are **liquid** **foams** solid?

QuickTime™ et un
décompresseur TIFF (LZW)
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Introduction to the rheology of disordered foams

after Andrew

rheology of complex fluids

before Sylvie

foam rheology

instead of Isabelle

maternity

Plan

- **motivations**
- **triple behaviour**
- **deformation**
- **stress**
- **synthesis ?**

bonus : 6 minutes movie

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- **motivations**
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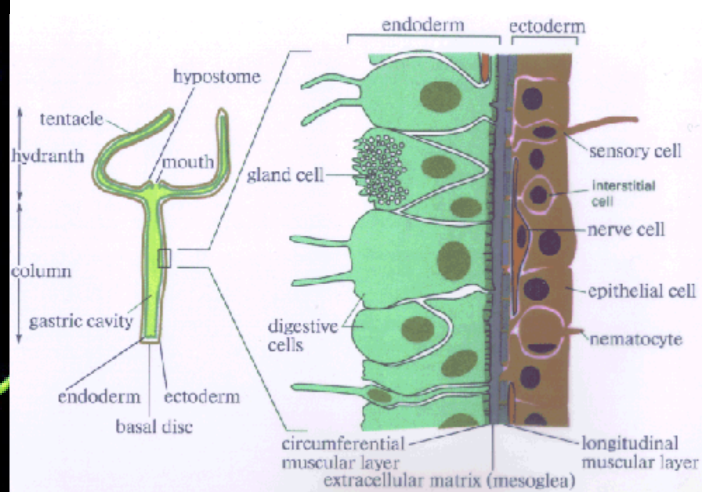
REARRANGEMENTS OF HYDRA CELLS

see also Jos' talk

Hydra



inner cells **skin**

cut into pieces
dissociate and mix all cells
it regenerates

1st day of regeneration
aggregate rounding
cell sorting

QuickTime™ et un
décompresseur TIFF (LZW)
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aggregate rounding
due to adhesion

cell sorting
due to adhesion
differences
(Steinberg)

Glazier & Graner

QuickTime™ et un
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sont requis pour visionner cette image.

Simulate (Potts)

- final state
  like liquids
  except that it
  can be stuck

- sequence
  of events
  determined
  by T1s

- quantitative
  measurements
  log not
  understood

2D foams

foams are models
for complex materials

2D foams are probably
both simple and rich :

- **triple behaviour** — elastic, plastic, fluid
- **imaging micro & macro** — individual object = bubble
- **no need of tracers** — pressure, velocity, deformation

A QUESTION

there are many others...

What dominates the macroscopic behaviour?

averages

continuous medium
like a solid or a liquid

stress and strain fields

a constitutive equation

fluctuations

localisation
in space
or time

answer will vary
with quantities — velocity, stress
or materials — foams, grains

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ıpresseur TIFF (LZW)
›ur visionner cette image.

