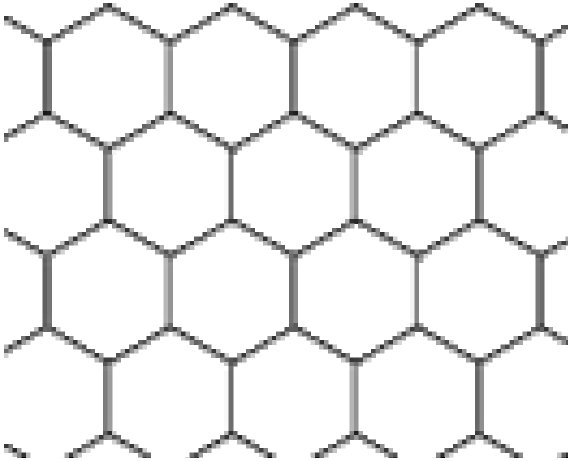
# Solid-like response

Linear viscoelasticity

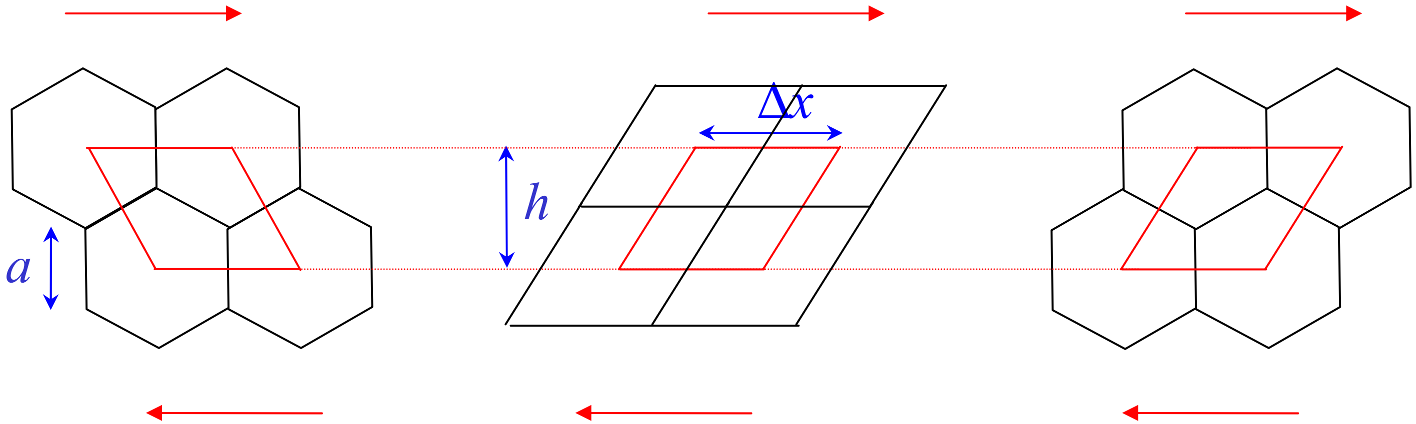
Non linear elasticity

Laboratoire de Physique des Matériaux Divisés et des Interfaces

# *Princen model*

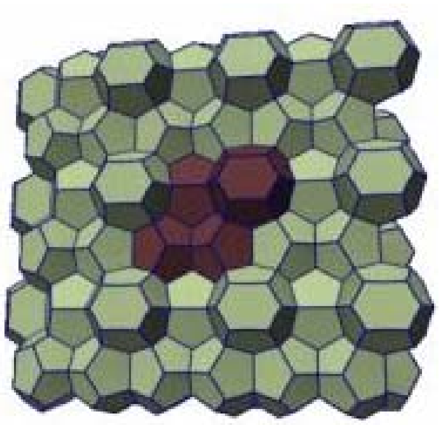


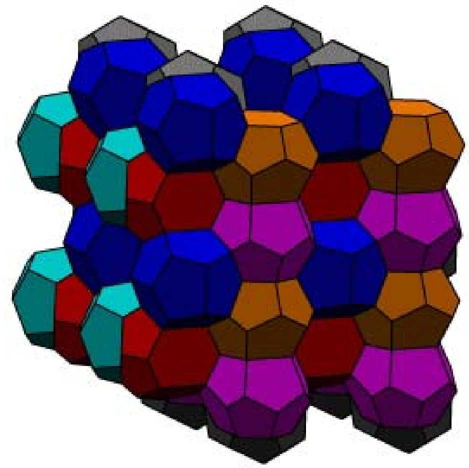
*Vincent-Bonnieu*

