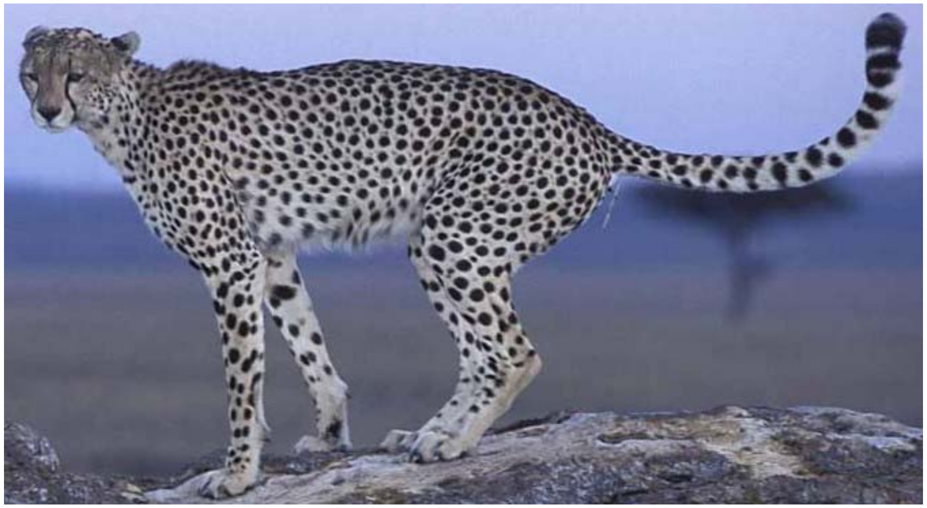
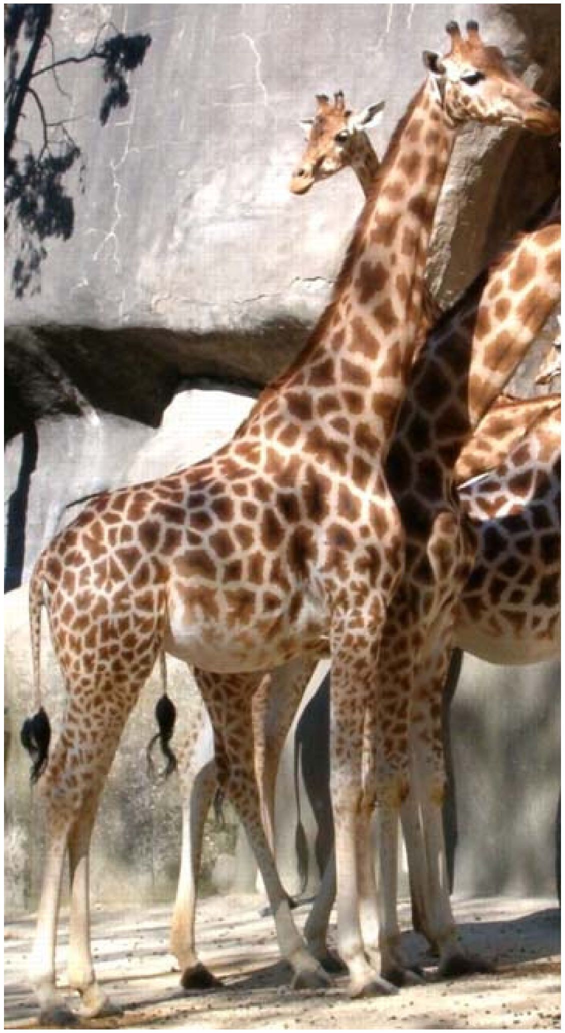
Some Patterns in Physics and in Nature

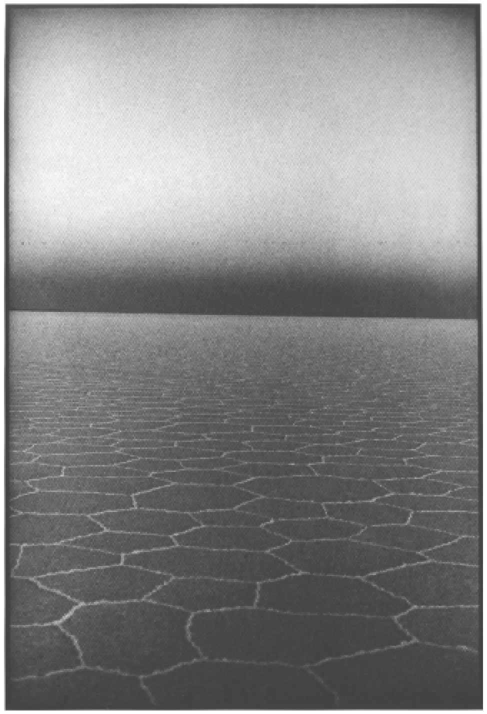
Introduction

PATTERNS IN NATURE

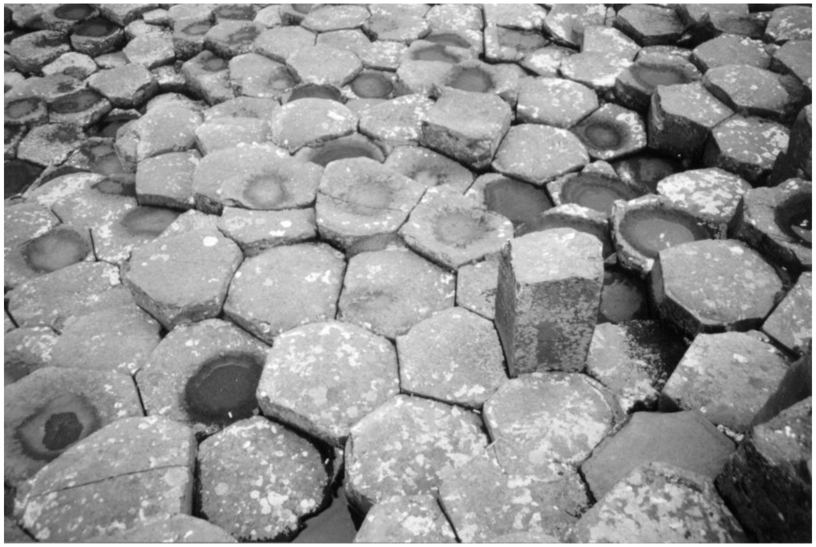
Foams are not only soap foams...







Uyuni salt lake (R. Depardon)



