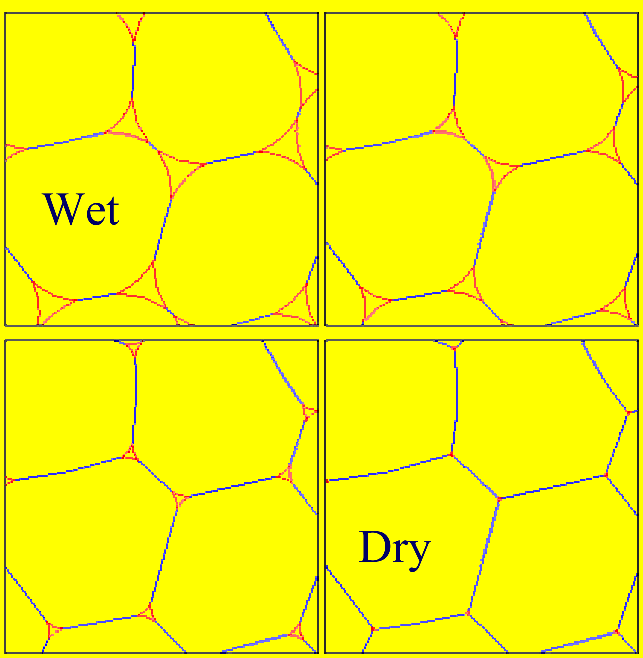


Potts Model (Raufaste)



Surface Evolver

Effect of liquid fraction on T_1s



T_1 initiated when Plateau borders touch.

Higher liquid fraction implies greater *critical distance* between vertices at which T_1s may occur.

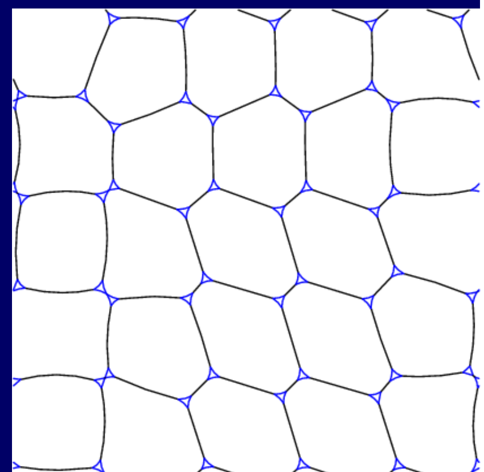
So ostensibly "dry" programs can be used to model wet foam

Wet quasi-static 2D foams

Edges represented as circular arcs;

Wet in the sense of having explicit PBs and being able to cope with four-sided PBs:

- PLAT, periodic , extensional shear (Bolton, Hutzler, Weaire)
- Surface Evolver (but unstable)



Quasi-static 3D foams

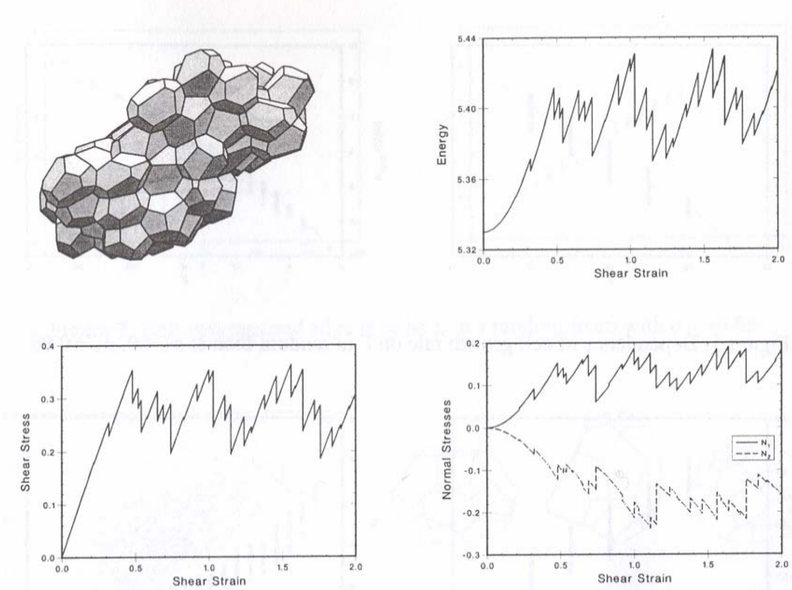


Figure 4: Evolution of the energy density, shear stress, and normal stresses during quasistatic simple shearing flow of a random monodisperse foam with N=64.