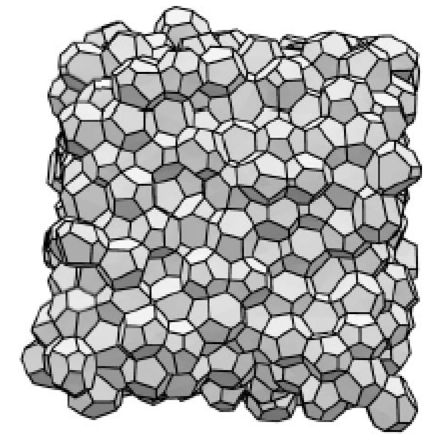



*Princen 1983*

# *3D dry foams*



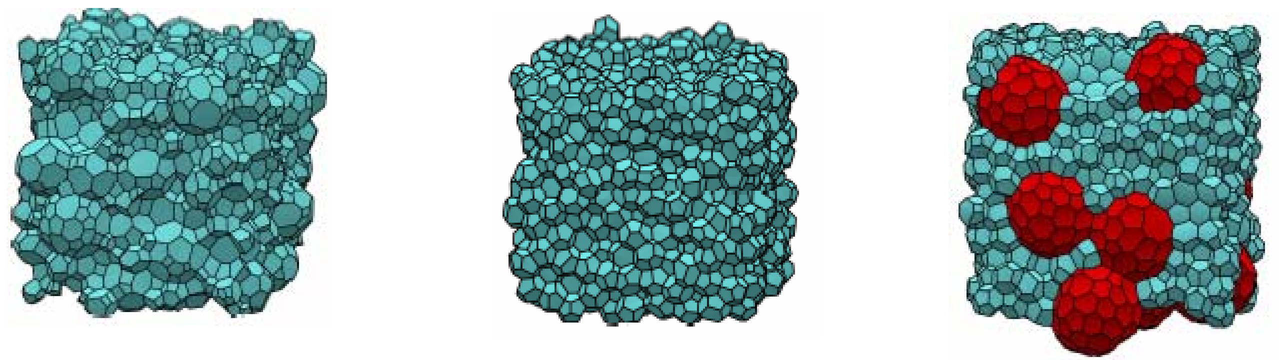




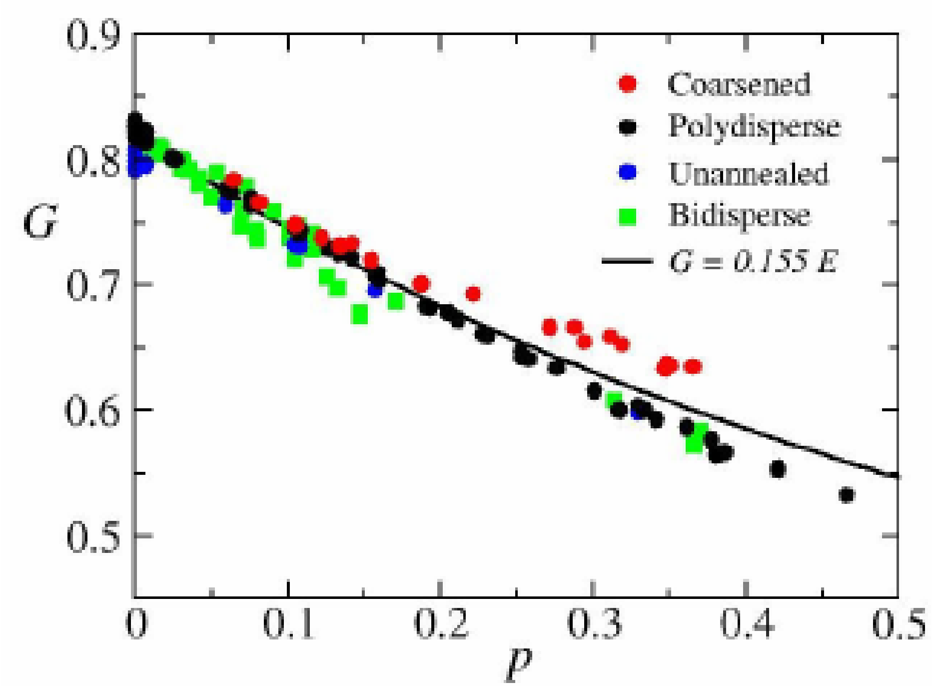
| | R G/T | ΔG/G |
|---|---|---|
| Kelvin | 0.50 | 0.5 |
| Williams | 0.49 | 0.5 |
| Weaire-Phelan | 0.54 | 0.04 |
| Disordered monodisperse | 0.51 | 0 |
| Derjaguin estimation | 0.8 | 0 |