Giant 's causeway

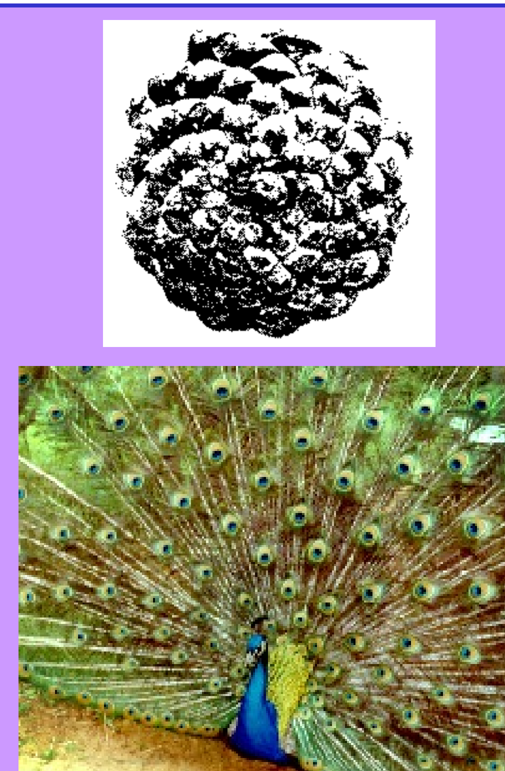
Nature is full of cellular structures

Those foams do not coarsen

Patterns in Nature (and Physics) are not only cellular structures

Stripe patterns

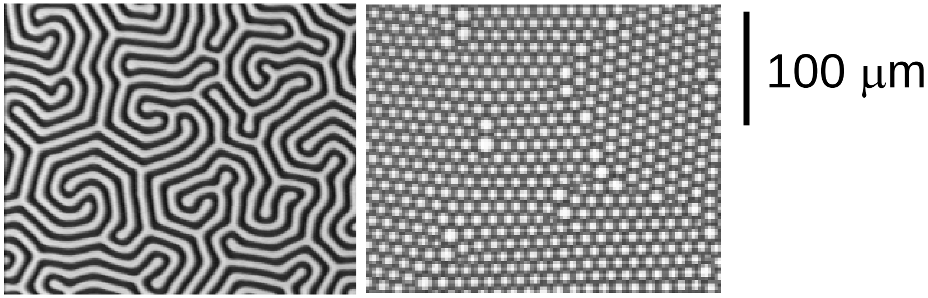
Bubble and cellular patterns



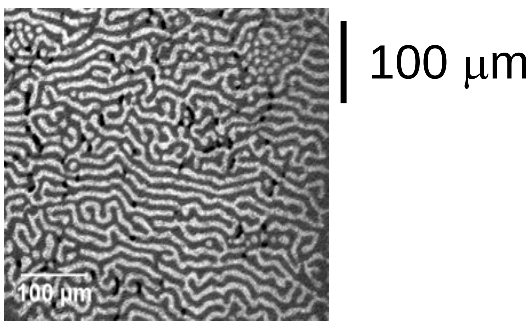
Spiral patterns

- **Binary systems**
- **Regular patterns, characteristic size**

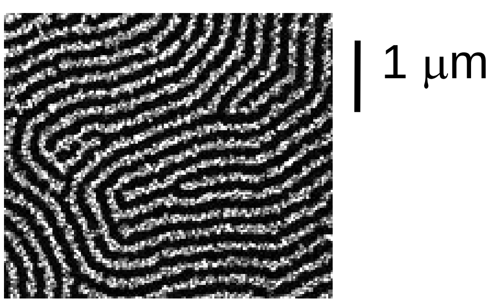
STRIPE / BUBBLE PATTERNS IN PHYSICS



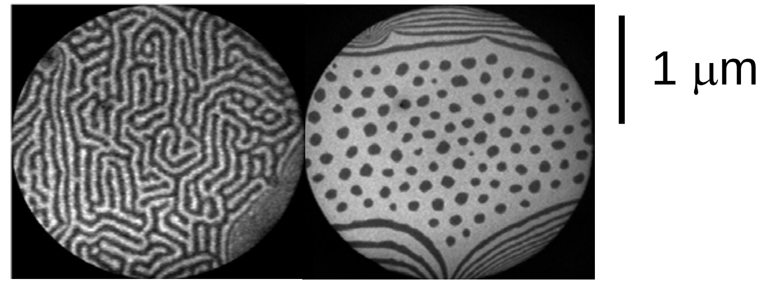
Magnetic garnets



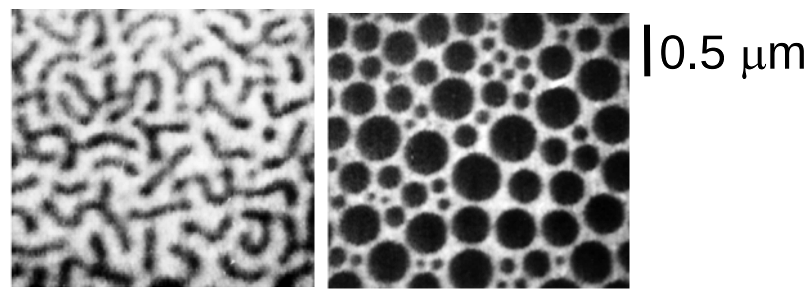
Superconductors



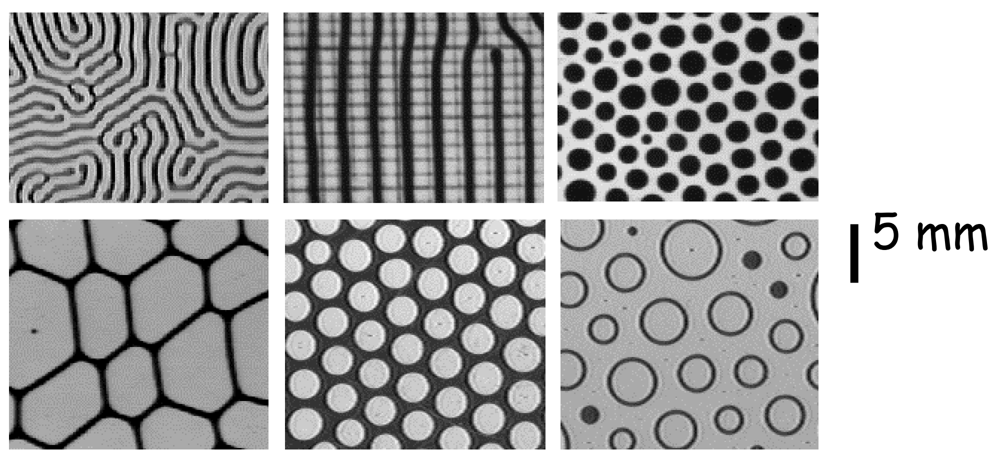
Diblocks copolymers



Pb atoms deposited on Cu

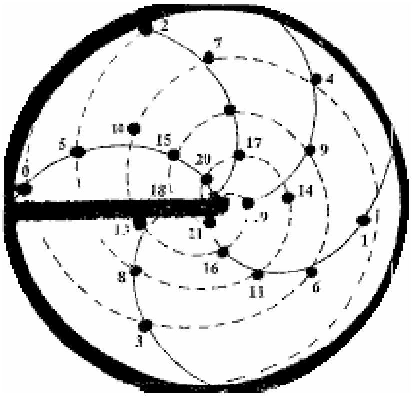


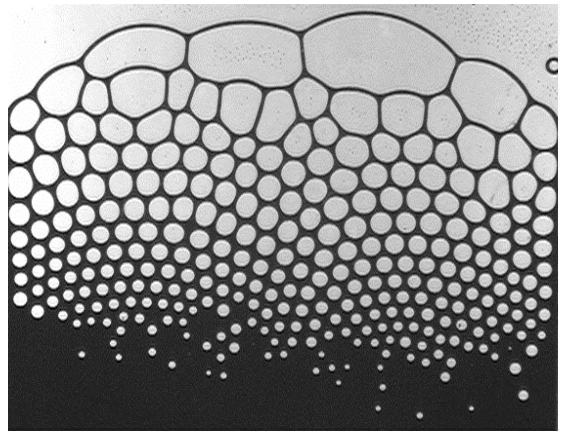
Langmuir monolayers



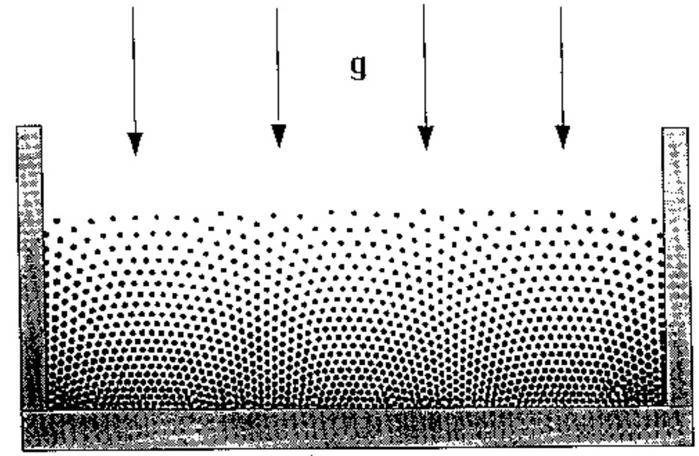
Ferrofluids

SPIRAL AND RELATED PATTERNS IN PHYSICS

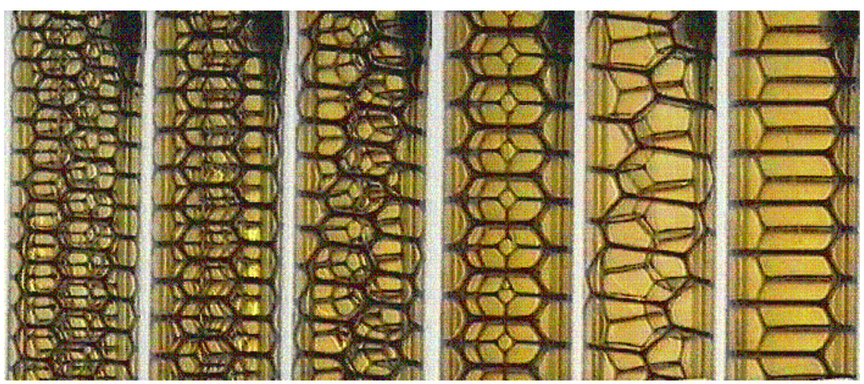


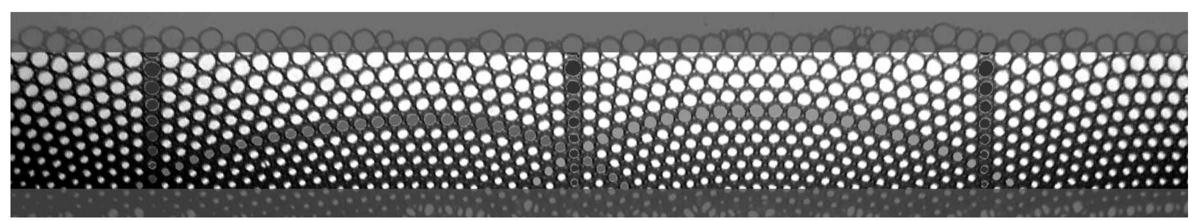


Ferrofluids



Magnetic spheres





Soap foams

A. Equilibrium 2D patterns in Physics

B. Spiral and related patterns in Physics and in Nature

A. Equilibrium 2D patterns in Physics

B. Spiral and related patterns in Physics and in Nature

What do we talk about?

- **Two-dimensional patterns**

- **Equilibrium patterns**

  Energy minimisation

  ⟶ No time-evolution, no coarsening

- **NB: We will not consider dynamical patterns :-(**

  Dissipative instabilities (Rayleigh-Bénard, Türing)

  Front instability (Taylor, solidification)

- ***A very few necessary ingredients:***
  - **Short-range repulsion**
  - **Middle-range attraction**
  - **Long-range repulsion**

A. Equilibrium 2D patterns in Physics

1. Overview of equilibrium patterns in Physics
2. The example of ferrofluids
3. The origin of equilibrium organisation

    Competing energies

    The characteristic size

4. Equilibrium and non-equilibrium cellular structure
5. Pattern selection

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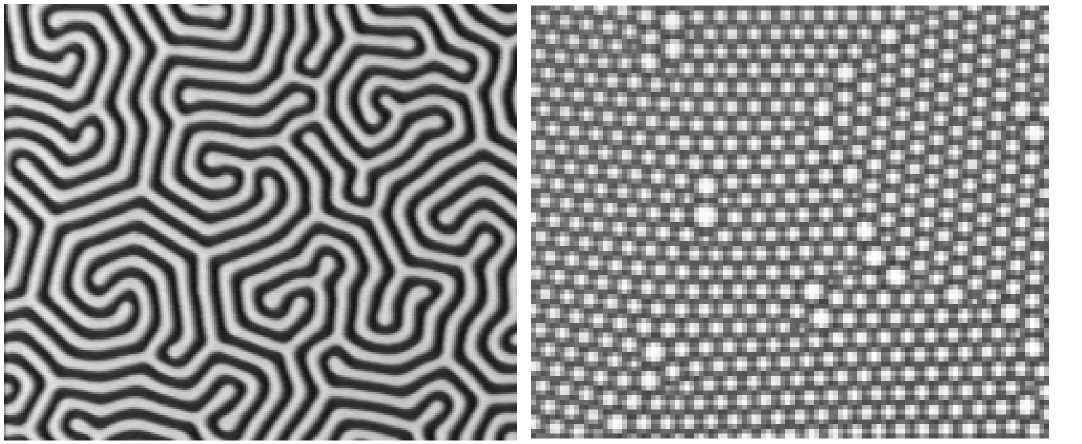
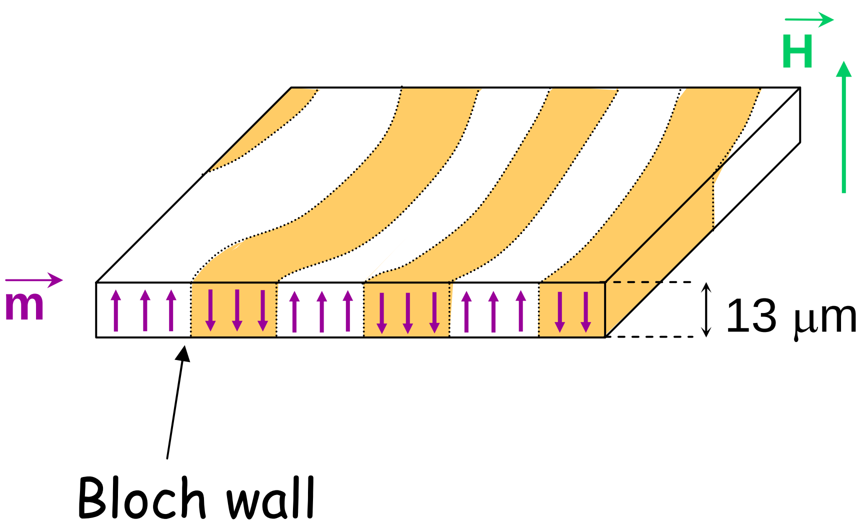
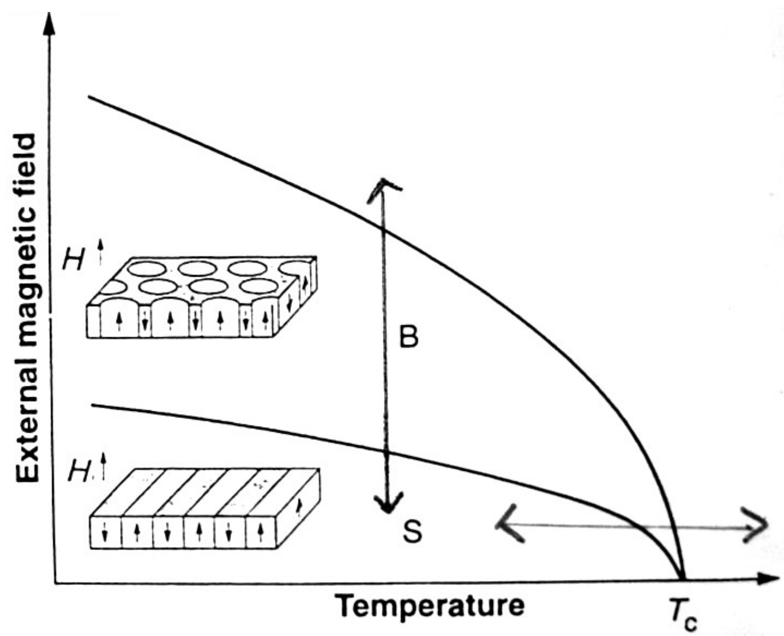
Thin magnetic garnet films

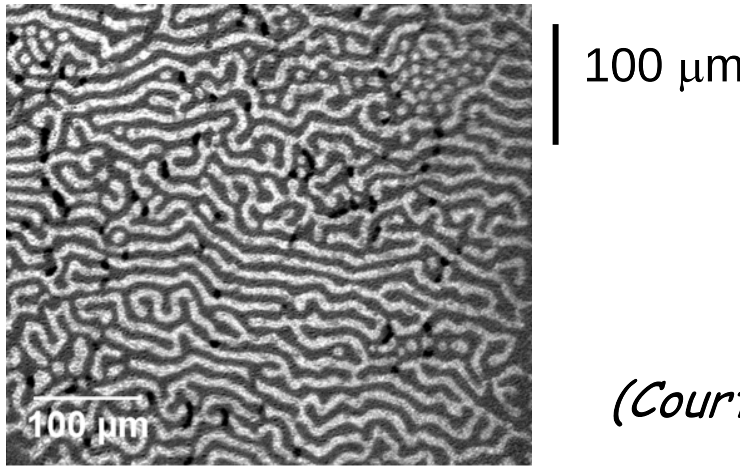


(Courtesy of P. Molho)

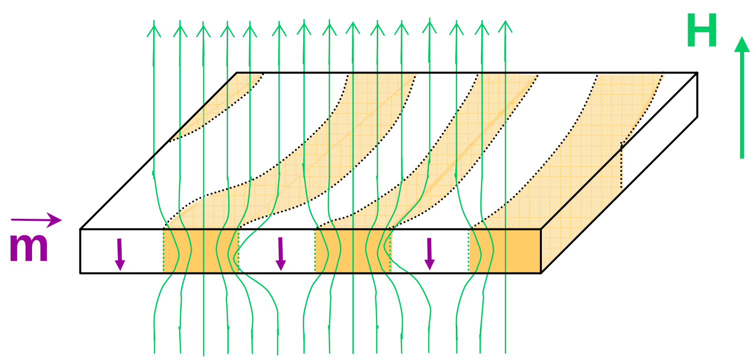


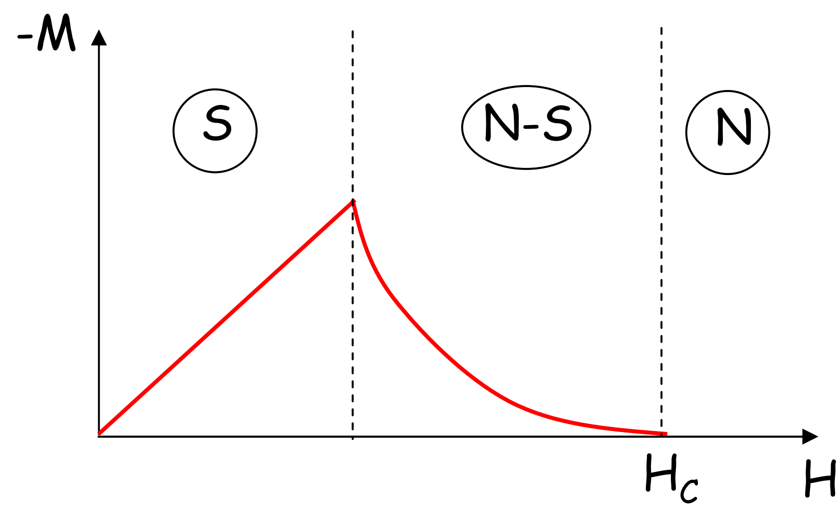


Superconductors



(Courtesy of V. Jeudy)





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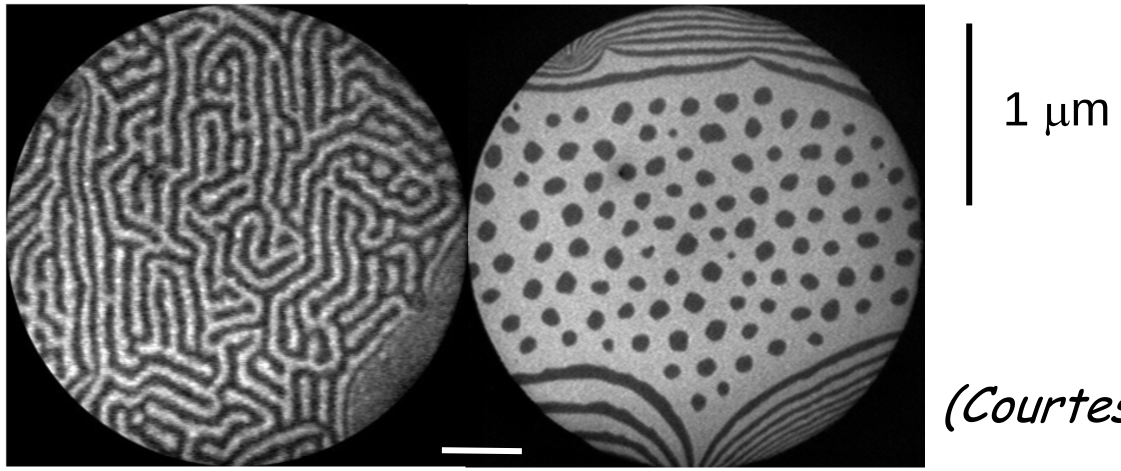
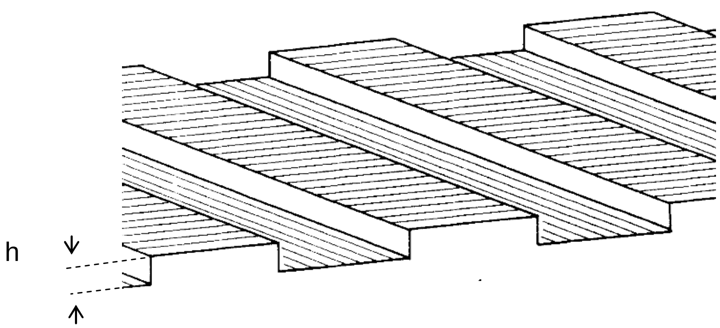
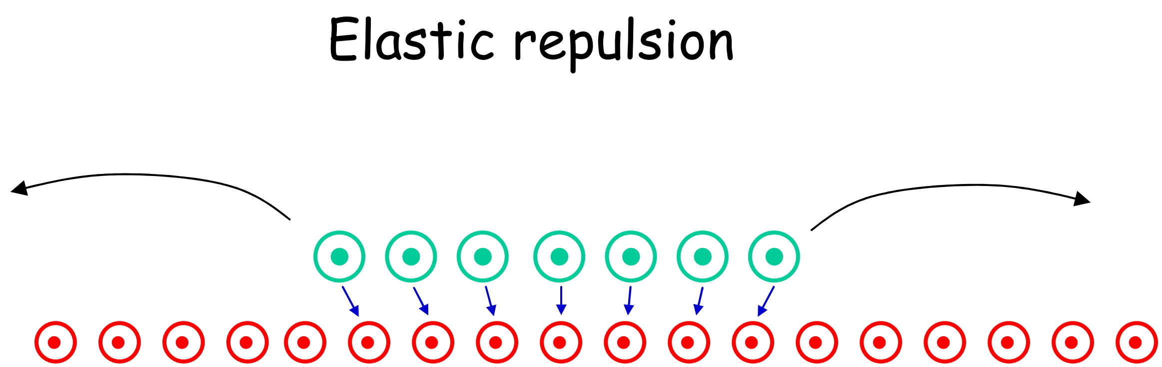
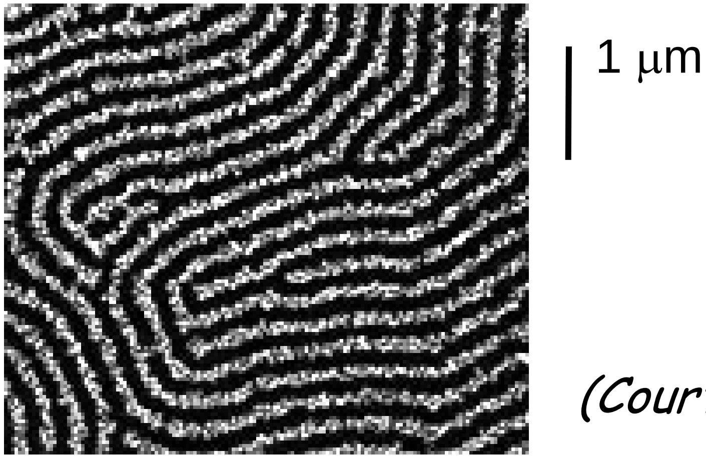