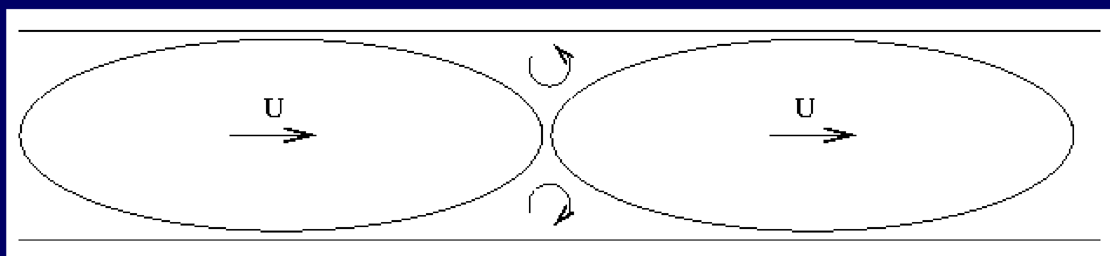
Kraynik
Reinelt

Surface
Evolver
(Potts)

Dry/Wet

Strain-rate effects (in 2D)

Move beyond quasi-statics by including e.g. external dissipation: drag of Plateau borders on bounding surfaces

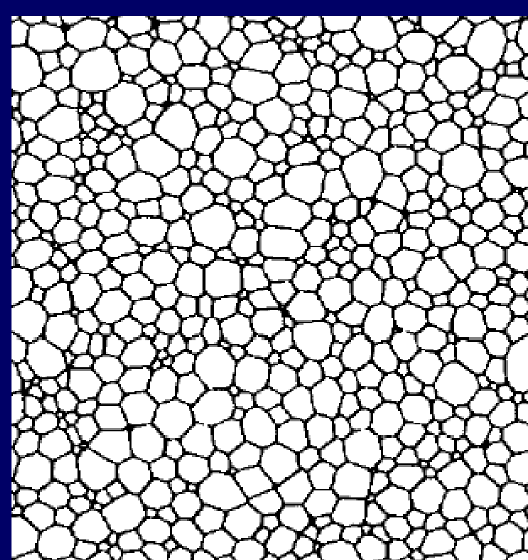


Including viscosity in dry 2D foams

Vertex model

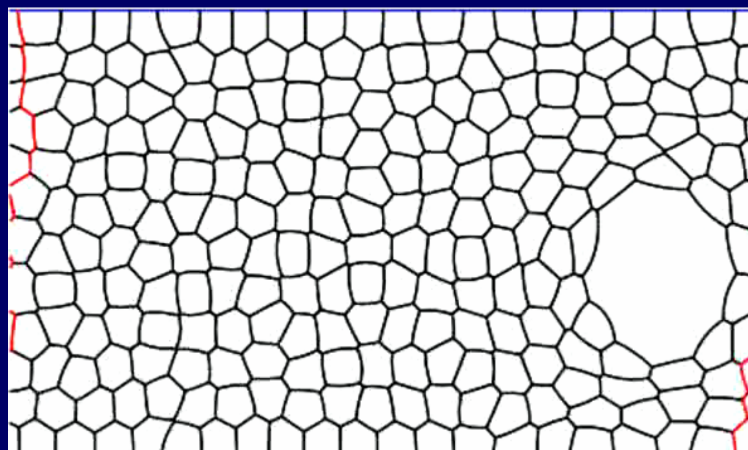
Kawasaki *et al.*

Cantat & Delannay.