Apparent behaviour

e.g. shaving foam
egg white

Photo : C. Goldenberg
Les Houches foam school

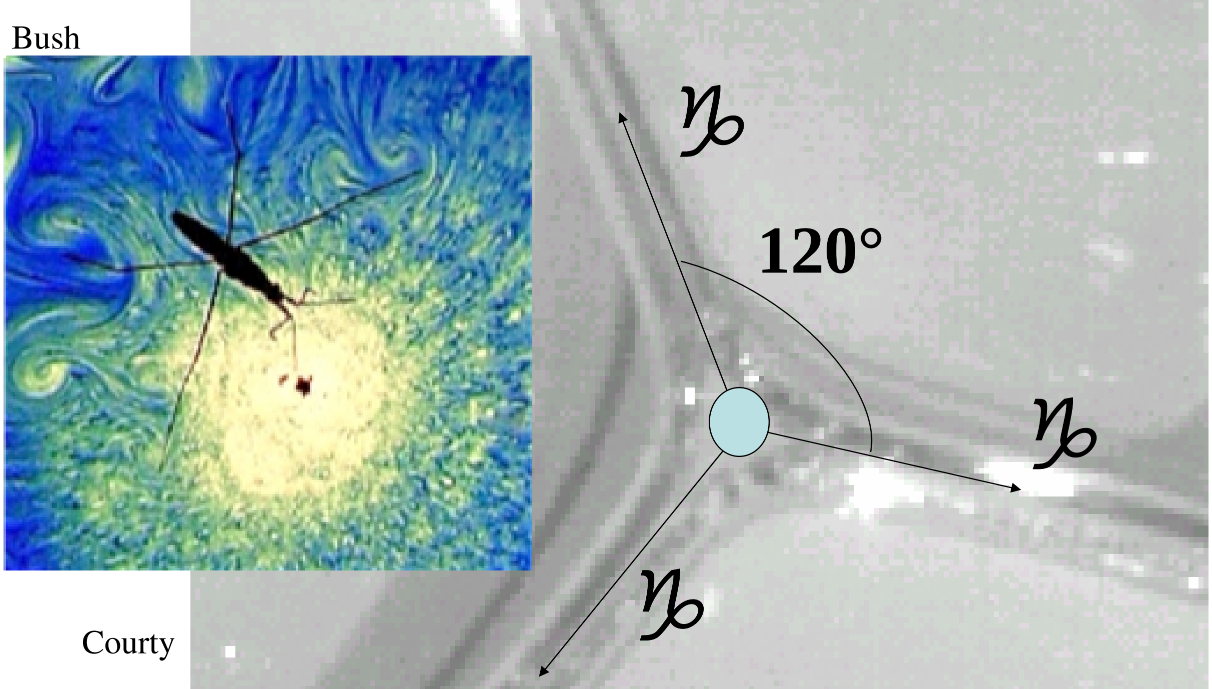
keeps a shape elastic solid

sculpt it plastic solid

flows under shear viscous liquid

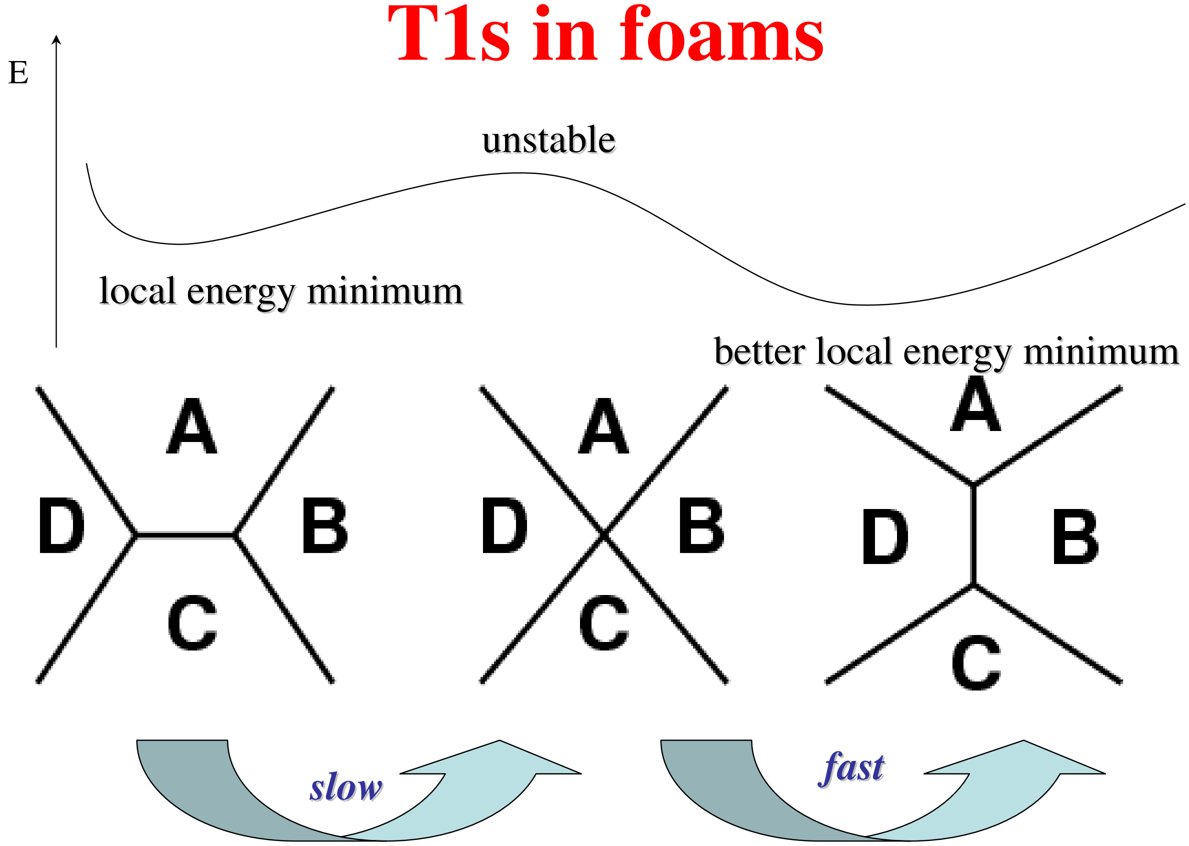
ENERGY MINIMA AND T1s

Bubble walls are under tension



bubbles tend to minimise their perimeter- see Rita

T1s in foams



Artificial rearrangements

F. Elias, C. Flament, J. A. Glazier, F. Graner, Y. Jiang, *Philos. Mag. B* 79, 729 (1999).

from a metastable state...

...to a more stable one...

QuickTime™ et un décompresseur
sont requis pour visualiser
cette image.

QuickTime™ et un décompresseur
sont requis pour visualiser
cette image.

...and back...

PHYSICAL DESCRIPTION

Elastic

- physical origin — energy minimum
- deformed bubble — back to its initial shape
- restoring force — due to surface tension
- reversible — not dissipative
- like an elastic solid — made of a liquid + a gas

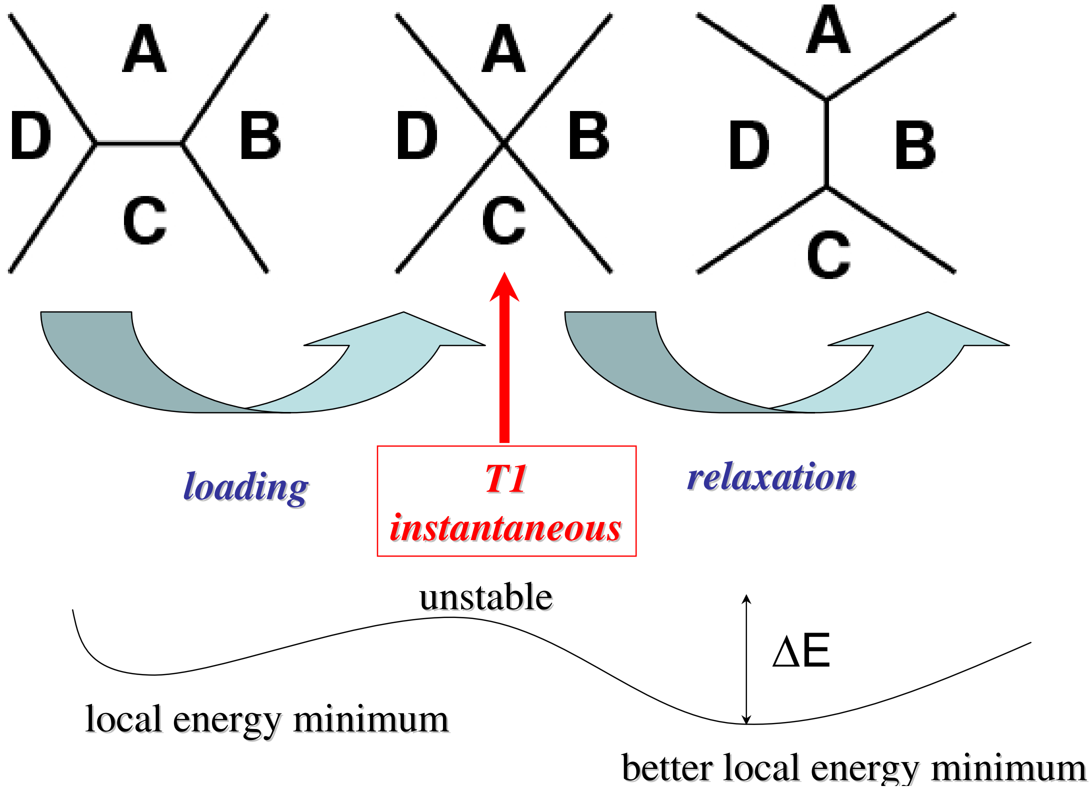
visco-elastic like polymers — *but plastic too ?*

Plastic ?

- physical origin — T1 change of topology
- a bubble falls in the next minimum
- irreversibly stays there
- at a T1 the energy remains continuous
- by itself, a T1 does not dissipate energy
- during the fast relaxation after a T1,
  the viscosity dissipates the energy barrier ΔE

  see Simon Cox

T1s in foams



Plastic !