(Courtesy of G. L. Kellogg)

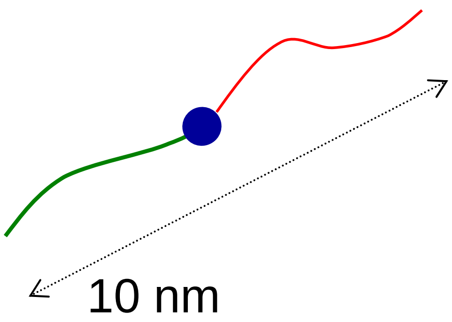


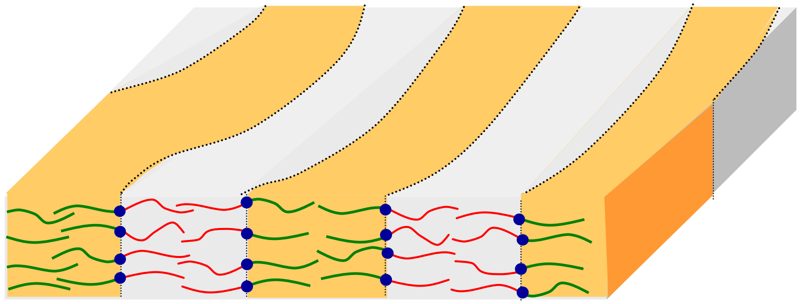


Diblock copolymers



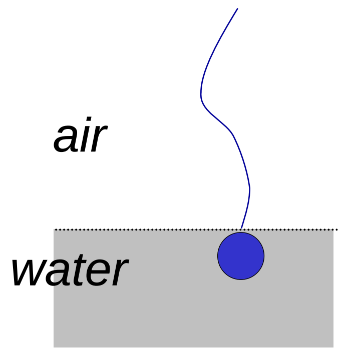
(Courtesy of H. Jaeger)



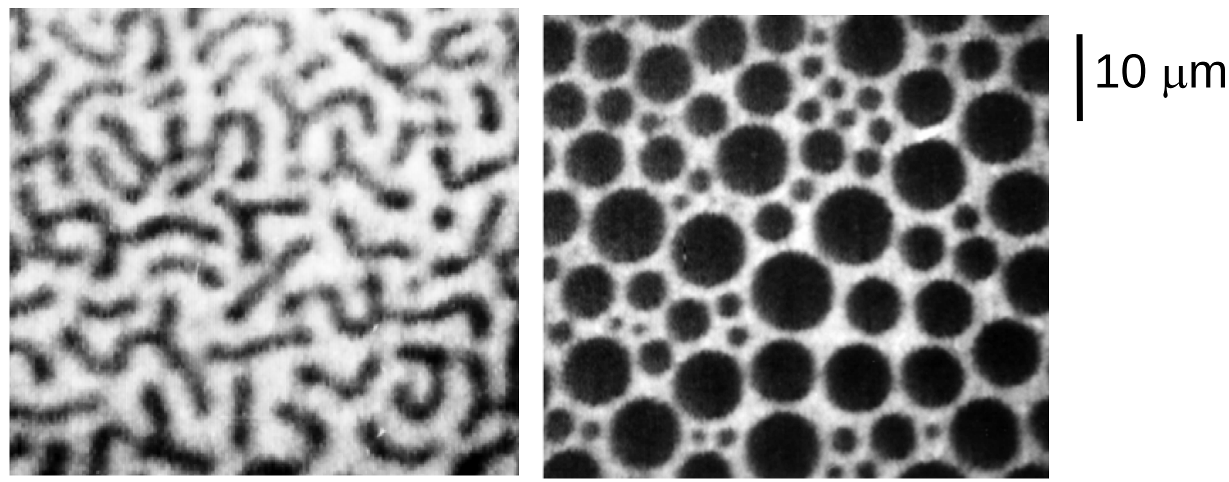


2D and 3D

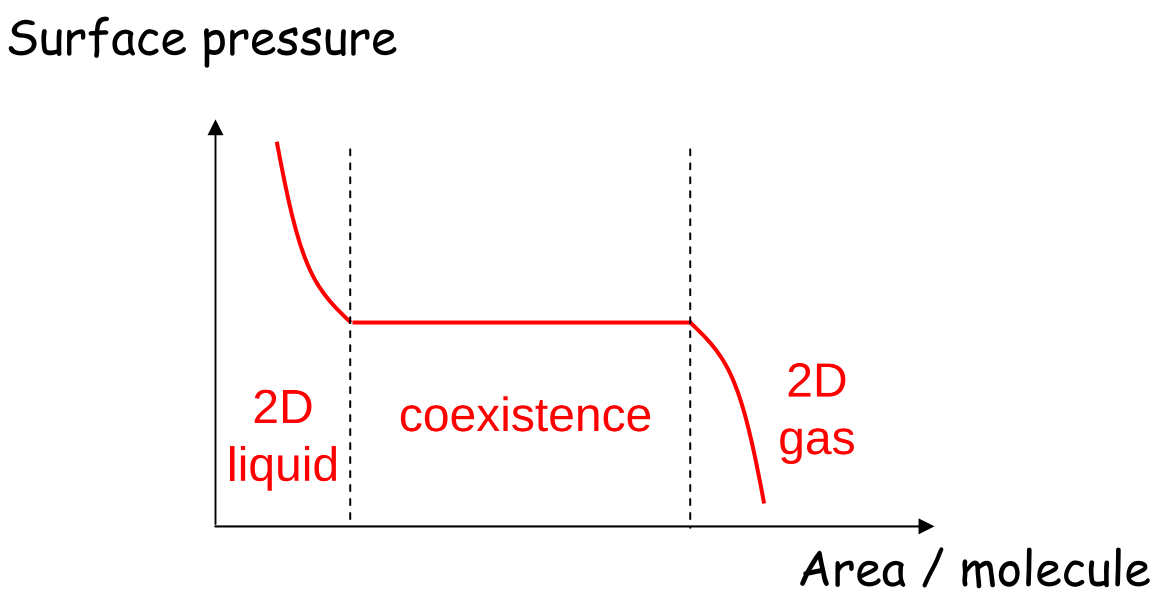
Langmuir films

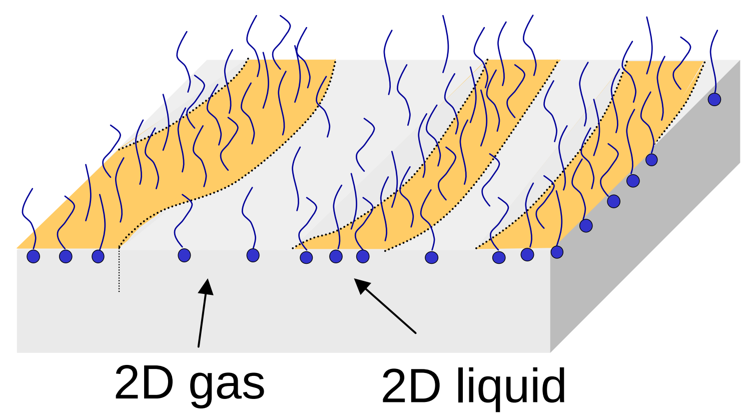


Surfactant molecule

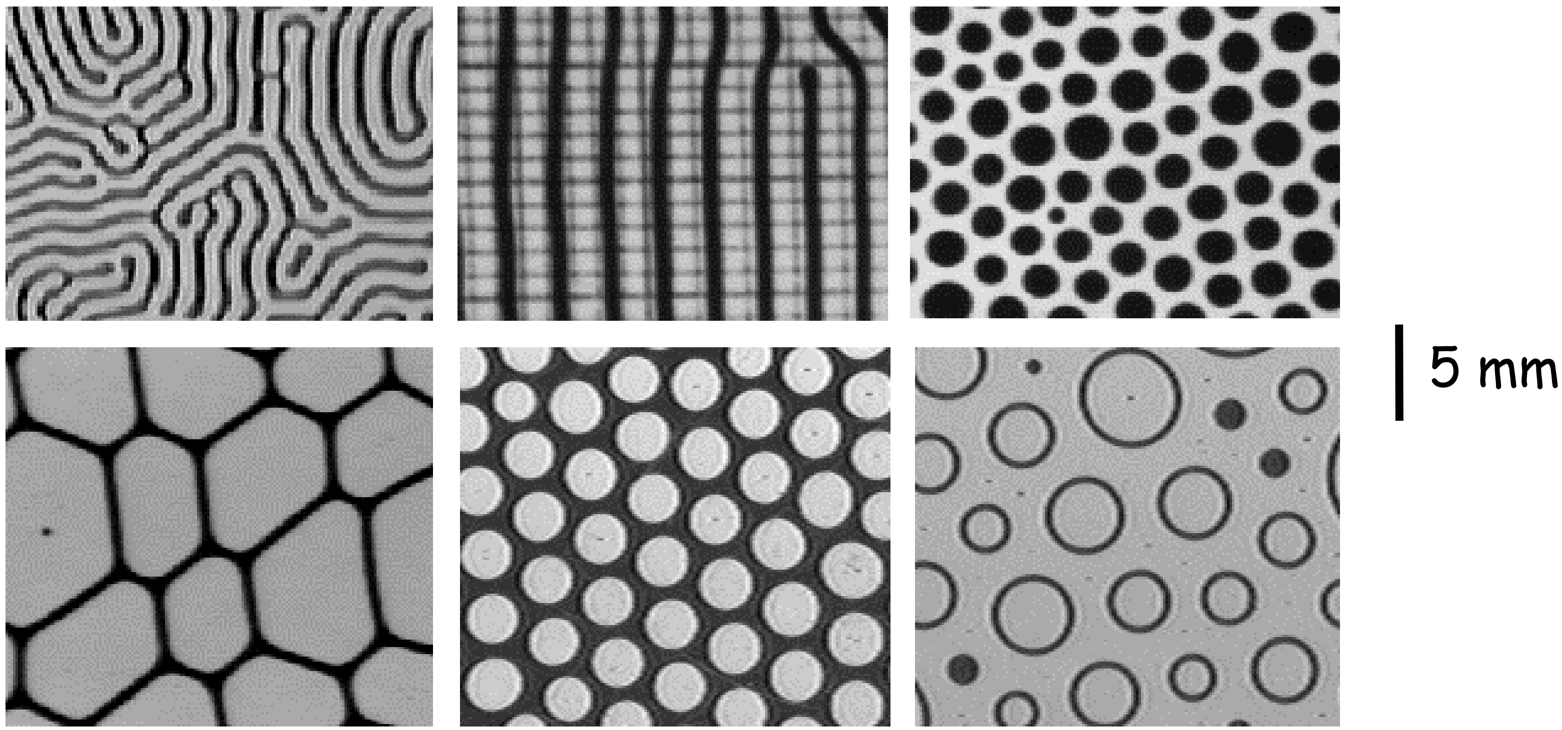


(Courtesy of S. Hakamatsu)





Ferrofluids



(Courtesy of F. Elias)

A. Equilibrium 2D patterns in Physics

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2. **The example of ferrofluids**

3. The origin of equilibrium organisation

   Competing energies

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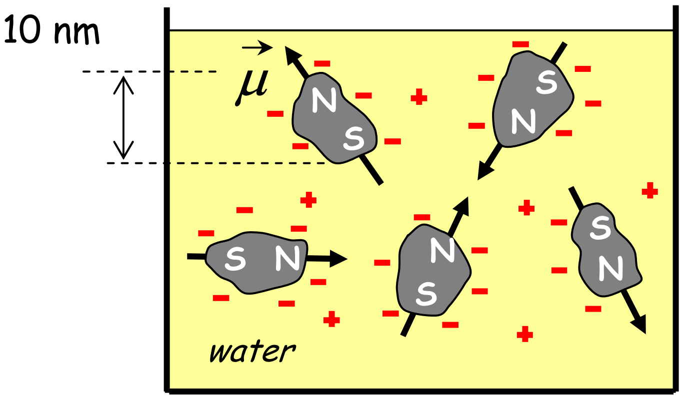
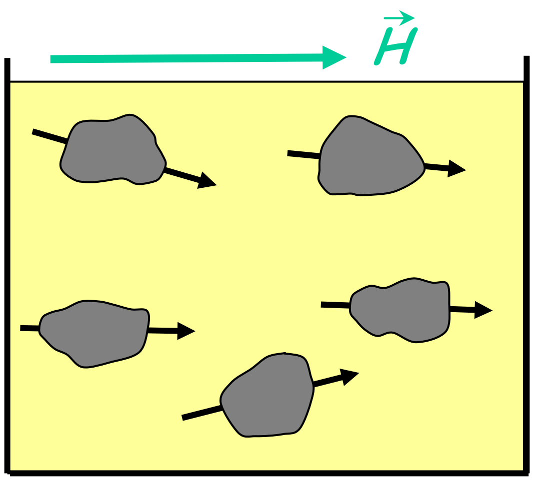
2. THE EXAMPLE OF FERROFLUIDS

Ferrofluid:

Microscopic scale: Colloidal suspension of magnetic particles.
Electrostatically stabilised

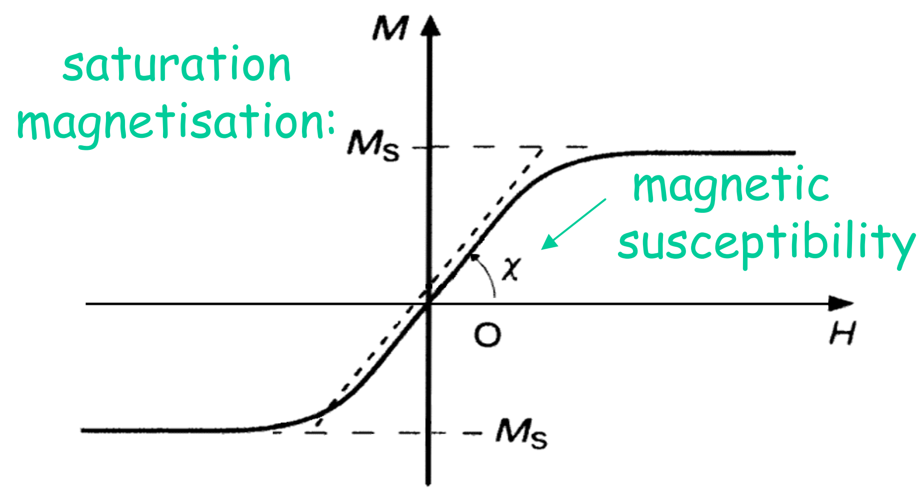


$\mu \sim 10^4 \mu_B$



Macroscopic scale: paramagnetic liquid.