- *straight films*
- *dissipation at vertices*
- *fast numerics*
- *solve force balance to give vertex velocities*

Large Bubbles



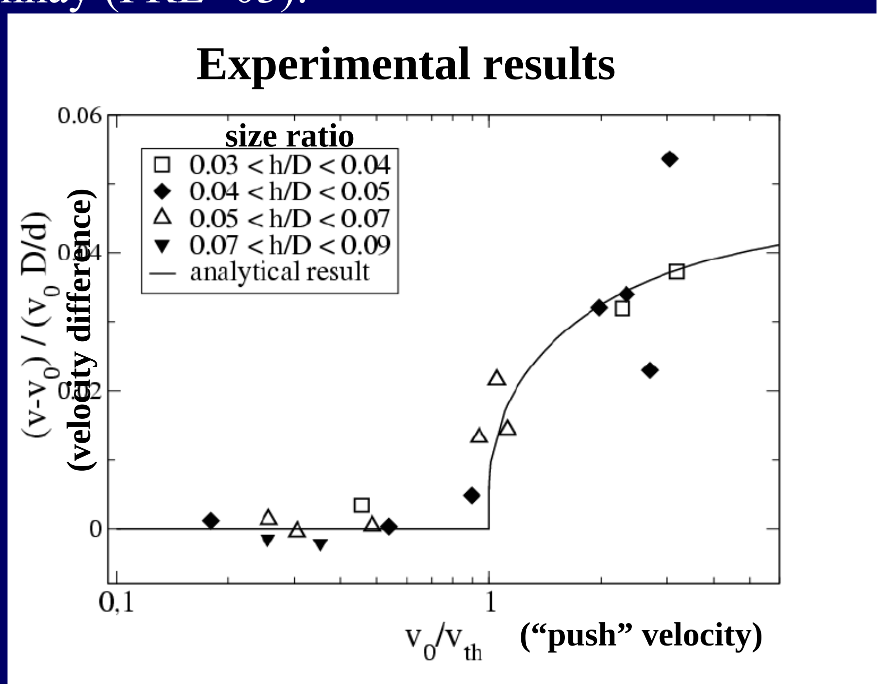
Area ratio =15; push foam with velocity v=2.6

Actually Surface Evolver with viscosity (see later), but effect the same.

Large Bubbles

Cantat & Delannay (PRE '03):

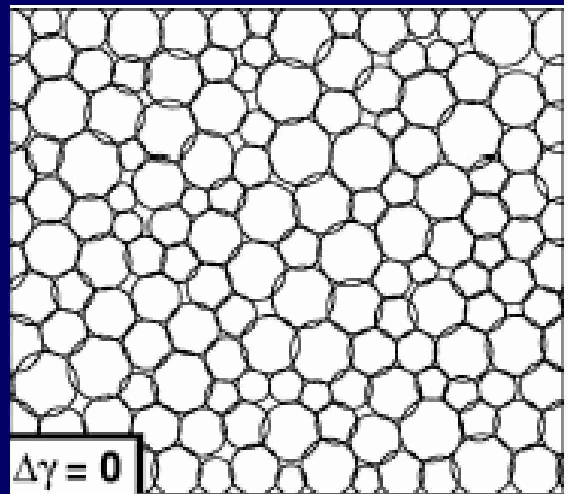
Also agreement with simulations using Vertex Model



Including viscosity in wet 2D foams

Durian's <u>*bubble model*</u>

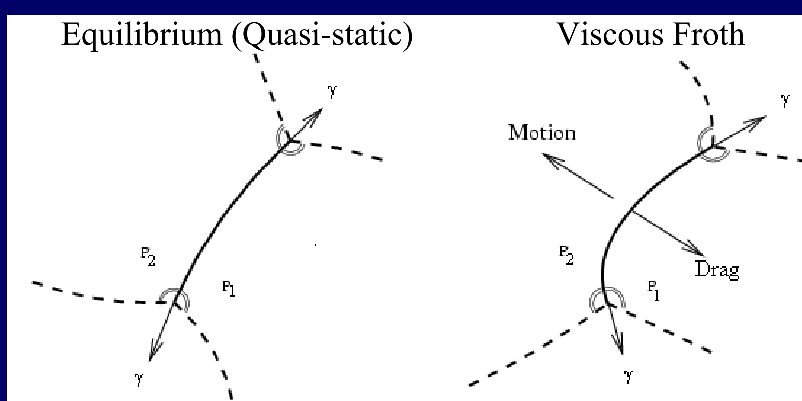
- *Circular bubbles with spring-like repulsion and viscous drag*
- *Appropriate to the wet-limit*
- *Also in 3D (Gardiner)*



"Viscous" Froth Model

Kern *et al.* PRE '04

Add external dissipative forces and retain *both* pressures & curvatures to give a realistic dry structure.