- rate of T1s ~ grad$v$
- dissipation power P_{diss} ~ ΔE grad$v$
- friction force independent of v

  F_{diss} ~ constant in average
- solid friction

new shape

sculpt it irreversibly
+ solid friction
= plastic solid

Cascade of rearrangements

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cette image.

Do you recognize whose hand it is ?

Fluid

- physical origin viscosity in films and interfaces
- opposes velocity gradients
- dissipative power $P_{diss} \sim (\text{grad } v)^2$
- dissipative force $F_{diss} \sim \text{grad } v$

viscous fluid

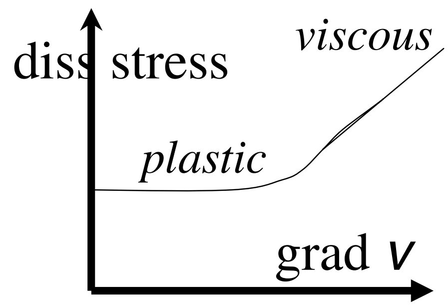
!!! in 2D foam the dissipation is usually external **!!!**
friction on the wall *V* rather than grad *V*
see Nicolai's lecture and Kapil's talk

Transitions between these

these three contribute to the total stress

- small deformation, low velocity, not many T1s
elasticity *dominates*

- many T1s,
quasistatic limit,
succession of equilibria
relaxations well separated
in time and space
plasticity *dominates*



-high velocity
viscous dissipation *dominates*

2D FLOW EXPERIMENTS

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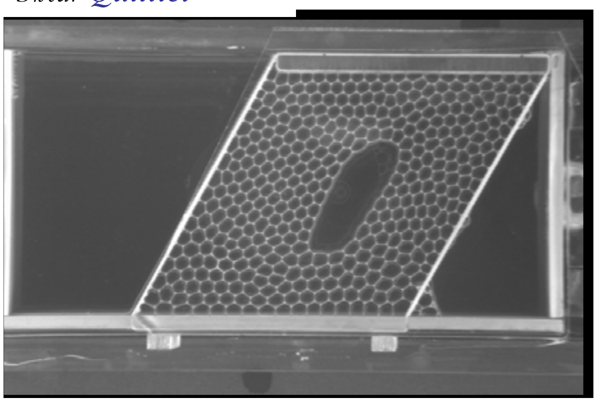
QuickTime™ et un décompresseur sont requis pour visionner cette image.

Constriction *Asipauskas* *Stokes* *Raufaste*

Shear *Quilliet*



Couette *Debrégeas*

QuickTime™ et un décompresseur codec YUV420 sont requis pour visualiser cette image.

Bubble shape model

derived from Potts model (J.A. Glazier)

QuickTime™ et un décompresseur GIF sont requis pour visionner cette image.

Correct bubble shapes
Quantitative measurements
Accounts for bubble shape fluctuations

Quick (lattice model)
Good statistics
2 or 3 dimensions
Statics or quasistatics

Flexible : can include many biological phenomena
see Jos' talk

Y. Jiang (Los Alamos)
C. Raufaste, S. Thomas

MEASUREMENTS

Aphrodynamics

which measurements ?

force = global

yield drag and effective viscosity

not intrinsic

it is sensitive to fluid fraction and geometry

geometry = local

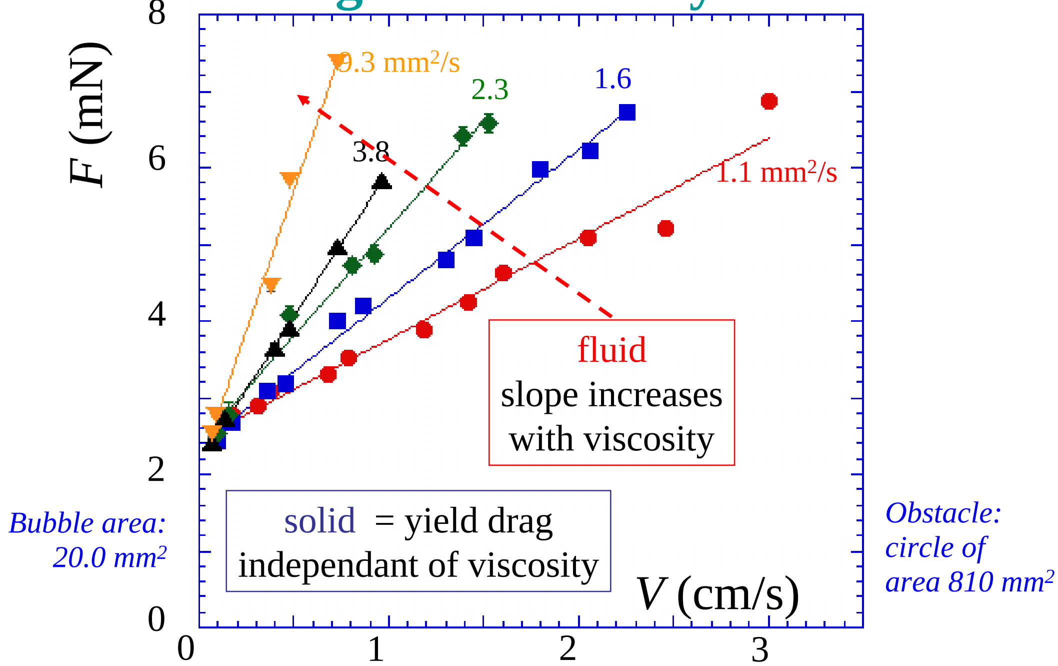
scalar	pressure, elastic energy, number of sides
vector	velocity
tensor	deformation, stress, T1