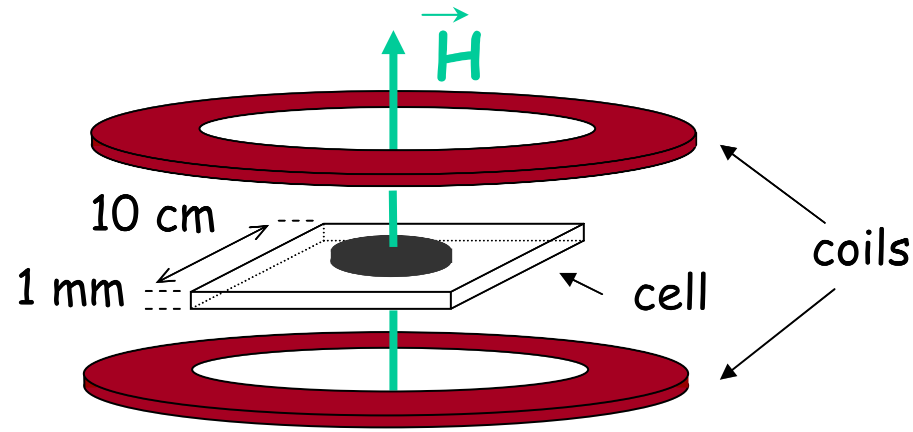
ferrofluid magnetisation: $\vec{M} \sim \Sigma \vec{\mu}_i$



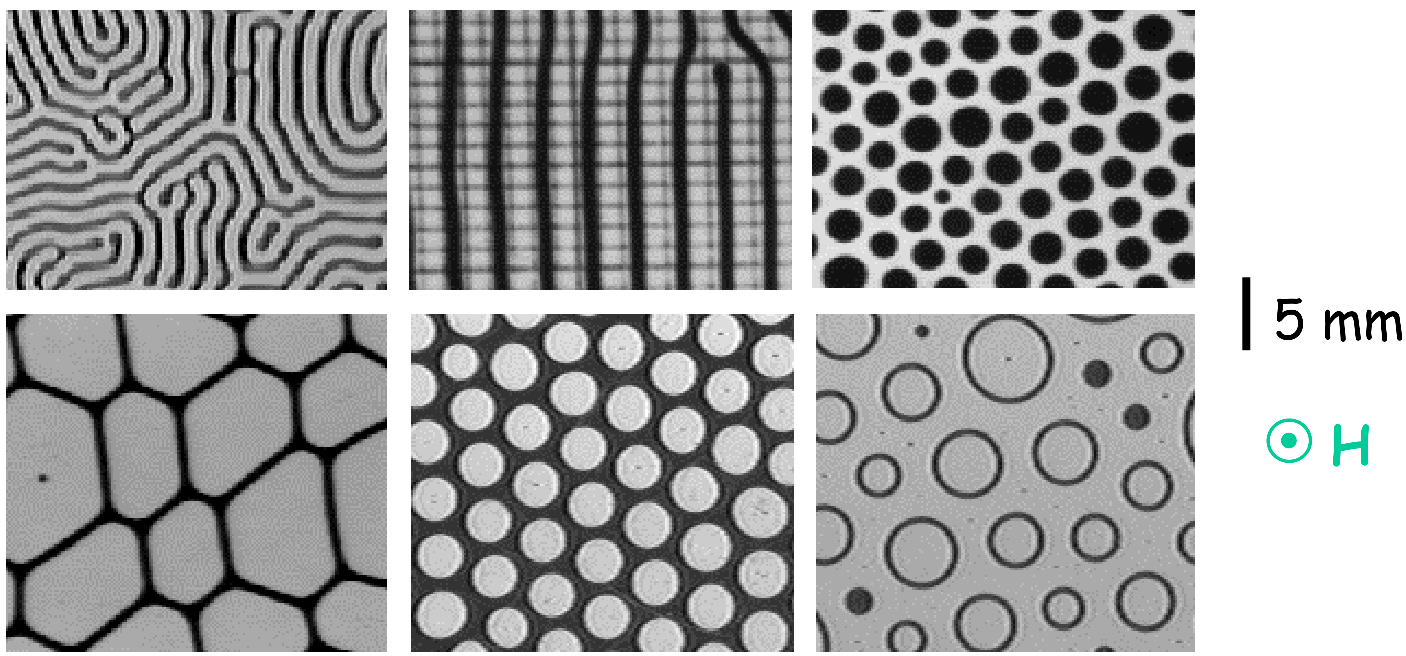
($\chi \approx 1$: *giant paramagnetism*)

Setup:

- Two phases:
  - Ferrofluid (water-based),
  - A transparent oil.
- In a flat, horizontal cell
- Perpendicular magnetic field



Patterns:



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3. THE ORIGIN OF EQUILIBRIUM ORDER

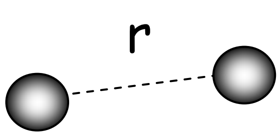
Equilibrium patterns ⟷ *energy balance*

1. (Very) short-range: repulsive energy

-> Either electrostatic interaction screened by counter-ions

-> or steric interaction

2. Middle-range: attractive energy



van der Waals interaction ⇨ *attractive*

Energy $E_{vdW} \sim - r^{-6}$ ⇨ *Middle-range*

Competition

→ Phase separation

→ ***Diphasic system***

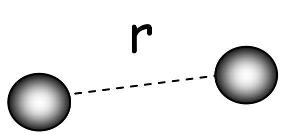
3. THE ORIGIN OF EQUILIBRIUM ORDER

Macroscopically:

1. (Very) short-range: repulsive energy

⇒ Volume of matter

2. Middle-range: attractive energy



⇒ Boundary energy

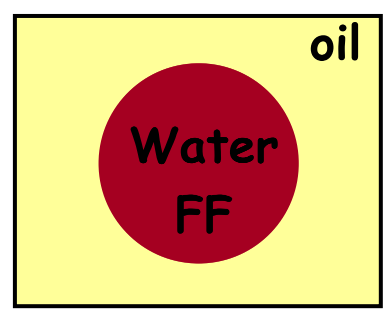
$$E_S = \sigma S$$

Minimisation of the surface of the interface

Competition

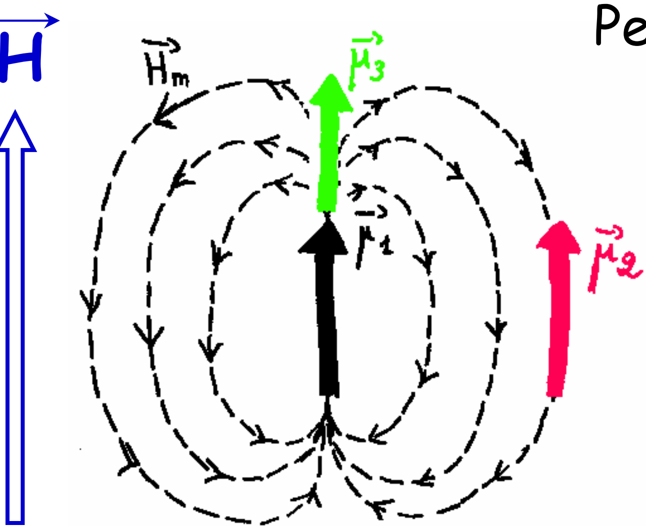
→ Phase separation

→ ***Diphasic system***



1 single circular domain

3. Long-range: repulsive energy: magnetic dipolar interaction



Permanent magnetic dipole

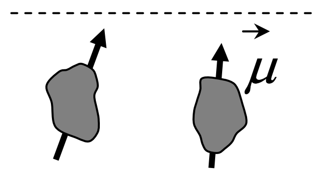
→ Induced magnetic field $\vec{H}_m$

→ Interaction with other magnetic dipoles

In the presence of the external field $\vec{H}$, $\vec{\mu}$ // $\vec{H}$

→ 2 cases → *attractive* / *repulsive*

Here: confined geometry ⊥ $\vec{H}$



Repulsive in average

Magnetic dipole interaction energy: $E_{dd} \sim r^{-4}$ ⇒ *Long - range*

Macroscopically: $E_m \approx \frac{1}{2} \mu_0 V_{FF} D \chi^2 H^2$

Demagnetising factor depends on the geometry

Summary:

1. Short-range repulsion:

steric or electrostatic

→ Oil / water phase separation.

2. Middle-range attraction:

van der Waals interaction

Magnetic particles: stable in the water phase.

→ Patterns

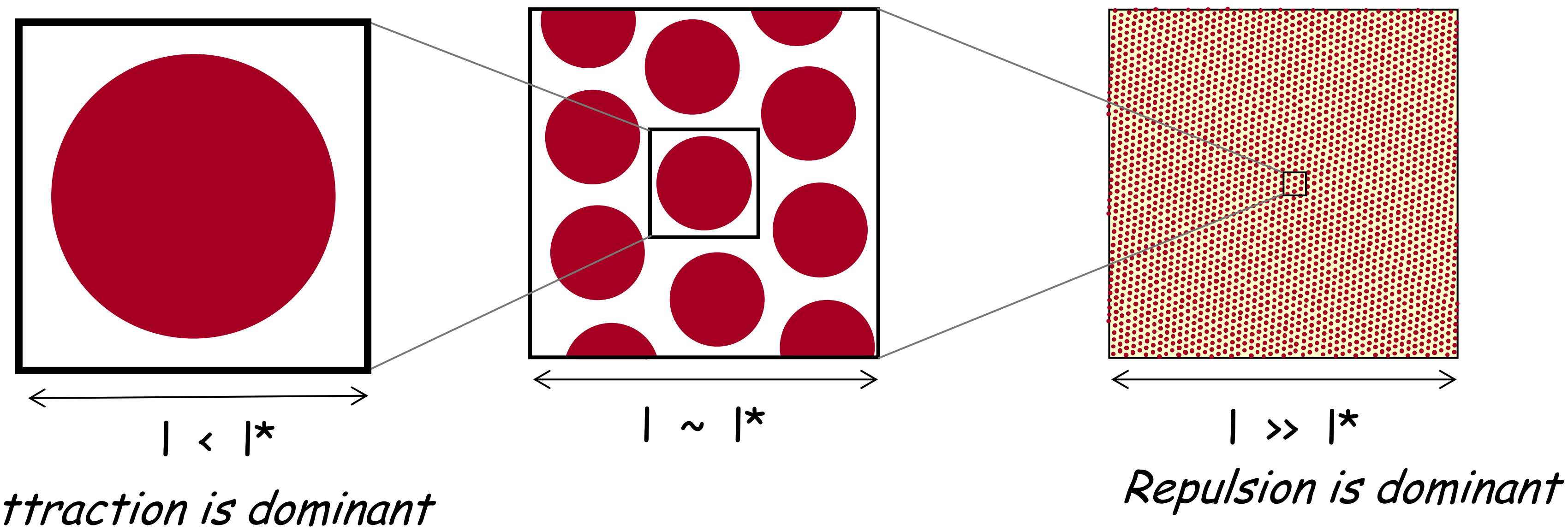
3. Long-range repulsion:

magnetic dipole interaction

The characteristic size

van der Waals energy: *middle - range*

Magnetic dipole interaction : *long - range* → Characteristic size *l**



Magnetic Bond number: $N_B = \dfrac{E_m}{E_s} \sim \dfrac{\mu_0 \chi^2 H^2 l^*}{4\sigma}$

$N_B \sim 1$ → $l^* \sim \dfrac{4\sigma}{\mu_0 \chi^2 H^2}$

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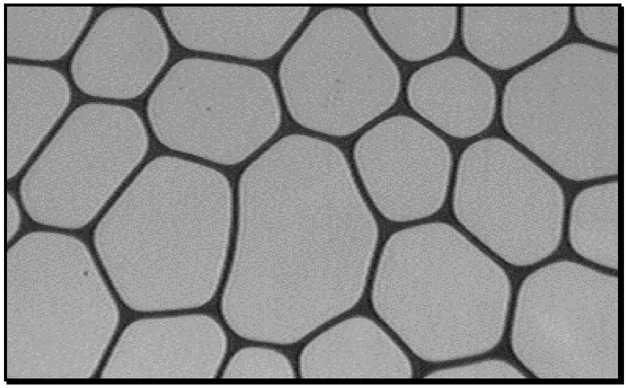
4. EQUILIBRIUM AND NON-EQUILIBRIUM CELLULAR STRUCTURES