*Derjaguin 1933; Kraynik, Reinelt 1996, 2004*

# *3D dry disordered polydisperse foams*



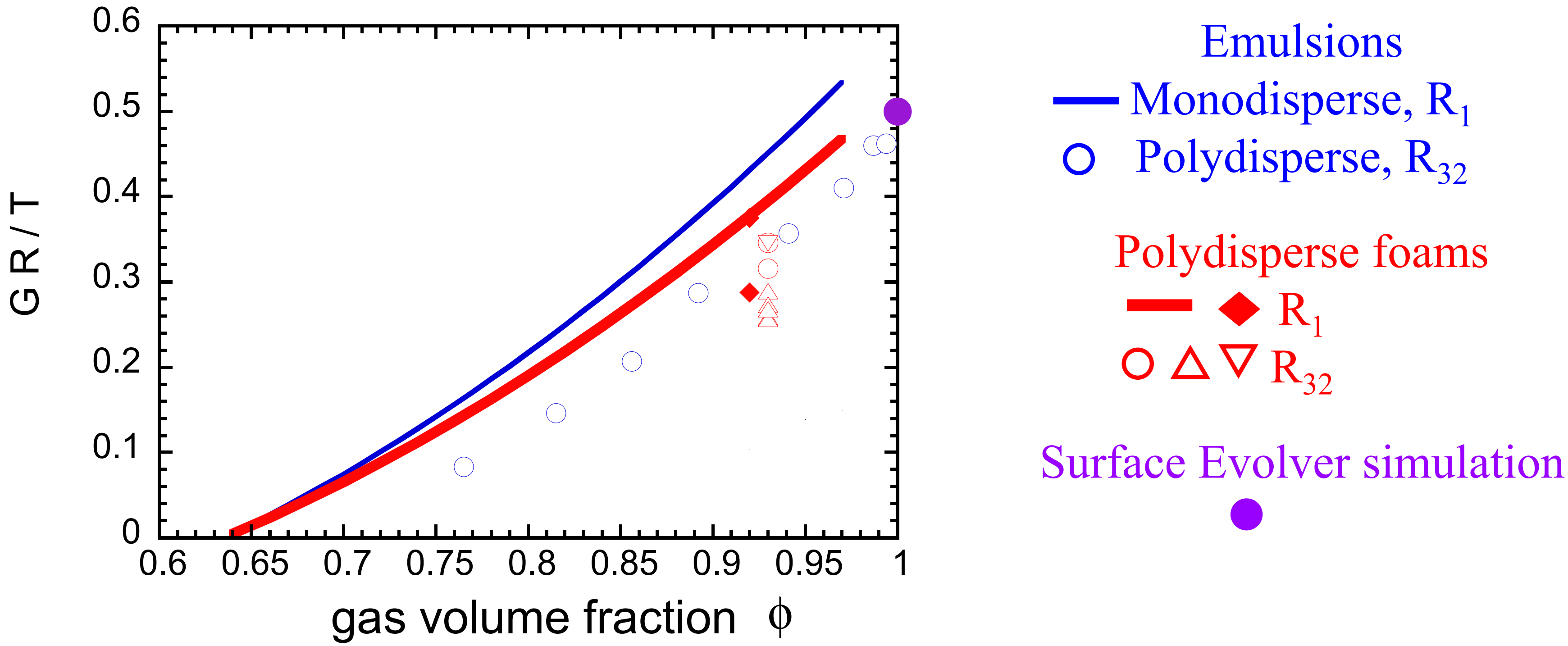
$$p = \frac{R_{32}}{< R^3 >^{1/3}} - 1$$