Force on a circular obstacle

Dollet

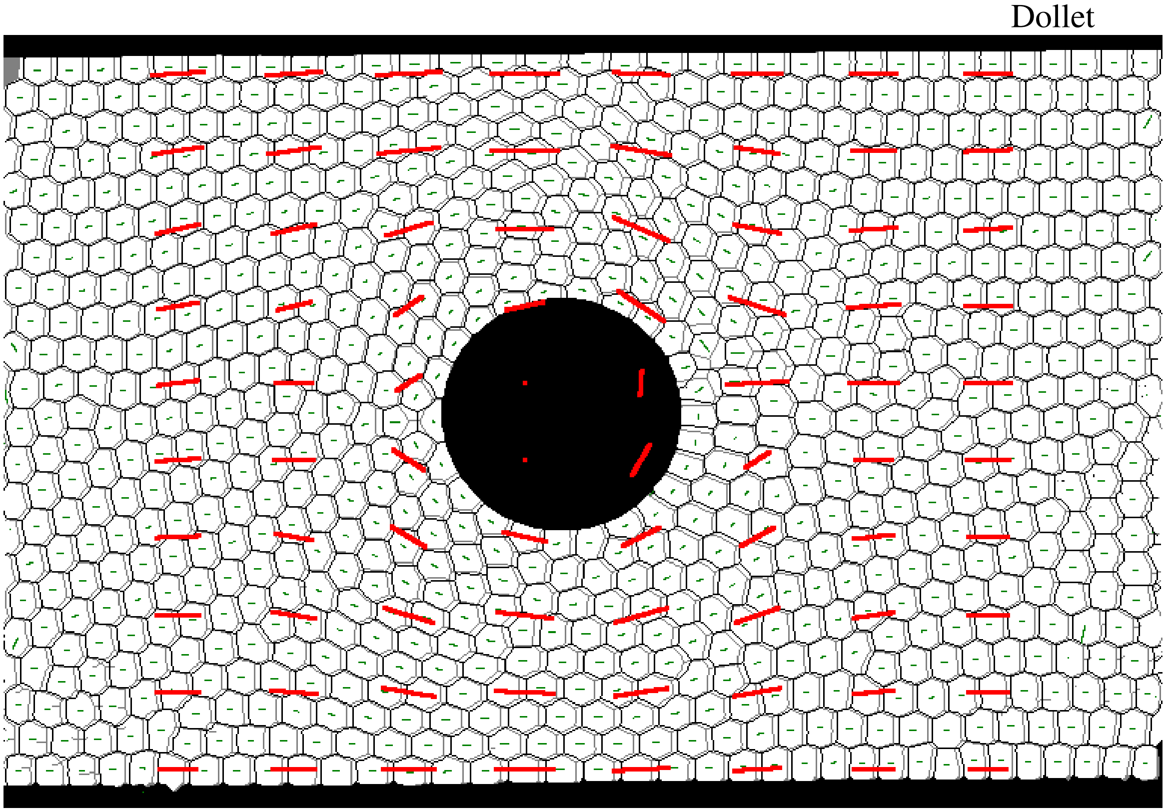
Global measurements

drag *versus* velocity



VELOCITY FIELD

Dollet



QuickTime™ et un décompresseur
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cette image.

M. Asipauskas

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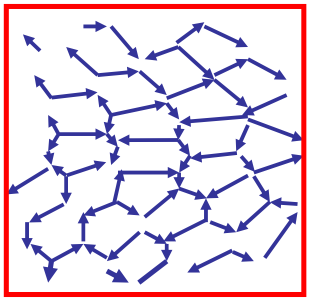
TEXTURE

M. Aubouy, Y. Jiang, J. A. Glazier, F. Graner, *Granular Matter*, 5, 67 (2003).

Texture tensor: definition

R.V.E.



link between bubbles = a vector $\vec{\ell}$

$$\overline{\overline{M}} = \langle \vec{\ell} \otimes \vec{\ell} \rangle = \begin{pmatrix} \langle \ell_x^2 \rangle & \langle \ell_y \ell_x \rangle & \dots \\ \langle \ell_x \ell_y \rangle & \langle \ell_y^2 \rangle & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

$$= \langle \vec{\ell}\, \vec{\ell}^T \rangle = \langle \ell_i \ell_j \rangle$$

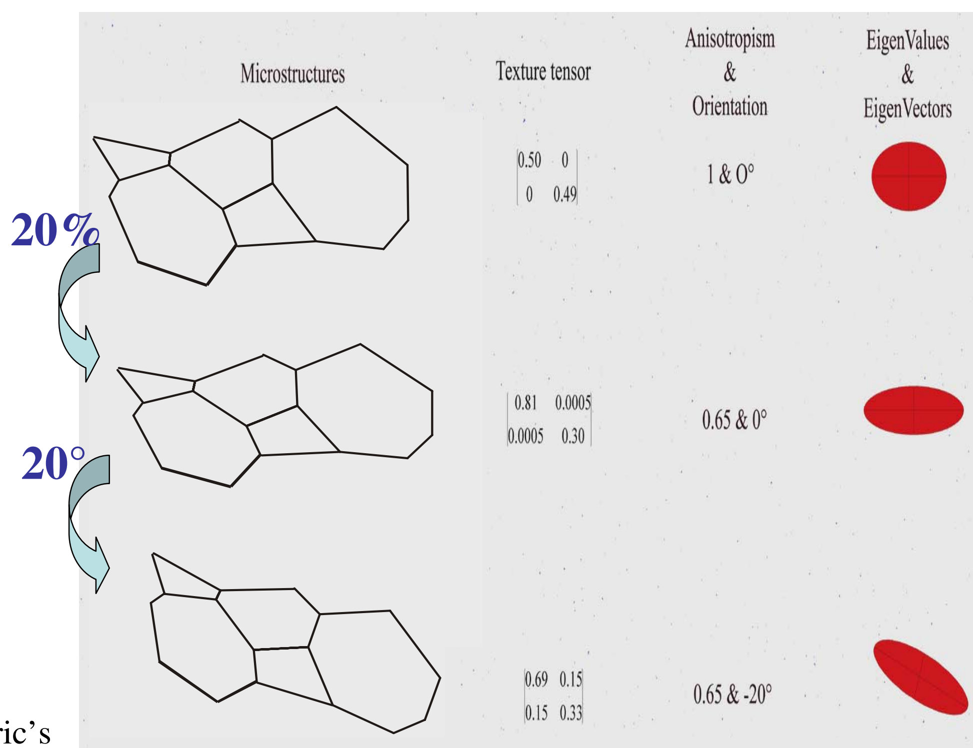
real, **symmetrical**
translation, rotation,
index permutation

insensitive to the
arbitrary orientation
chosen for the vectors

texture tensor, for granular materials
order parameter for nematic polymers
de Gennes
Terentjev
fabric tensor, **conformational** tensor...

general : a state function of the material
can be used at all scales: micro, meso, macroscopic