- Equilibrium foam energy: $E = E_{surface} + E_{repulsive}$

$$(dE/dS)_{Seq} = 0$$

S = surface of the interface

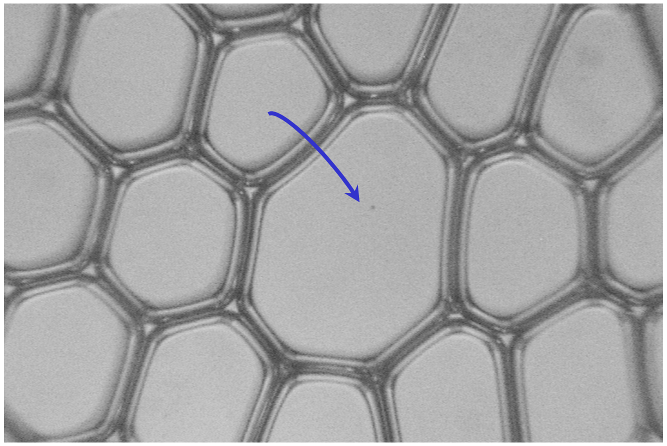


⟶ *Static foam*

- Non-equilibrium foam: $E = \mathbf{E}_{\mathbf{surface}} + E_{repulsive}$

$$dE/dS \neq 0: \quad S > S_{eq}$$

⟶ *Evolving foam*



2D Soap foam: Von Neumann :

$$\frac{d A_n}{dt} = \kappa (n - 6)$$

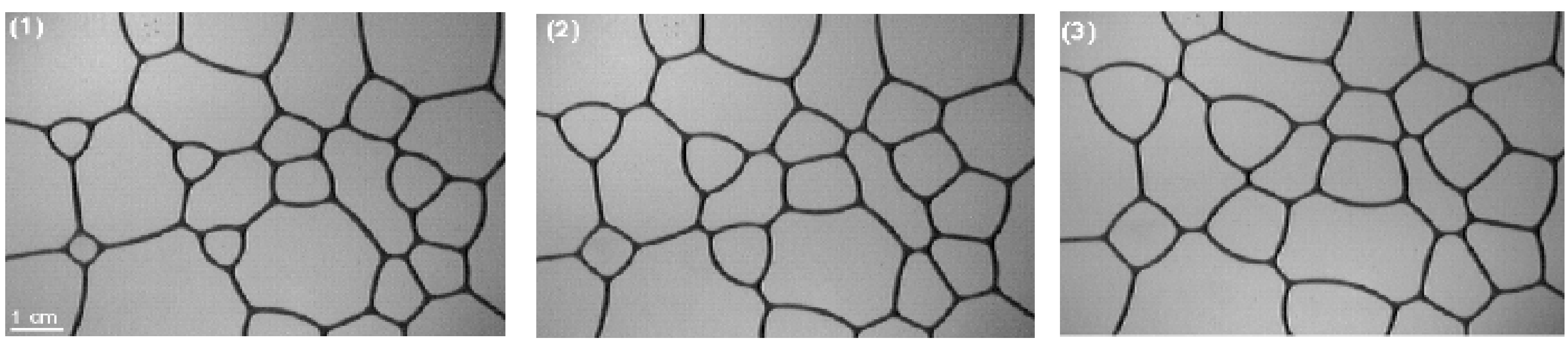
- Non-equilibrium foam: $E = E_{surface} + E_{repulsive}(H)$

$dE/dS \neq 0: \quad S < S_{eq}$

→ *Evolving foam*

2D ferrofluid foam: « anti-Von Neumann »: $\frac{dA_n}{dt} \begin{cases} > 0 \text{ if } n < 6 \\ = 0 \text{ if } n = 6 \\ < 0 \text{ if } n > 6 \end{cases}$

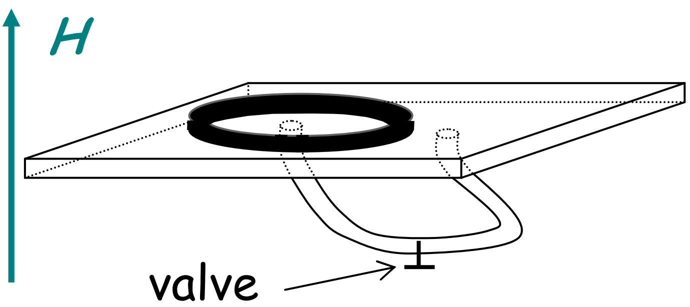
t = 0 | t = 34 min | t = 82 min

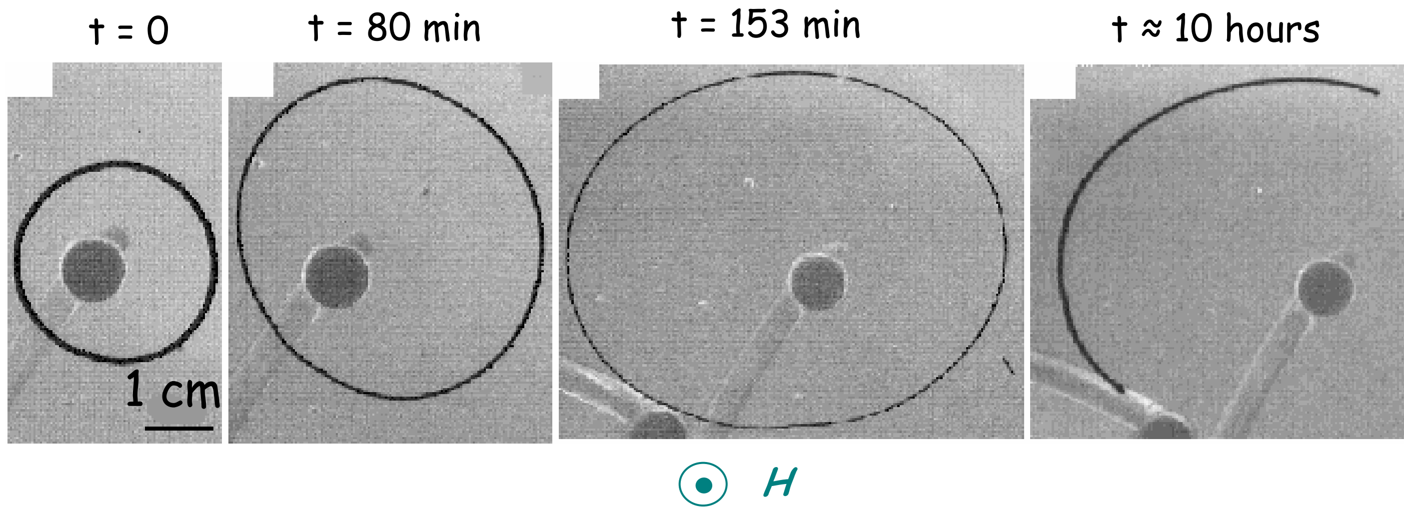


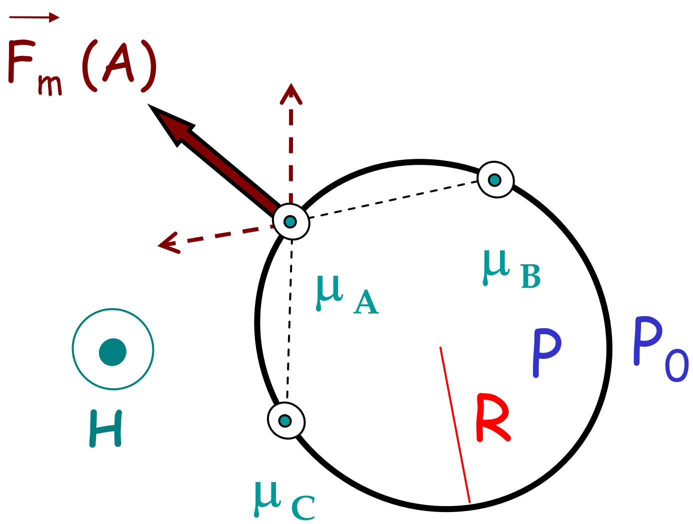
⊙ *H*

(Elias et al 1999)

2D ferrofluid ring







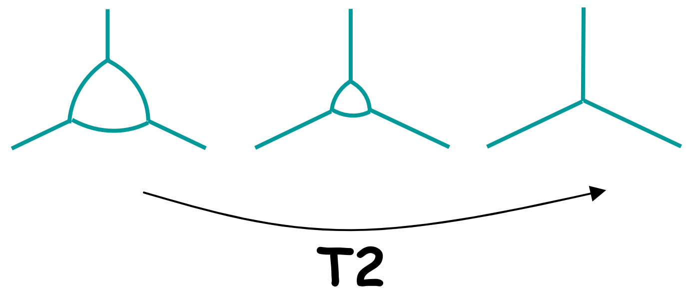
$$\Delta P = P - P_0 = \frac{2\sigma}{R} - \frac{K_m(H)}{2R^3}$$

with $K_m(H) \sim H^2$

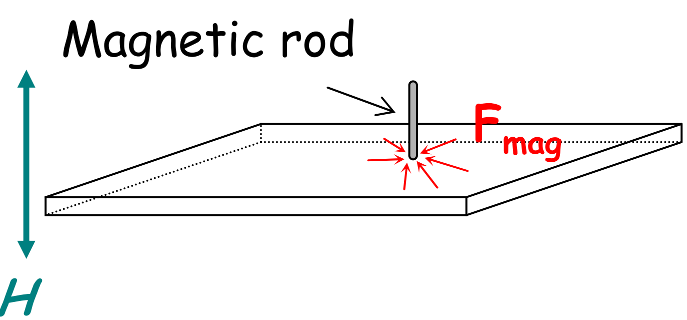
« Strong » H -> $\Delta P < 0$ ➡ the ring grows

2D ferrofluid foam: « anti-T2 »:

Surface energy dominant -> cell disparition



Magnetic energy dominant -> cell creation



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**Anti -T2
in a 2D
ferrofluid foam**

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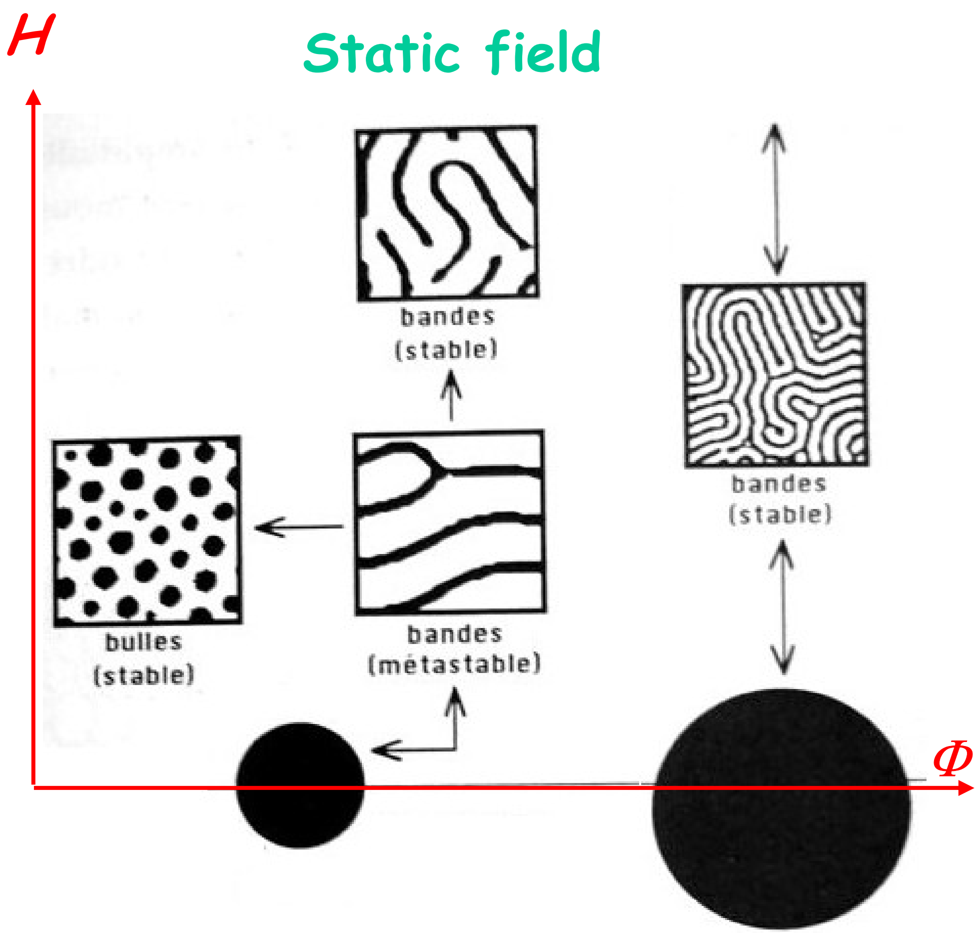
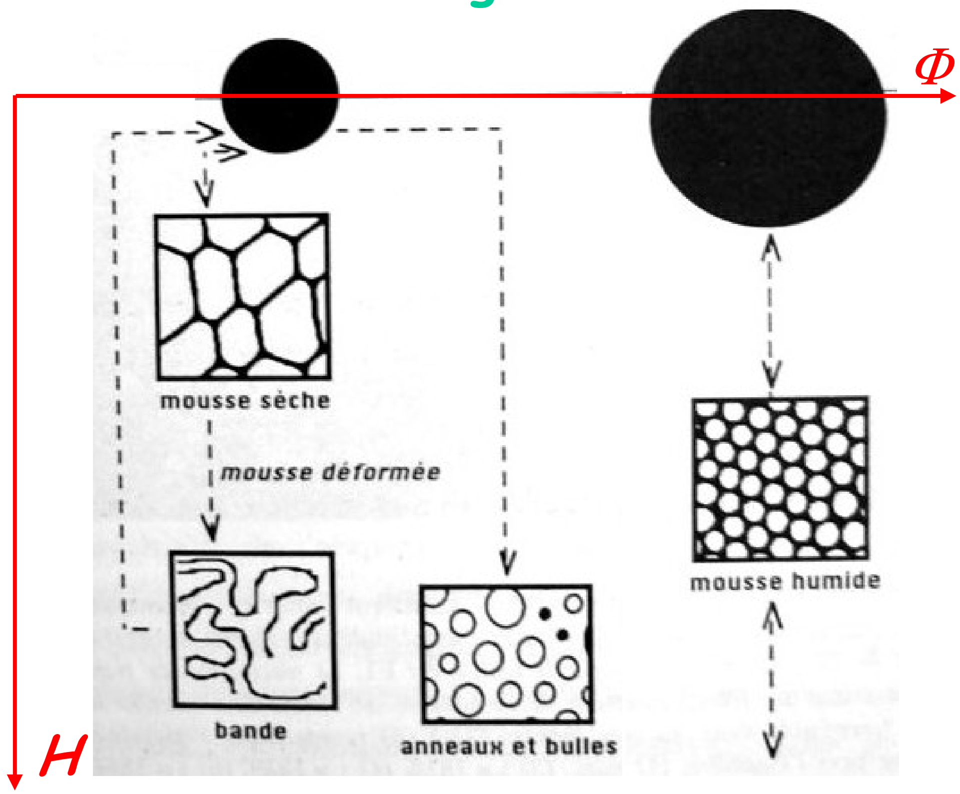
5. PATTERN SELECTION

Approximative phase diagram

Static field



Alternating field



But: metastable states, large energy barriers

→ Importance of the history

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B. Spiral and related patterns
in Physics and in Nature

B. Spiral and related patterns in Physics and in Nature

1. Macroscopic deformations of the cellular pattern

Gravity rainbow

Disclinations in foams

Conformal transformation

2. Phylotaxis

INTRODUCTION

How to map a convex surface with a cellular structure (120° angles) ?