$$G \cong 0.5 \frac{T}{R_{32}}$$

*Kraynik, Reinelt 2004*

# *Effect of liquid content*



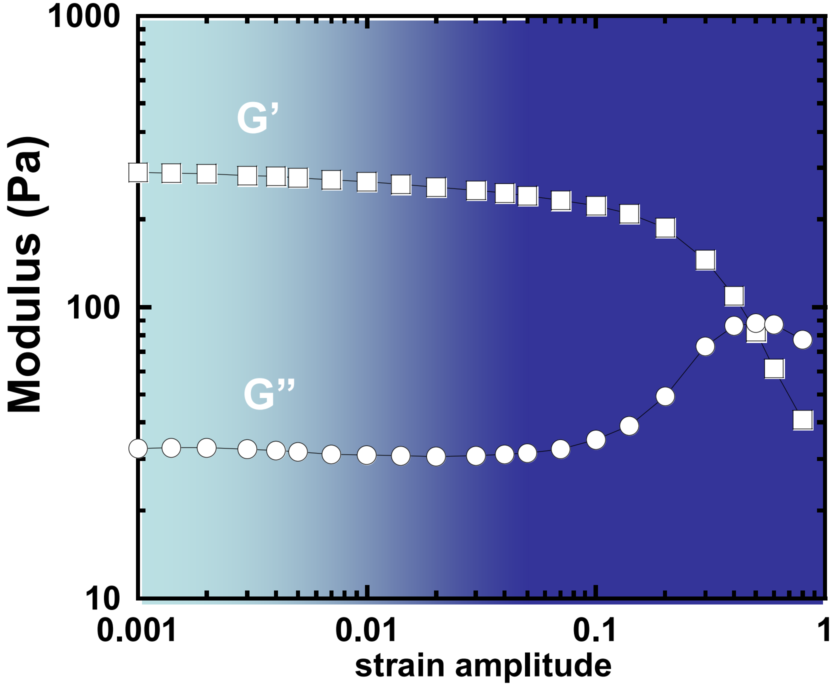

$$G = \alpha \phi (\phi - \phi_c) \frac{T}{R_{32}} \qquad \alpha \cong 1.4$$

*Princen, Kiss 1986; Mason et al 1995; Saint-Jalmes, Durian 1999; Gopal, Durian 1999; Höhler, CA 2005; Kraynik, Reinelt 2004 Bolton, Weaire 1990*