Keep 120º angles

Viscous Froth Model

Laplace Law modified to:

$$\lambda \, \mathrm{v}^{\alpha} = \gamma \, C - \Delta p$$

with drag λ, normal velocity v,
curvature C and surface tension γ.

Bretherton: α=2/3; expts: α=1 OK.

Limiting cases include

- Ideal soap froth
- Grain growth

For more justification of use of normal velocity and α=1, see Cantat, Kern & Delannay (2004)

Model Time-scales

Relaxation: $T_\lambda = \dfrac{\lambda \mathrm{R}^2}{\gamma}$ (e.g. after T_1)

Coarsening: $T_\kappa = \dfrac{\mathrm{R}^2}{\kappa\gamma}$ (gas diffusion)

Shear: $T_\zeta = \left(\dot{\zeta}\right)^{-1}$

length-scale R
permeability κ
strain ζ

Numerical Algorithm

Discretize network of films and calculate point-wise curvature.

Choose α=1 and integrate equation of motion around the boundary of bubble *i* using Gauss' theorem:

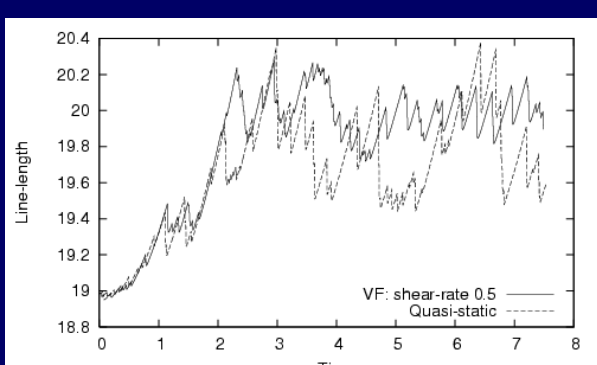
$$\frac{dA_i}{dt} = \frac{\pi}{3}(n_i - 6) + \sum_j (P_i - P_j) l_{ij}$$

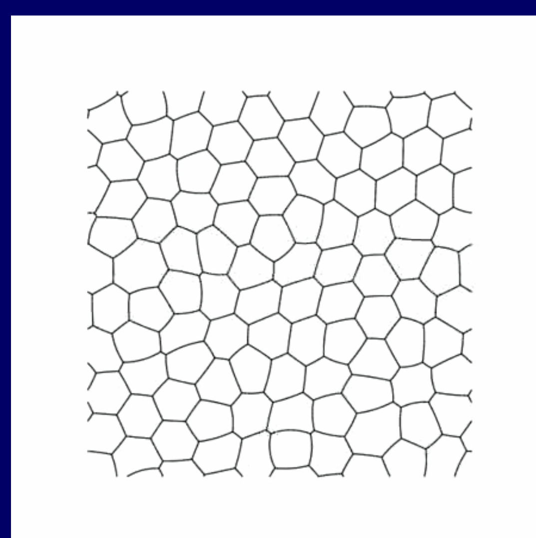
Gives matrix equation for the pressures at each step.

Use (adapted) Surface Evolver, especially for finite foams in constrained geometries, and/or home-grown C-code for periodic foams.

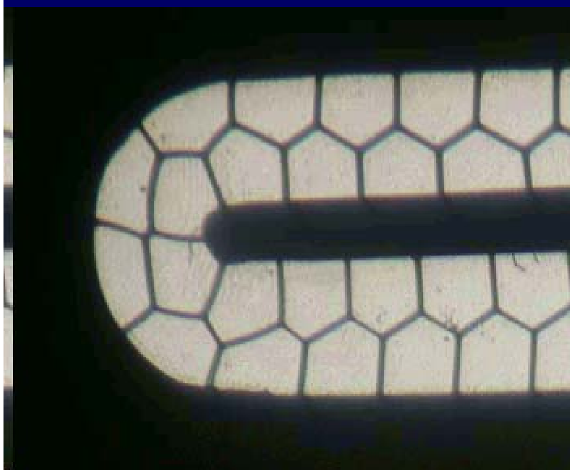
Simultaneous coarsening (gas-exchange) straightforward.