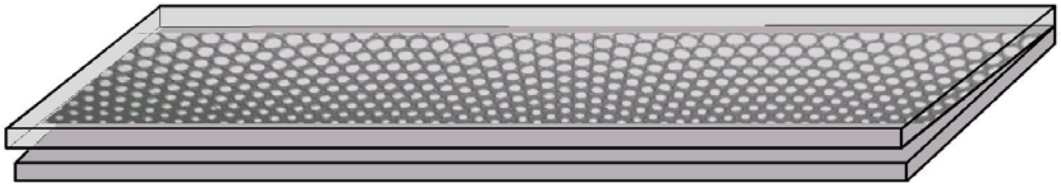
- Socker (football) ball: introduces 5-sided cells
- Macroscopic deformation of the structure

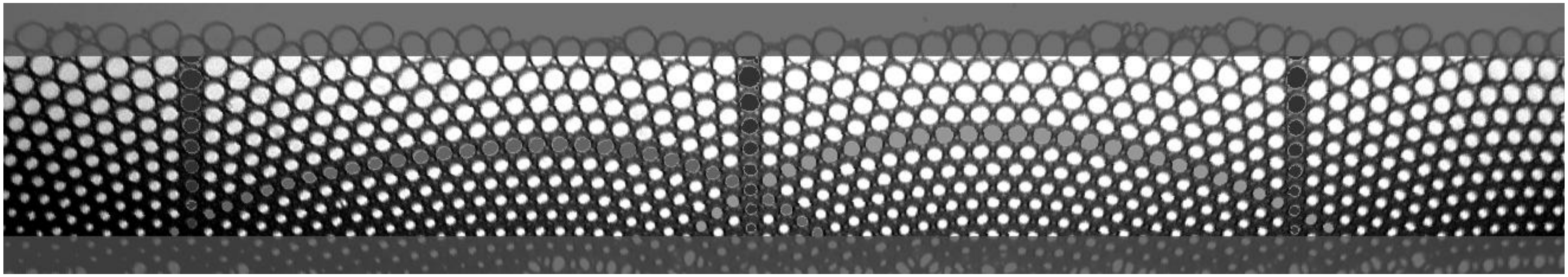
Museum of Industry and Design (Budapest, Jugendstil)

1. MACROSCOPIC DEFORMATIONS OF THE CELLULAR PATTERN

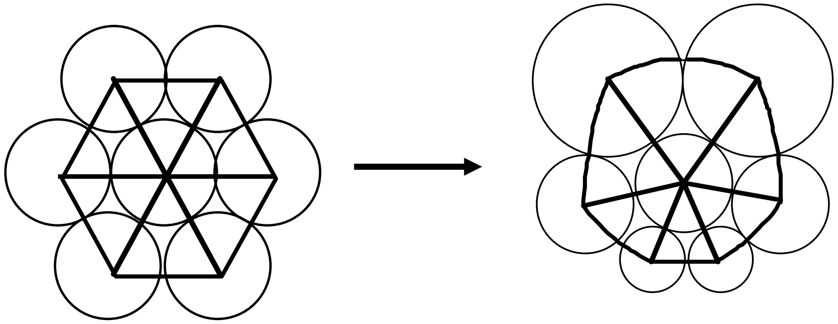
a) The "gravity rainbow"

- **Monodisperse 2D soap foam between inclined plates** *(Drenckhan et al 2004)*

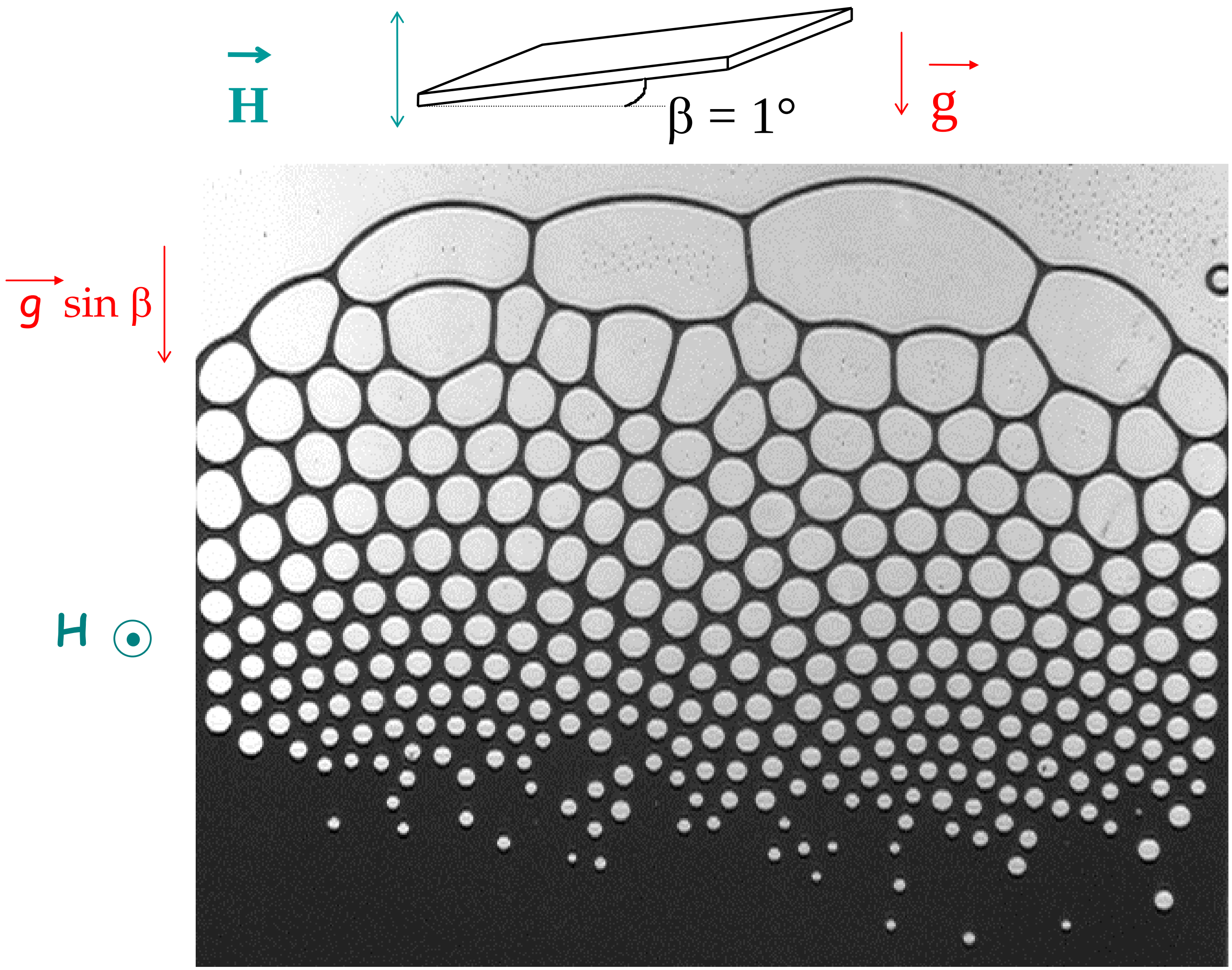




The 120° angles are preserved

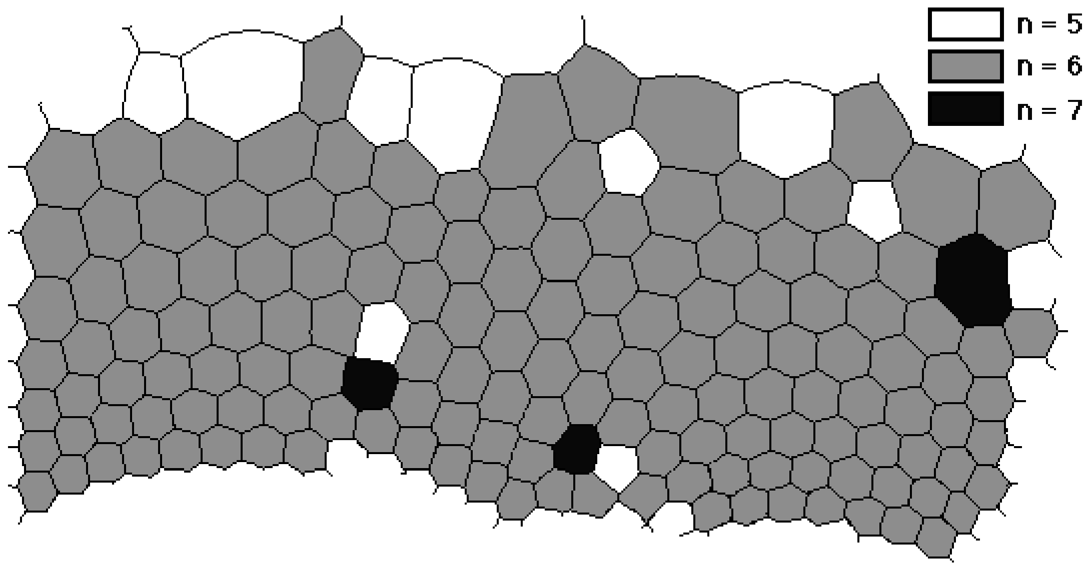


- **2D Ferrofluid foam under gravity: gravity rainbow** *(Elias et al 1998)*



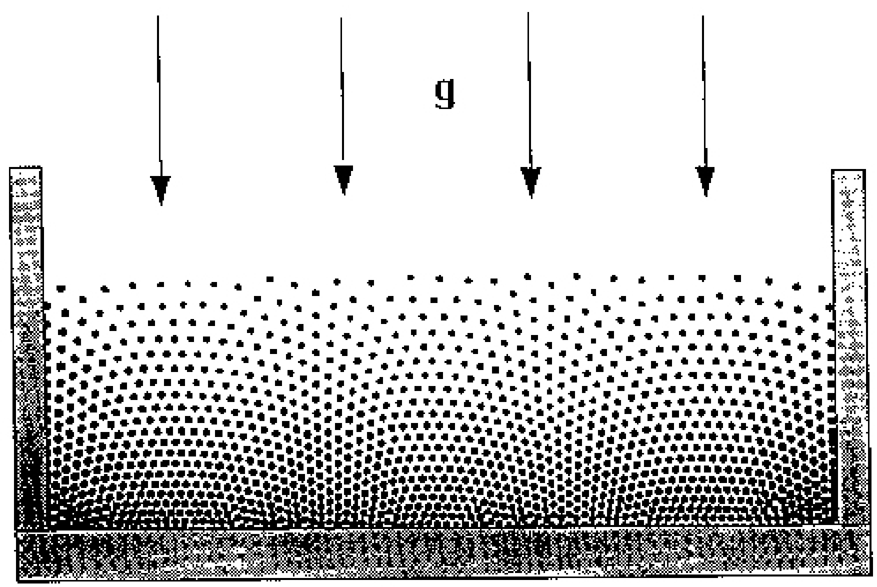
∇ (ferrofluid volume fraction) ⟶ ∇ (cell area)

<n> ≈ 6



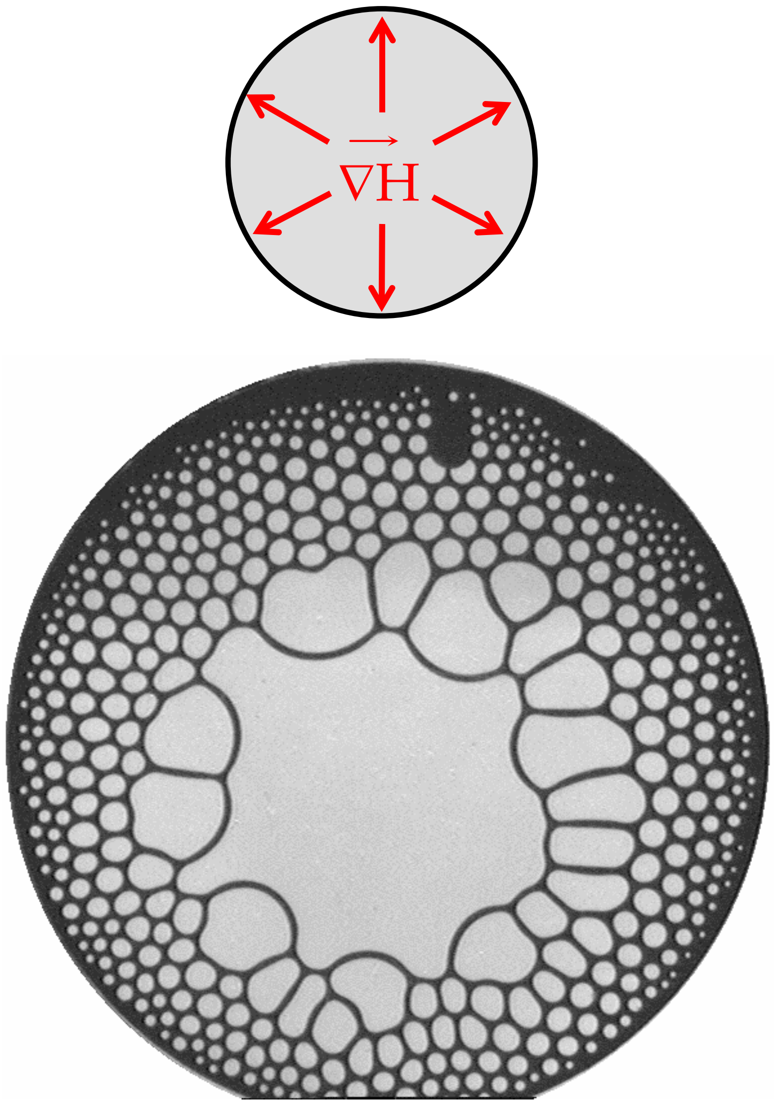
- **Repelling magnetic spheres subjected to gravity** *(Rothen et al 1993)*