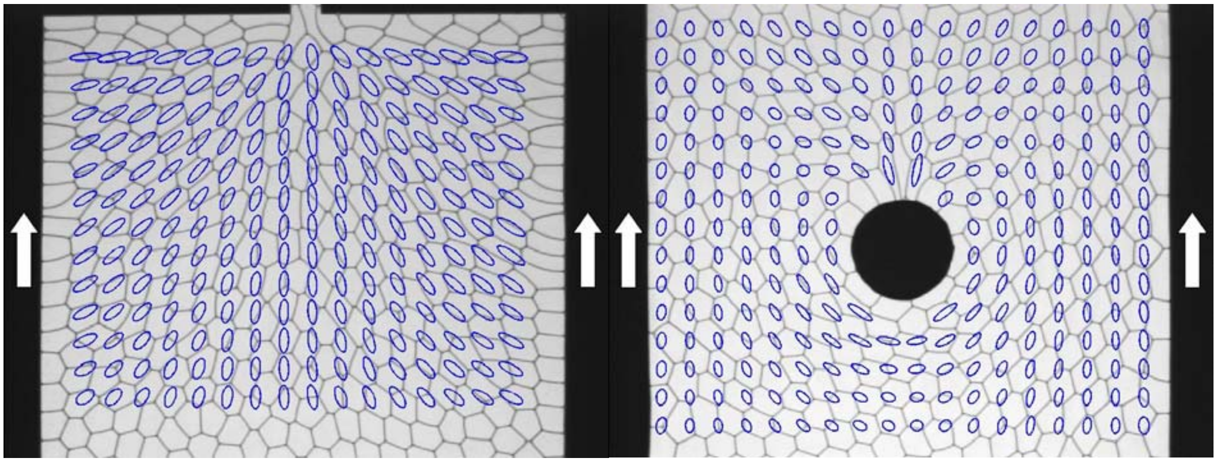
Texture tensor: representation



Information = size, direction, anisotropy

see Eric's demo?

Texture tensor field



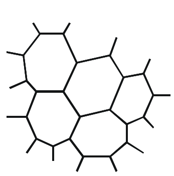
$\overline{\overline{M}}(\vec{r})$ averaged over 3000 images

continuous !

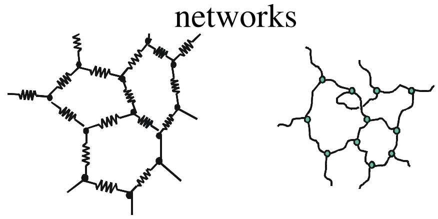
M. Asipauskas, M. Aubouy, J.A. Glazier, F. Graner, Y. Jiang
Eufoam (2000), and *Granular Matter,* 5, 71 (2003).

3D & 2D (rearranging) patterns

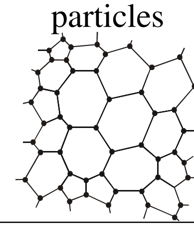
cells



networks



particles



	cells	networks	particles
	foam *biological tissue* *microstructure of crystals* *dense grains*	*non-covalent gels* *rearranging springs*	*atoms* *molecules* *colloids* *loose grains*
site	center	node	center
link	neighbor	connection	Voronoi
	2D : duality = choice		

ELASTIC STRAIN

M. Aubouy, Y. Jiang, J. A. Glazier, F. Graner, *Granular Matter*, 5, 67 (2003).

Strain : elastic & plastic

applied strain = **elastic** strain + **plastic** strain

applied strain → *(grad v) t*

elastic strain → current state; *reversibly stored deformation*; *constant in steady flow*

plastic strain → increases steadily with time; *much larger*

Operational definition of elastic strain

$$\overline{\overline{U}} := \frac{1}{2}\left[\mathrm{Log}\,\overline{\overline{M}} - \mathrm{Log}\,\overline{\overline{M}}_0\right]$$

coincides with classical definitions
when applicable

if M, M_0 co-diagonal

$$\begin{pmatrix} \mathrm{Log}\sqrt{\frac{\lambda_1}{\lambda_1^0}} & 0 & 0 \\ 0 & \mathrm{Log}\sqrt{\frac{\lambda_2}{\lambda_2^0}} & 0 \\ 0 & 0 & \mathrm{Log}\sqrt{\frac{\lambda_3}{\lambda_3^0}} \end{pmatrix}$$

eigenvalues of M **> 0**
U always defined

dimensionless function of M and M_0

U has same **symmetries** as M and M_0
translation, rotation, index permutation

no division for tensors
(here M_0^{-1} in linear limit)

M. Aubouy, Y. Jiang, J. A. Glazier, F. Graner, *Granular Matter*, 5, 67 (2003).

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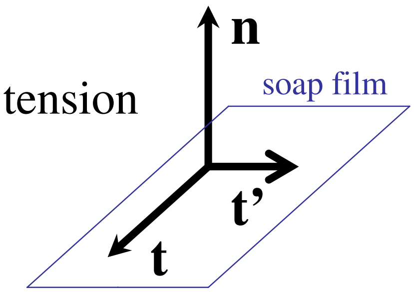
STRESS

Contributions to stress

Dissipative : see above
Elastic : gas pressure + liquid wall tension



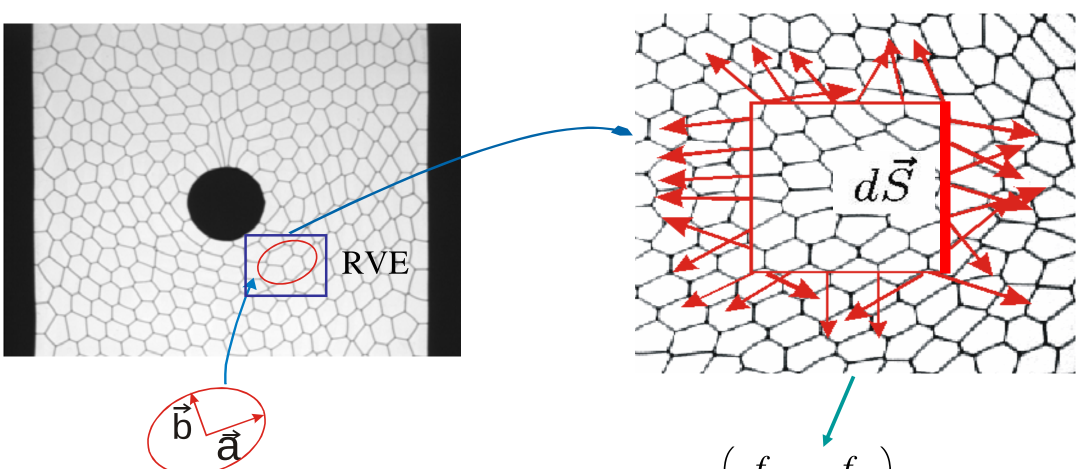
$$\left(\,{}^{\gamma}\gamma_{\;0}\right) = \gamma\,(t\otimes t + t'\otimes t')$$
$$= \gamma\,(I - n\otimes n)$$