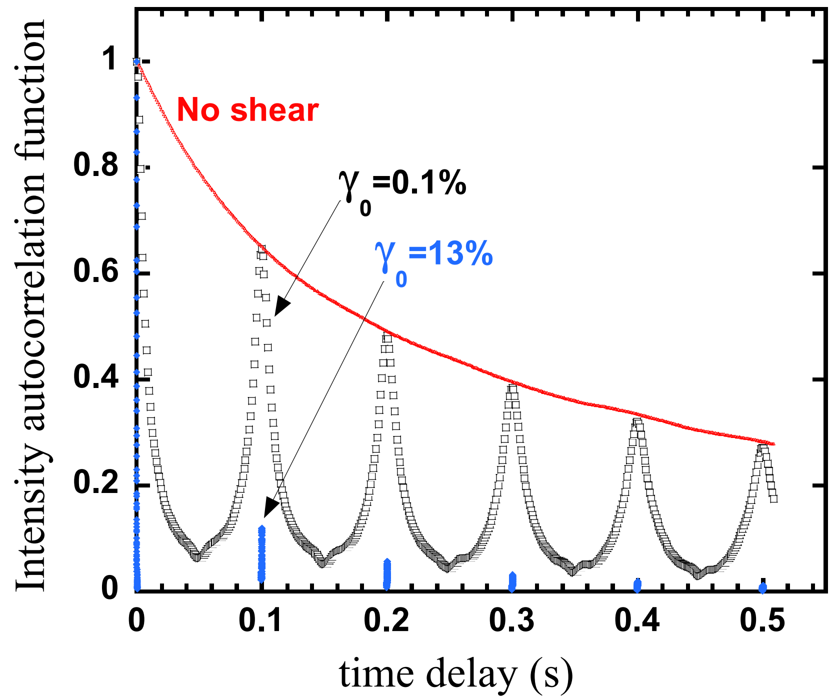
# *Does a linear regime exist ?*

*Weaire, Fortes 1994*

Rheometry

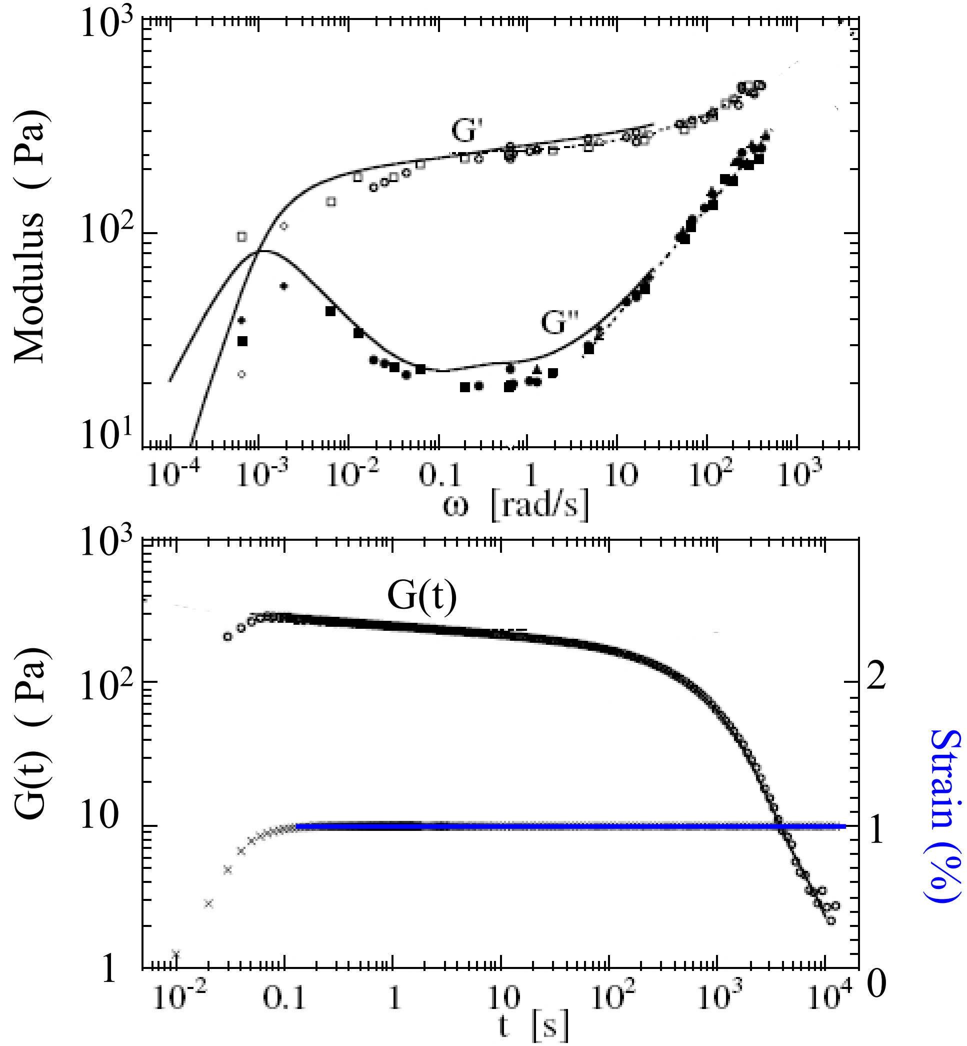



DWS echoes




Gillette foam
Age = 60 min
Frequency = 10 Hz

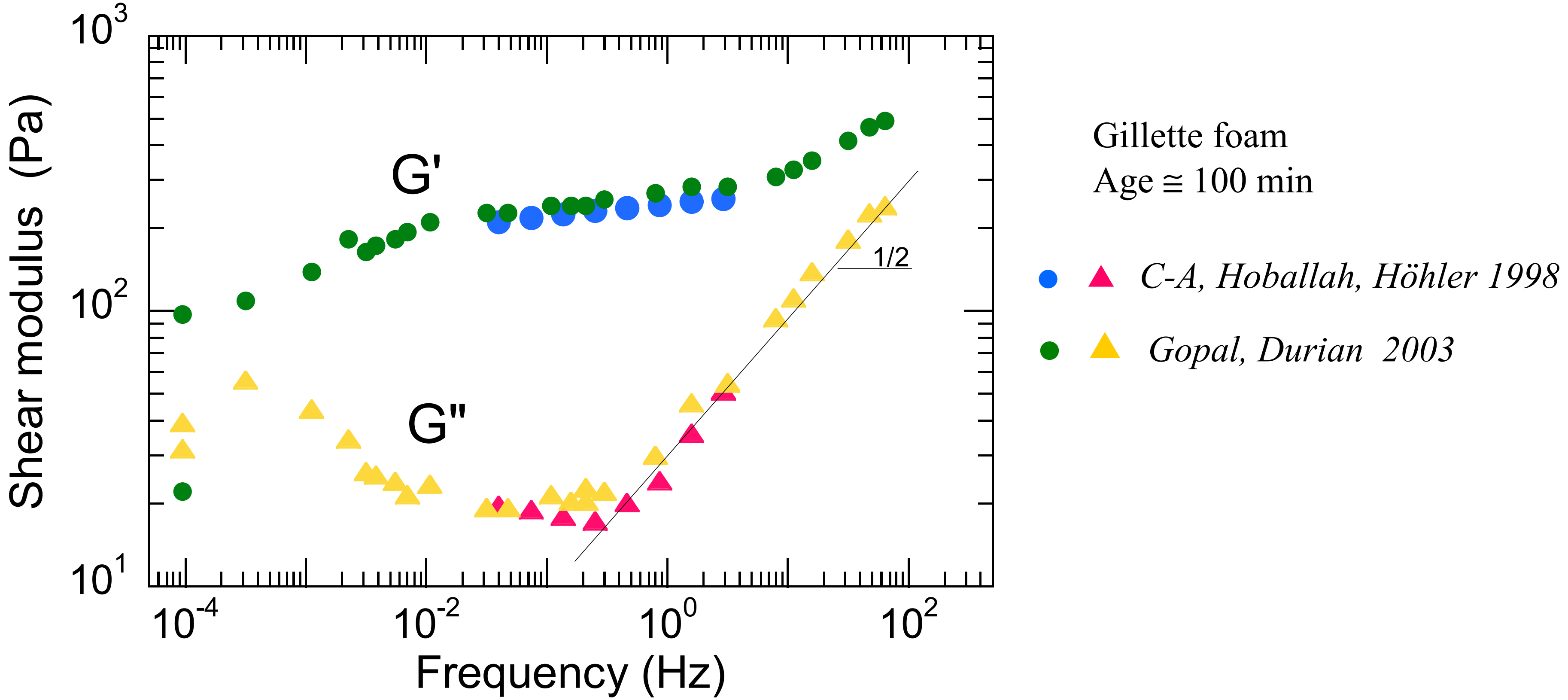
# *Equivalence between temporal and frequency responses*




$$G^*(\omega) = i\omega\,\mathcal{L}[G(t)][i\omega]$$