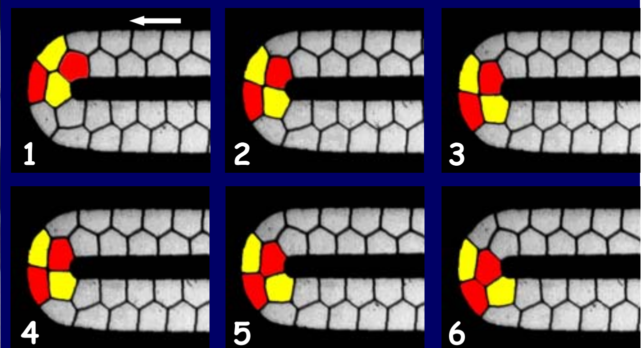
Simple shear of a periodic foam



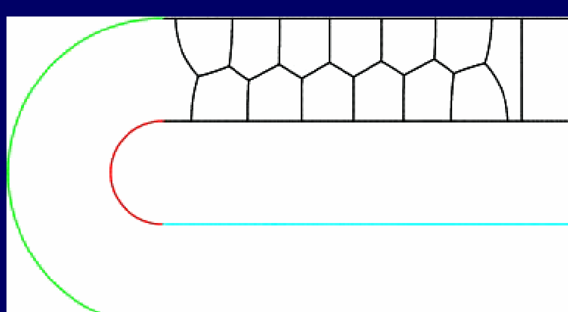


T_1 generator – the first test

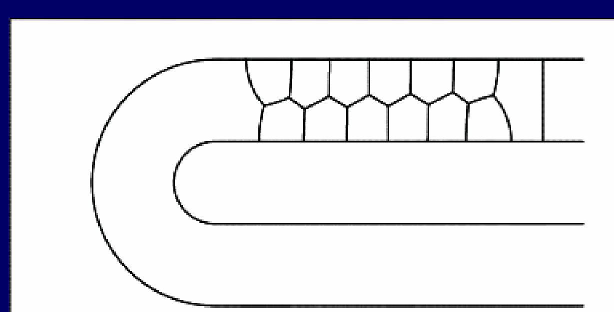




Velocity is important

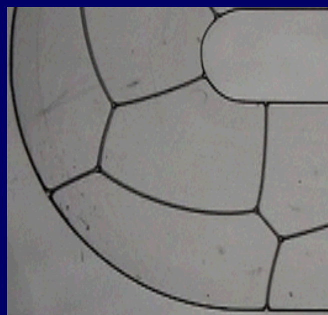


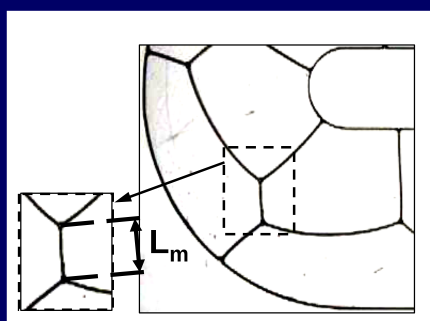
Quasi-static
Surface Evolver

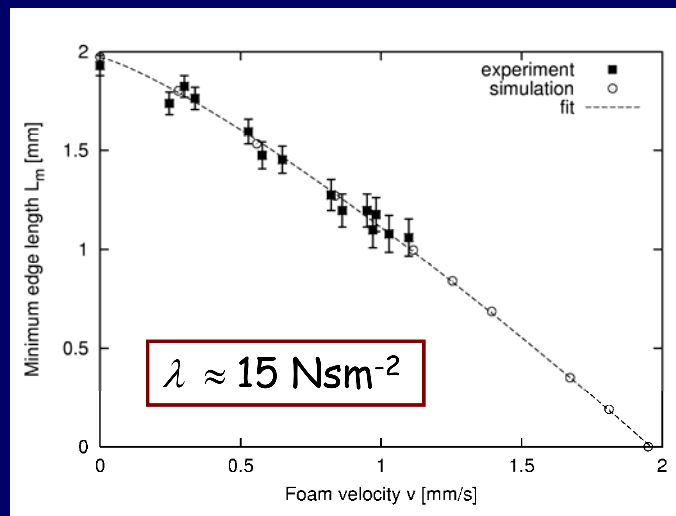