Elastic ~ - p I + γ ∑ ∫ dS (I - n⊗n)
Dissipative ~ Young + viscosity gradV

viscosity / Young $= t_{relax}$
transition when t_{relax} grad V ~1

no hidden variable - no indetermination

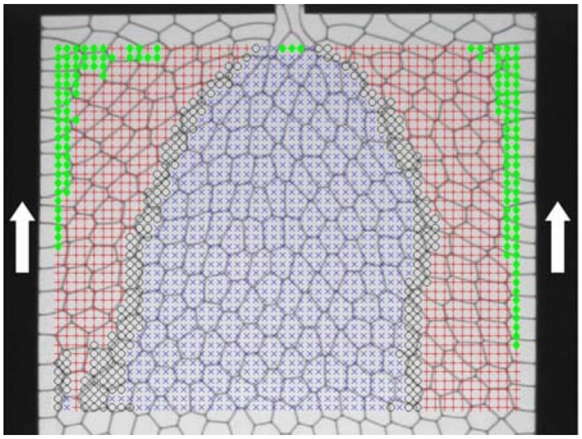
(network) stress: measurement

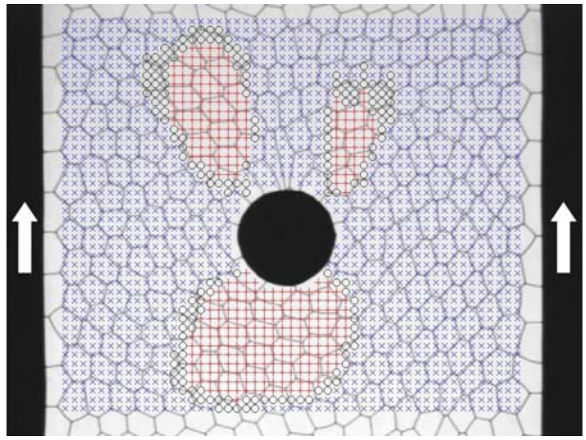


$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \longleftarrow \quad \overline{\overline{\sigma}} = \begin{pmatrix} \dfrac{f_x}{\mathrm{d}S_x} & \dfrac{f_x}{\mathrm{d}S_y} \\ \dfrac{f_y}{\mathrm{d}S_x} & \dfrac{f_y}{\mathrm{d}S_y} \end{pmatrix}$$

$\overline{\overline{\sigma}} \approx \rho \left\langle \vec{f} \otimes \vec{\ell} \right\rangle$ depends on specific physics
forces, density of links, dimension of space
independent on M except for polymers $f \approx l$

(network) stress: tensor field





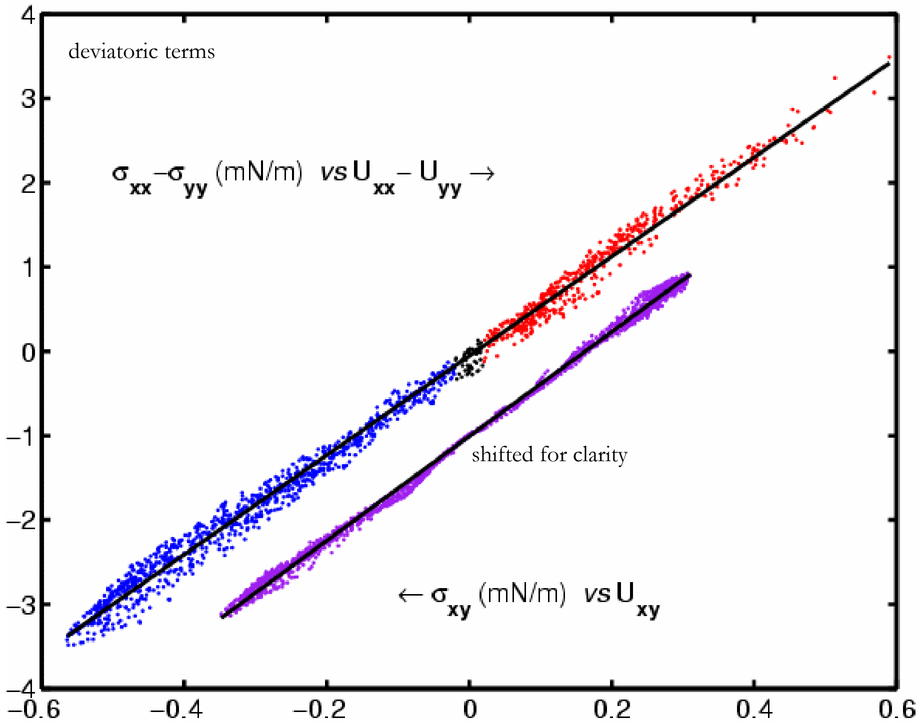
$$\sigma_{xx} - \sigma_{yy} \begin{cases} \text{positive} \\ \text{zero, within errors} \\ \text{negative} \end{cases}$$

Asipauskas et al.

deviatoric elastic STRAIN - STRESS

Elastic strain-stress relation

measured on a 2D flowing foam *1 RVE = 1 data point*



-continuous
-isotropic
-linear
-slope 2**μ**

Asipauskas et al.