*Gopal, Durian 2003*

# *Viscoelastic response*



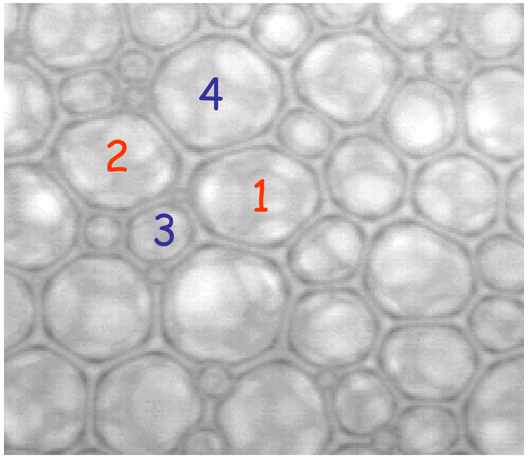

**Dissipation at the film scale**

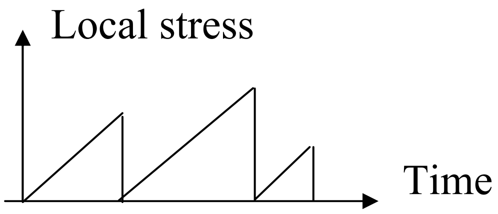
- Fluid transport in films and borders — *Buzza, Lu, Cates, J Phys II 1995*
- Intrinsic surface viscosity of the interfaces — *Edwards, Wasan 1996*