Viscous Froth
Surface Evolver

Quantitative comparison

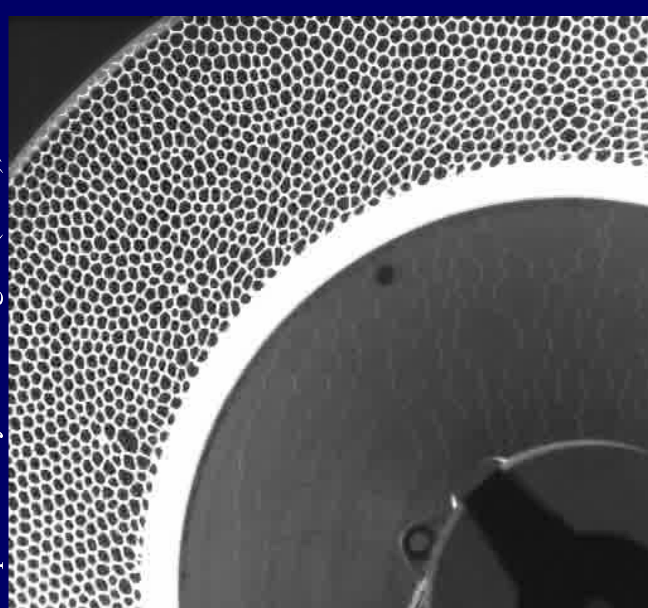






Couette Shear (Experiment)

Experiment by G. Debregeas (Paris), PRL '01

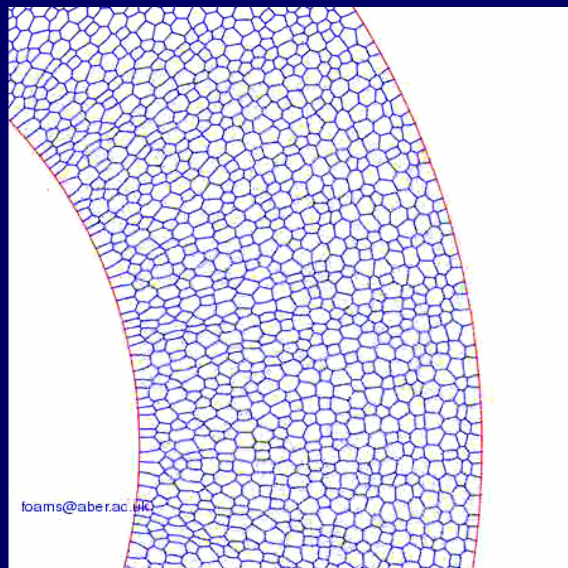


Much faster than real-time.

Shear banding? Localization?

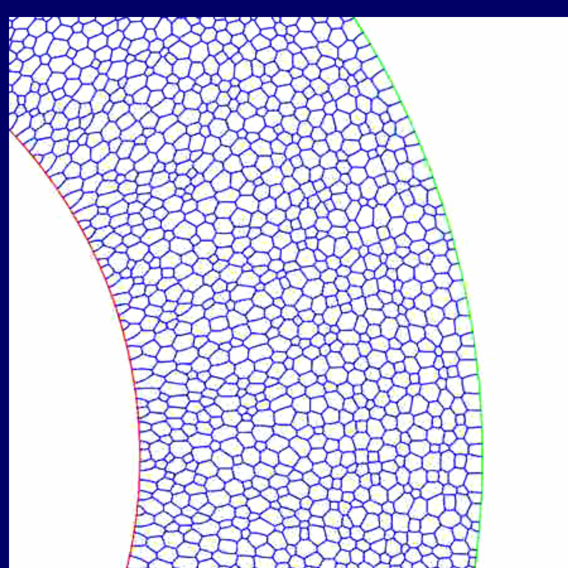
cf Lauridsen *et al.* PRL '02

Couette shear (simulations)