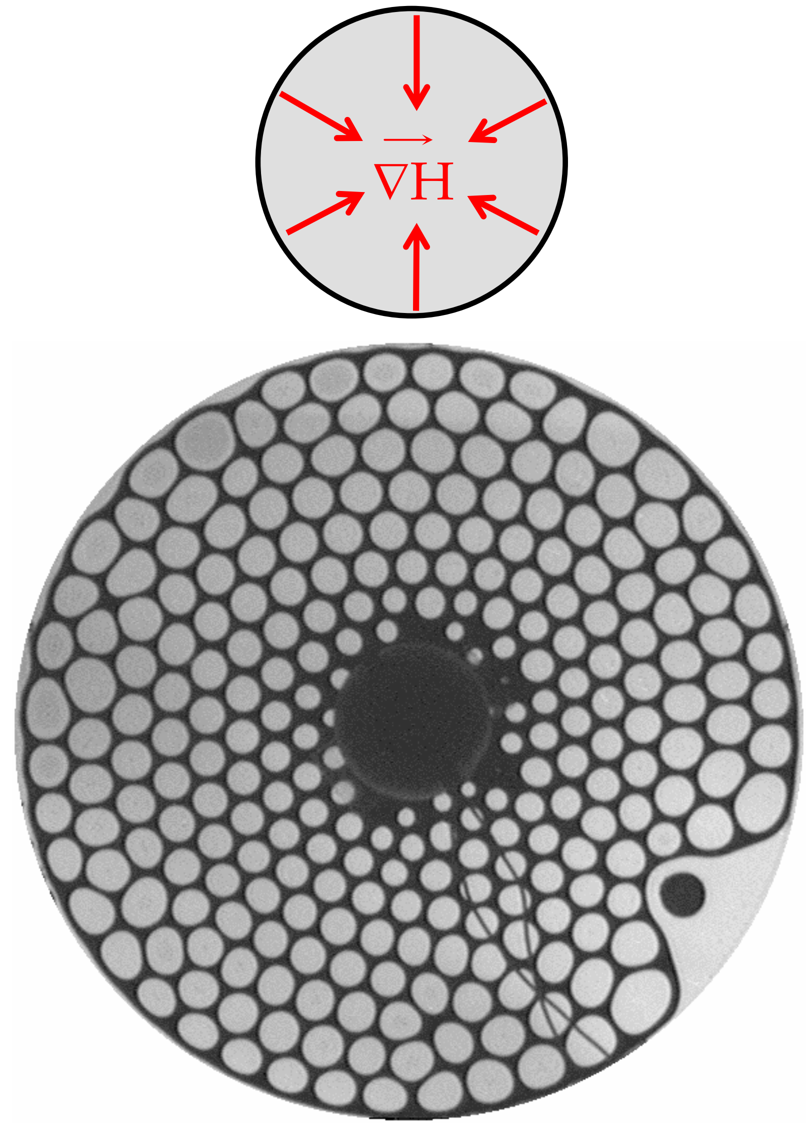
H ⊙



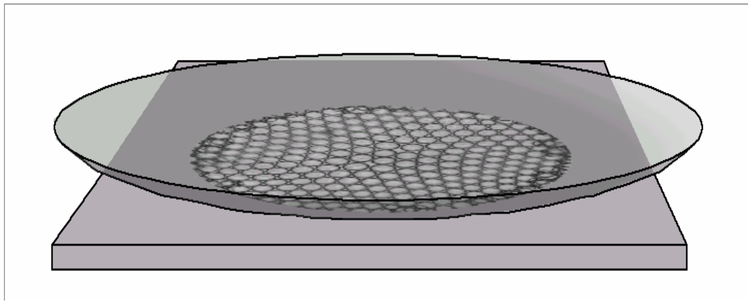
b) Disclinations in foams

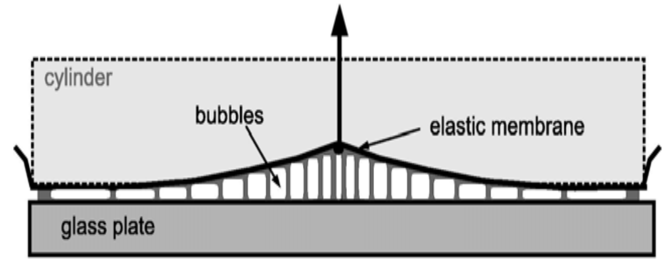
- 2D Ferrofluid foam under a radial force field

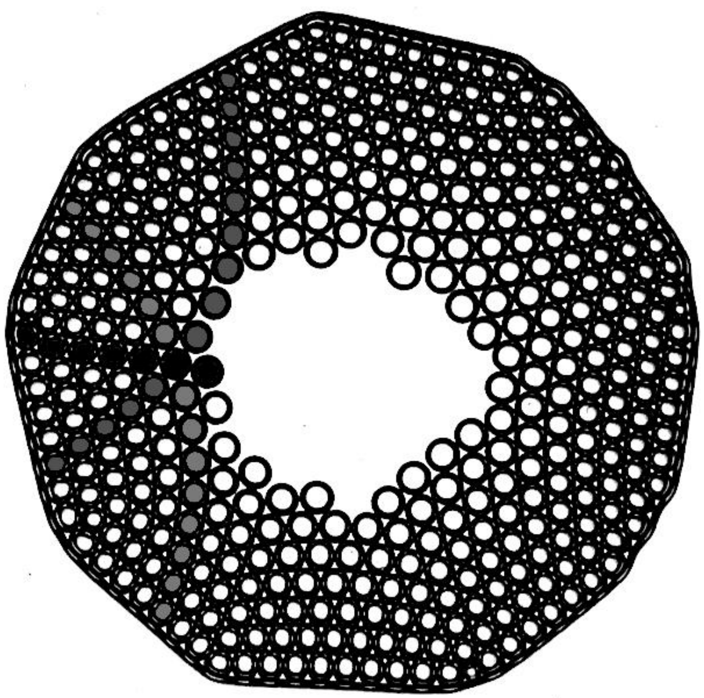


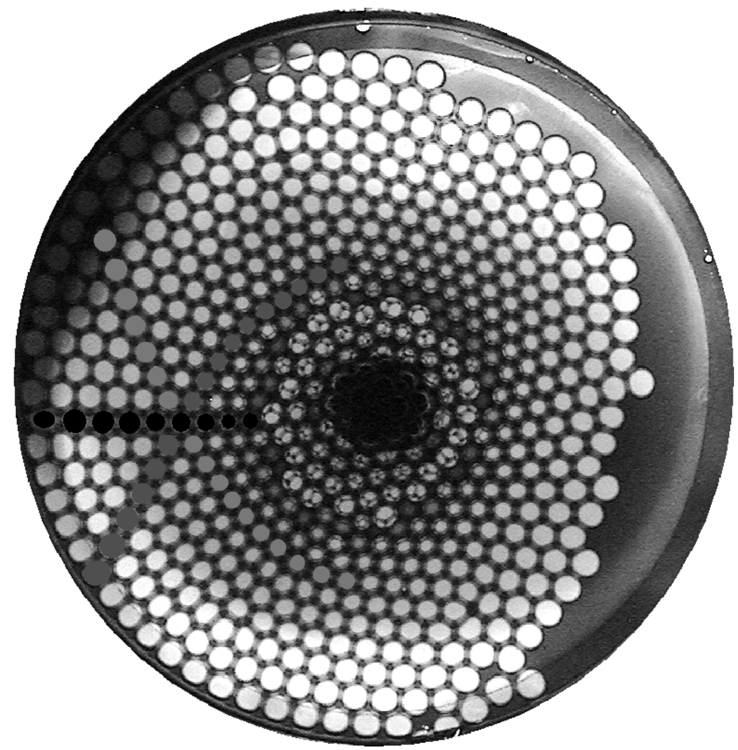


- **2D soap foam under radially deformed plates**

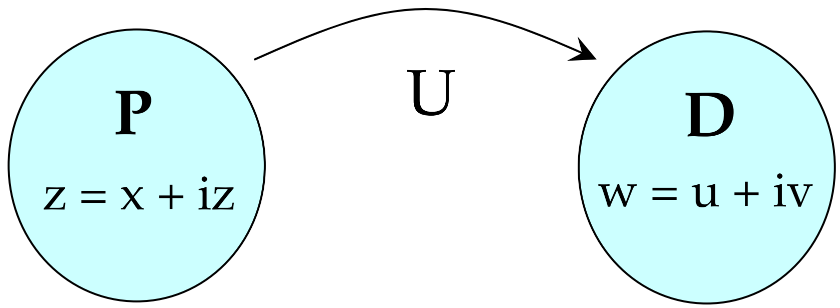








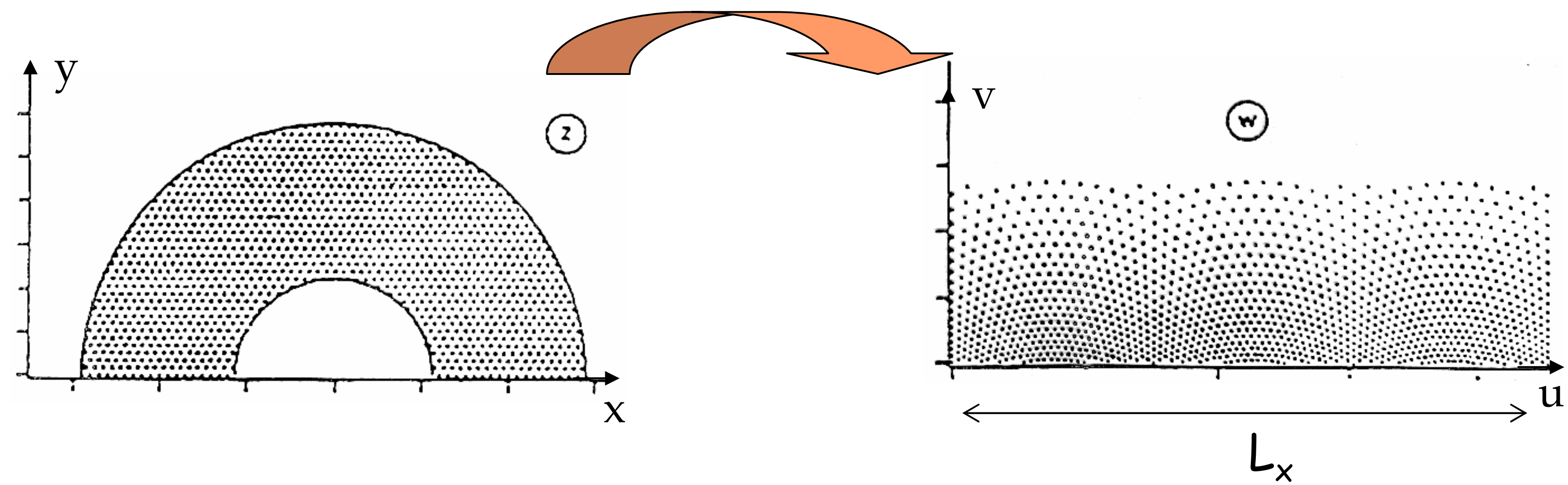
c) Conformal transformations



- Transformation U described by an *analytic* function
- U *preserves the angles*

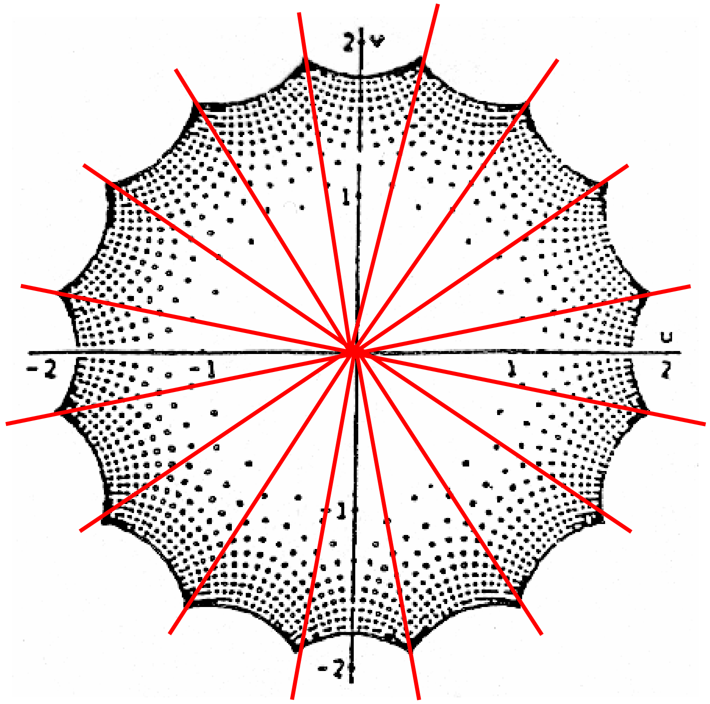
- **In the plane: one solution:**

$$w(z) = -\frac{i}{\alpha}\ln(i\alpha z) \quad \text{with} \quad \alpha = \frac{n}{3}\frac{\pi}{L_x}$$



Conformal transformation in a radial geometry:

$$w(z) = \gamma\ z^{\left(\frac{1}{1-\delta}\right)} \quad \text{with} \quad \delta = \frac{p}{3}$$