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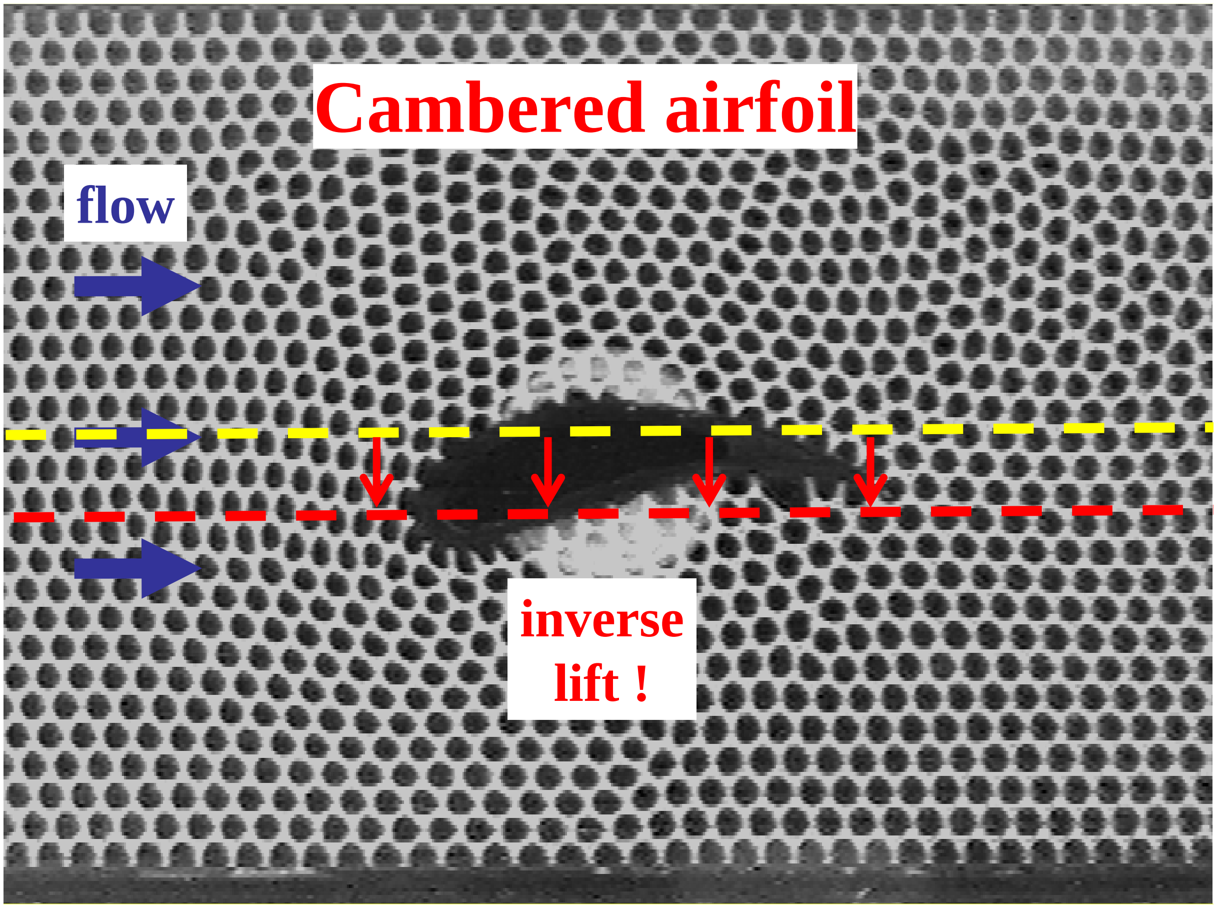
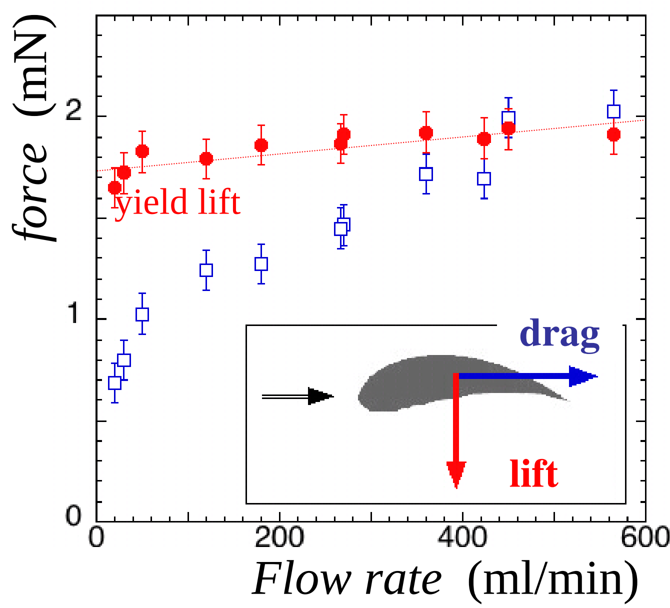
LIFT ?



Lift : global measurements



Lift almost independent of flow rate!

B. Dollet, PRL 2005

QuickTime™ et un
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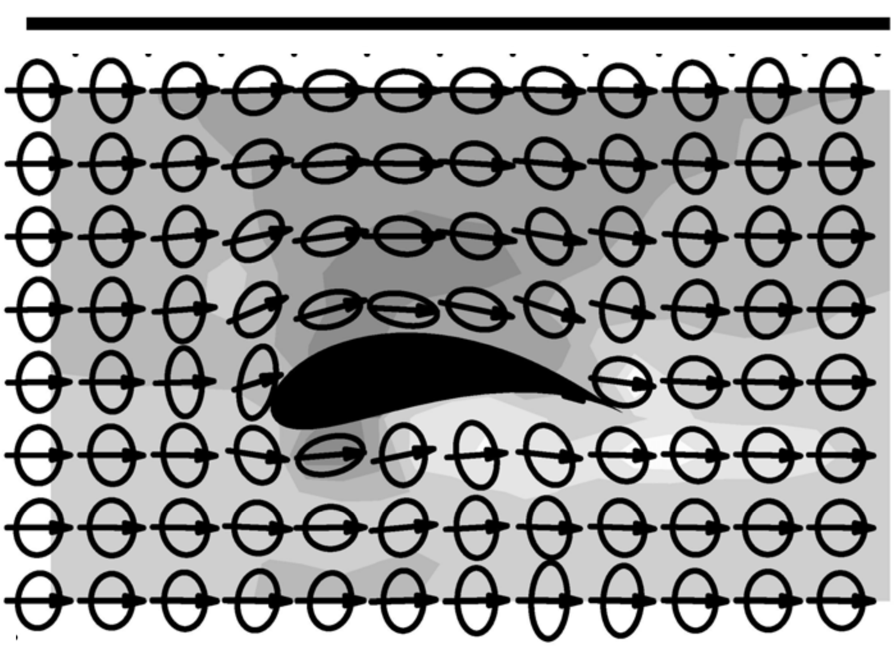
sont requis pour visionner cette image.