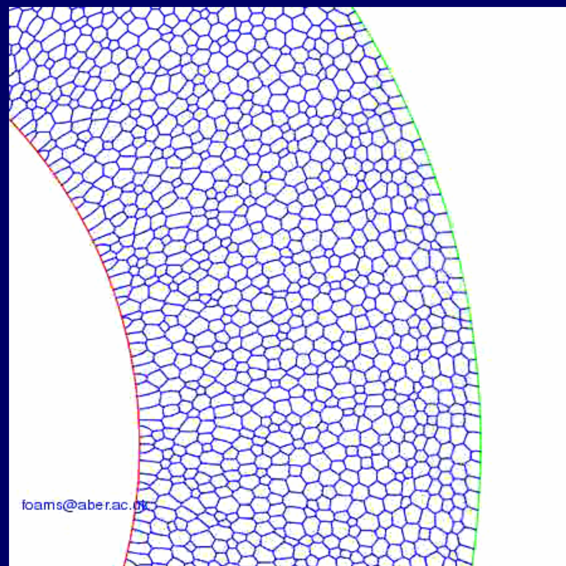
Quasi-static

Couette shear (simulations)



Viscous Froth (low strain-rate)

Couette shear (simulations)

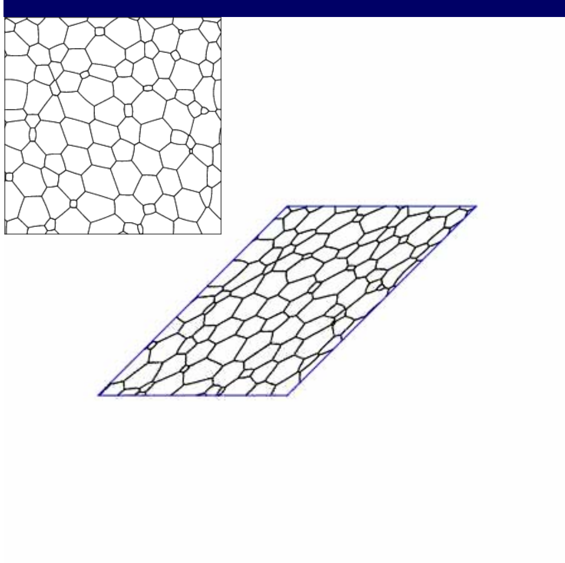


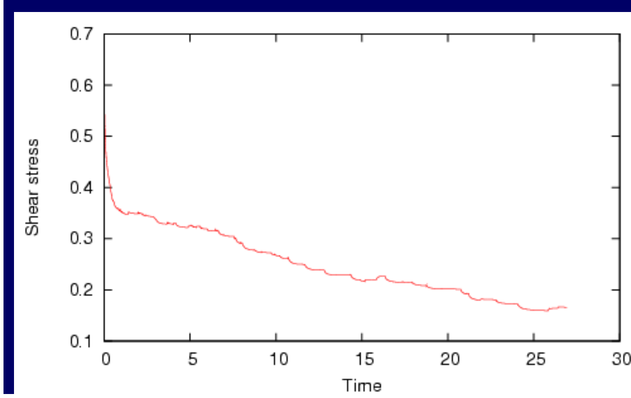
Viscous Froth (high strain-rate)

Stress relaxation, with coarsening

Von Neumann's Law - rate of change of area due to gas diffusion depends only upon number of sides:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = k(n-6)$$





Initial Conditions

- Often inconvenient to construct an initial data file by hand.
- Most popular algorithm is a Voronoi construction:

  - scatter a number of points in space;

  - each point S_i generates a Voronoi cell: those points that are closer to S_i than any other S_j.

- Use software such as Sullivan's VCS (http://torus.math.uiuc.edu/jms/software/).
- Then relax to equilibrium in Potts Model, Surface Evolver, etc.

Other possibilities - lattice? Annealing?

Summary

A range of simulations allow us to probe the dynamics of 2D and 3D foams, while retaining all structural information.

Includes variations in bubble volume, liquid content and viscous drag.

Challenges: wet or dissipative dry 3D foams.