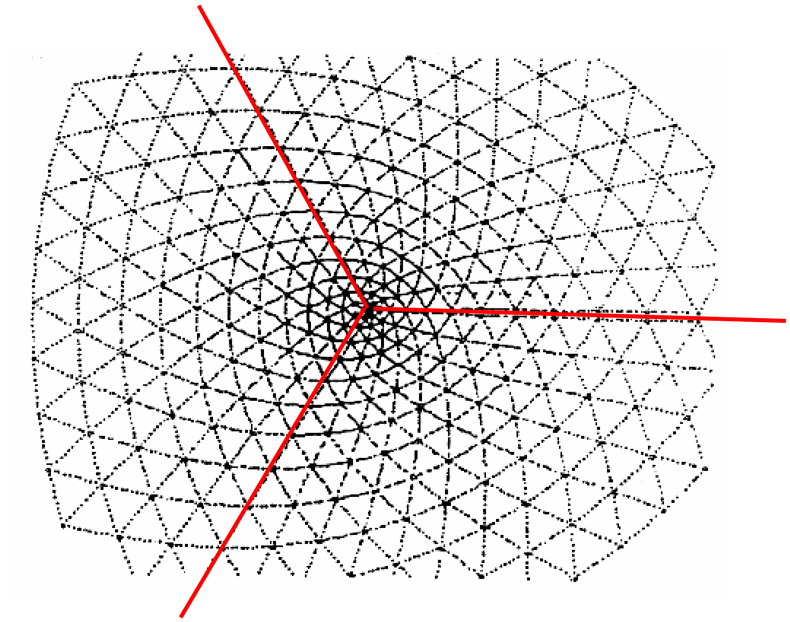


$\delta < 0$:

disclination

$p\pi / 3$

p integer < 0



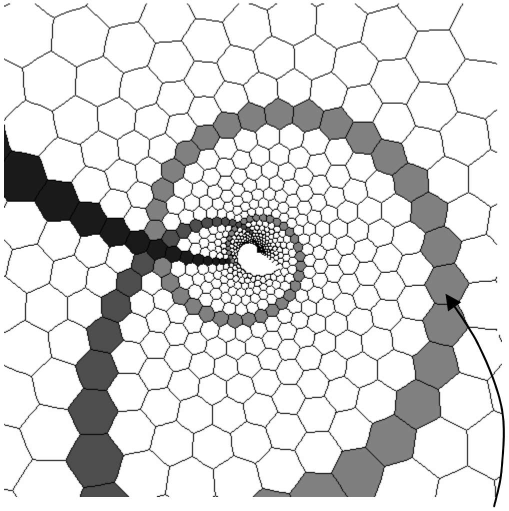
$\delta < 0$:

disclination

$p\pi / 3$

p integer > 0

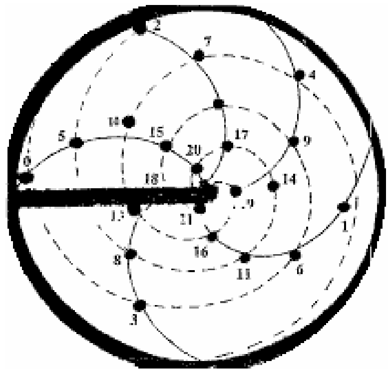
2. PHYLOTAXIS



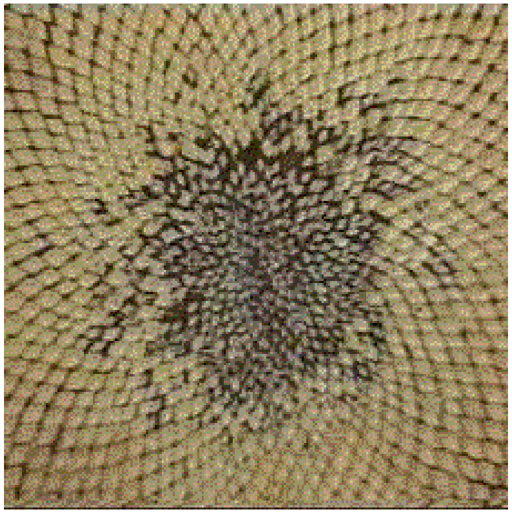
$w(z) \sim e^z$

3 logarithmic spirals

(S. Cox)



repelling drops
of ferrofluid
(Douady, Couder)



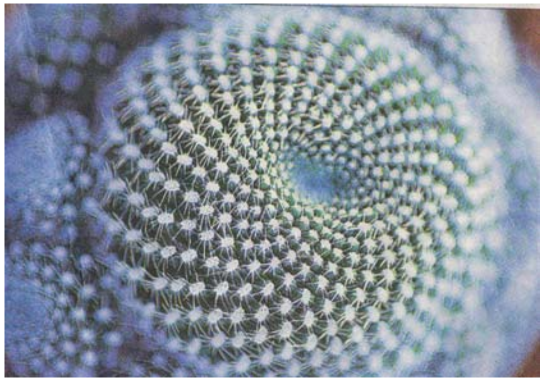
Sunflower

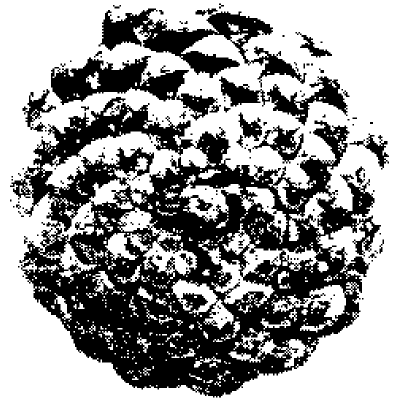
peacock

Number of each spiral type that cover the plane -> [i j k] consecutive numbers of FIBONACCI SEQUENCE

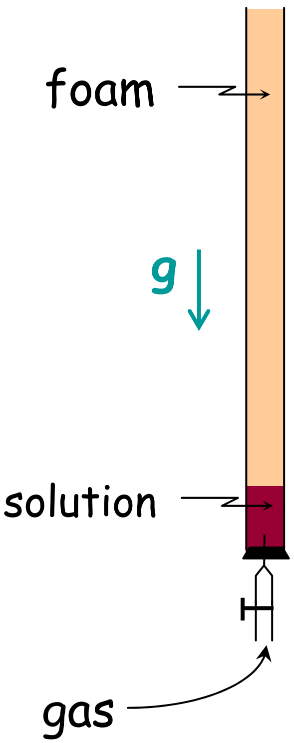
Phylotaxis at the surface of a cylinder:

Cactus

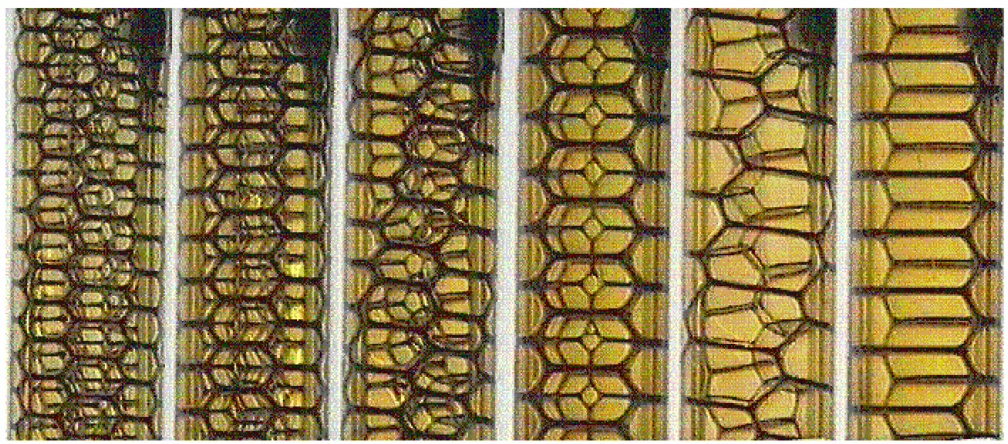




Pine cone

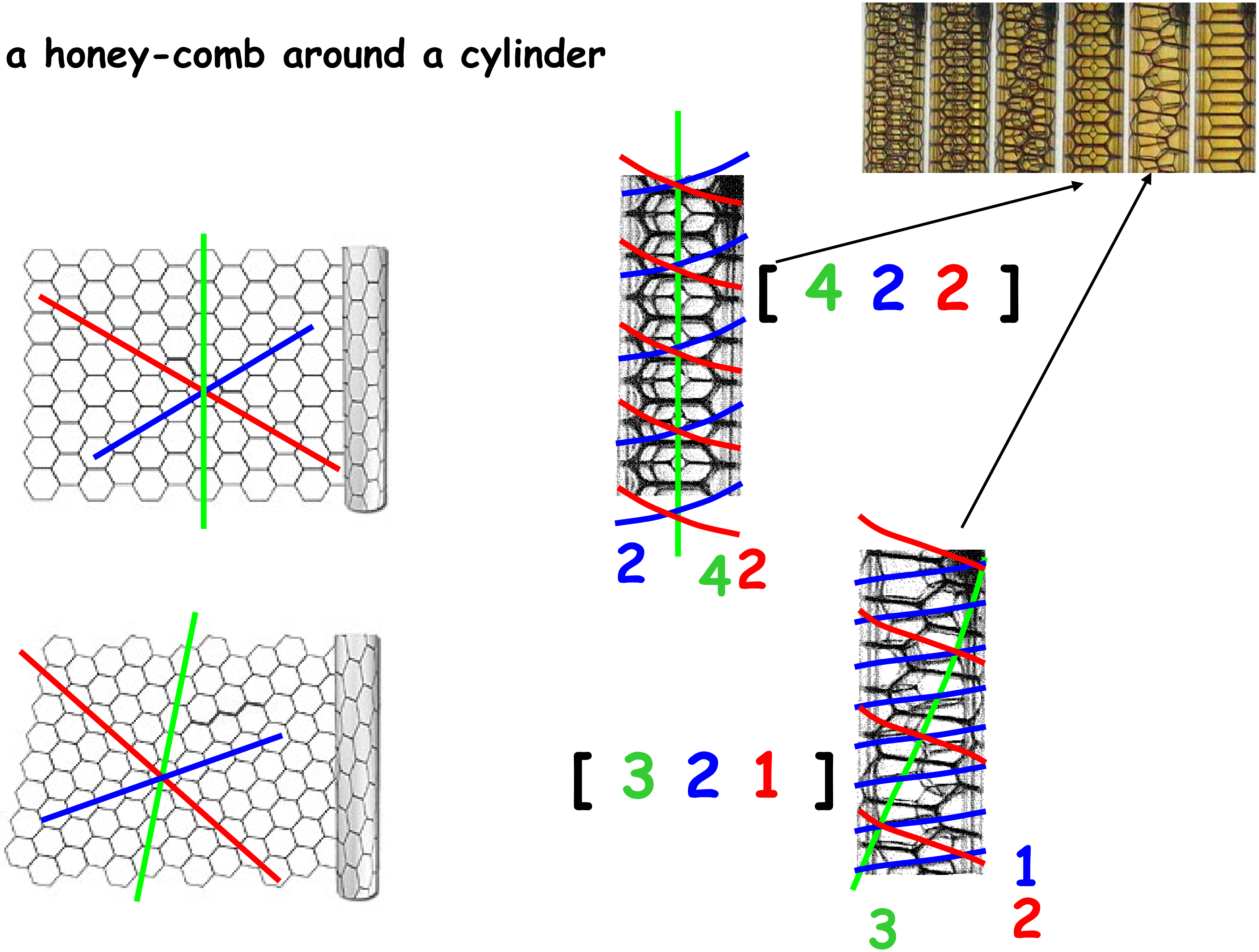


Mono-disperse bubbles in cylinders
Small ratio bubble diameter / cylinder diameter

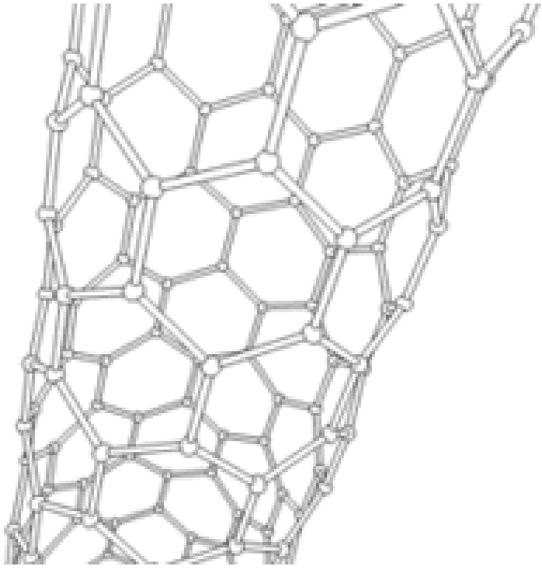


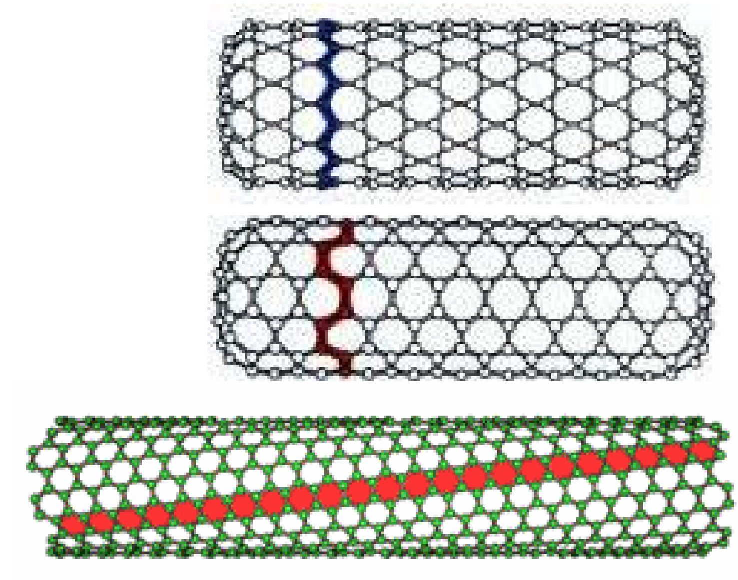
See Antje's tutorial

Wrapping a honey-comb around a cylinder

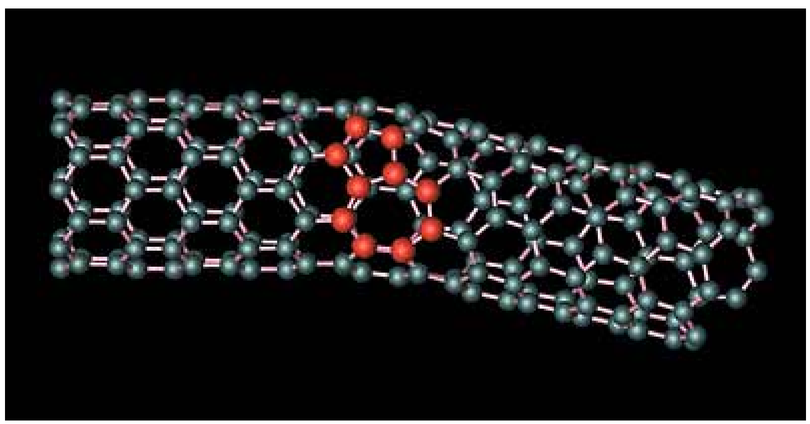


[i j k] – ALWAYS consecutive numbers of the FIBONACCI SEQUENCE





Analogy to carbon nano-tubes
Depending on the “wrapping” either conductor or insulator



(5,7) defect

Joining different types introduces defects like in 2D-foams with very interesting properties (i.e. make diodes)

CONCLUSION

A. Necessary condition for equilibrium self-organised patterns

- *Short-range repulsive* interaction,
- *Middle-range attractive* interaction,
- *Long-range repulsive* interaction.

Patterns = metastable states ; importance of pattern formation

B. Hexagonal patterns in a field -> Conformal transformation

Foams as a model system for natural arrangements

Thank you ...

MSC Paris 7

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TCD Dublin

Wiebke DRENCKHAN