Lift : interpretation

qualitatively independent from distance to channel boundaries
based on local measurements



obstacle
⇓
shear, elongate
⇓
stretch, compress
⇓
elastic and
pressure stresses
~ 0.8 mN + 0.8 mN
⇓
downwards lift
~ 1.6 mN

Dollet et al PRL 2005

TOWARDS A CONSTITUTIVE EQUATION ?

continuous

macroscopic

Complex material

I place my camera over a fixed region of the flowing foam :

shear rate = $\frac{d}{dt}$ (**stored strain**) + **T1 rate**

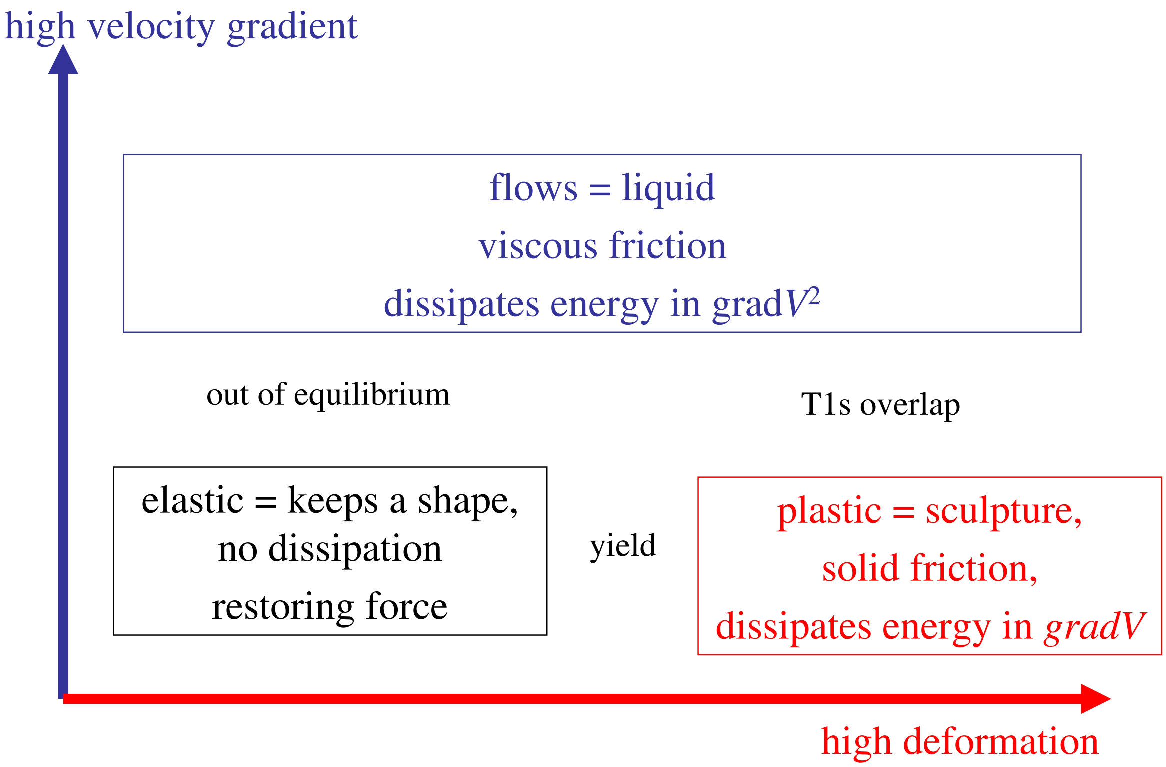
elastic regime **permanent flow** **quasistatic plastic regime**

very general

complex = at least 2 independent variables

viscous stress **elastic stress** **0**

2 independent variables



Map of T1 rearrangements

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décompresseur TIFF (LZW)
sont requis pour visionner cette image.

measurements *P. Marmottant*

creation and
destruction
of sides

predict number & orientation ?

stored strain x velocity gradient

T1: 1 link destroyed, 1 link created

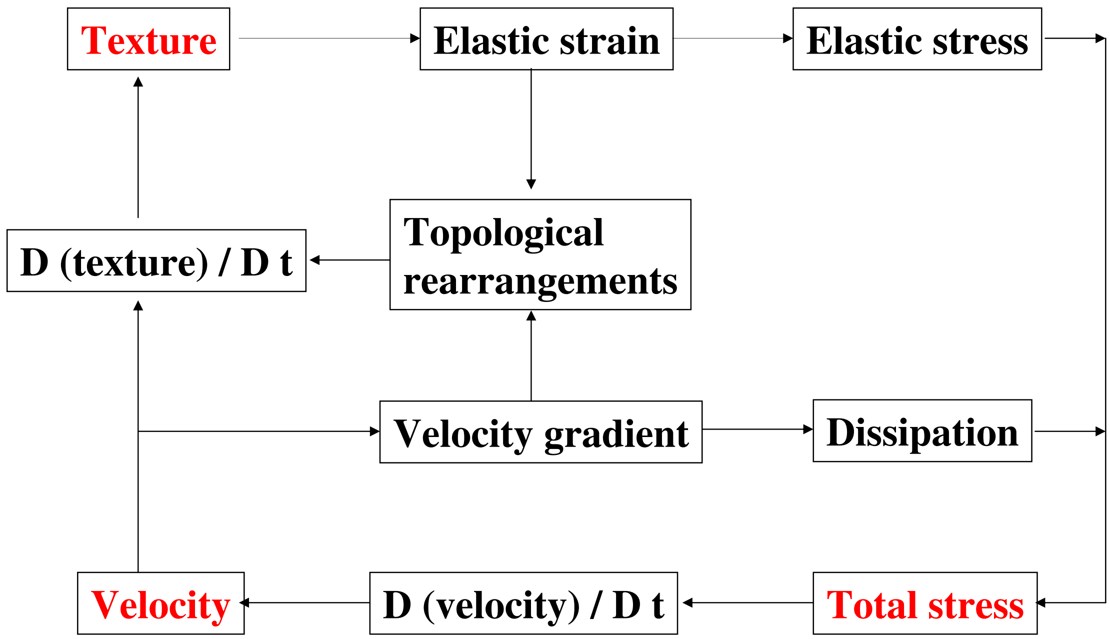
$\vec{l}_o$ link vector

$$\overline{\overline{t}} \equiv \langle \vec{l}_o \otimes \vec{l}_o \rangle = \langle l_{oi} l_{oj} \rangle$$

This toplogical tensor
can be represented as an ellipse
with major/minor axe in the
same direction as eigenvectors
and proportional to eigenvalues

QuickTime™ et un
décompresseur TIFF (LZW)
sont requis pour visionner cette image.

Conjectured equations



closed system of equations

currently testing and solving them

Summary

- mechanics of foams
  - yes, continuous description
  - yes, liquid foams are solid, but not only

- useful tensors
  - texture : applies to many systems
  - elastic strain : stored deformation
  - topological rearrangements : rate of T1s

- future models
  - unify elasticity, plasticity, hydrodynamics
  - back to biological cells ?

a 6 minutes movie...

... a brief overview ...

The solidity of liquid foams

... in french ...

... with english subtitles

realised by Université Joseph-Fourier

sorry, no pop corn