**Dissipation at a mesoscopic scale "Soft Glassy Materials"**

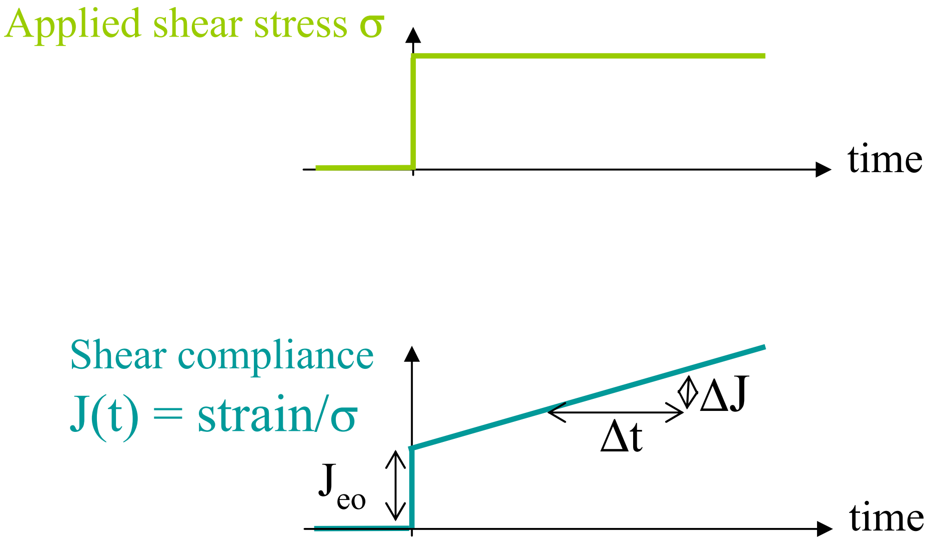
- Irreversible loss of elastic energy upon bubble rearrangements — *Sollich et al PRL 1997*, *Hébraud, Lequeux PRL 1998*

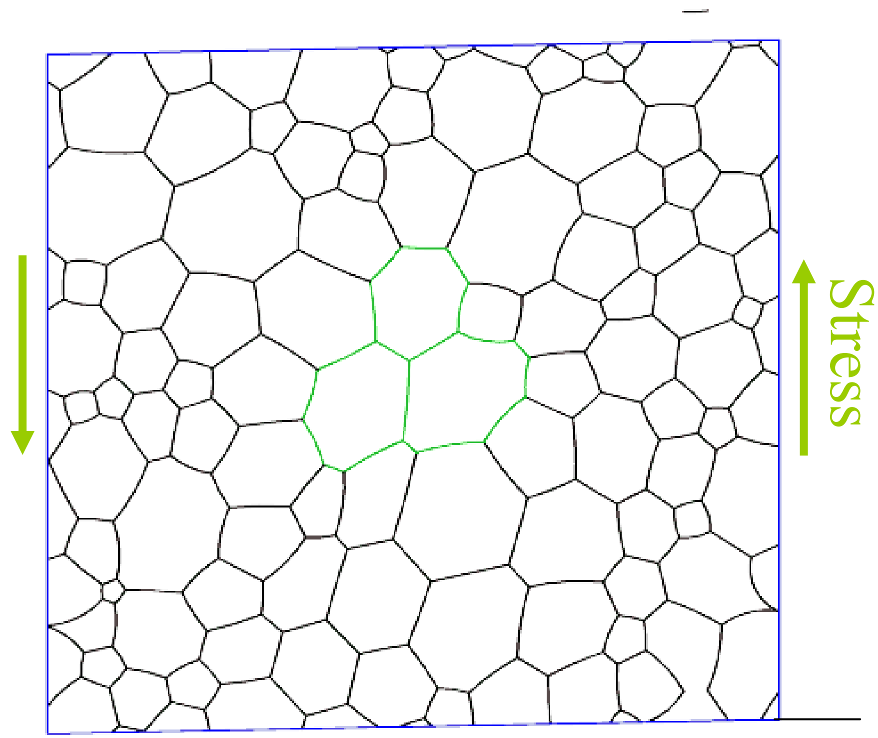
# *Coarsening is an intrinsic source of dynamics in foams*



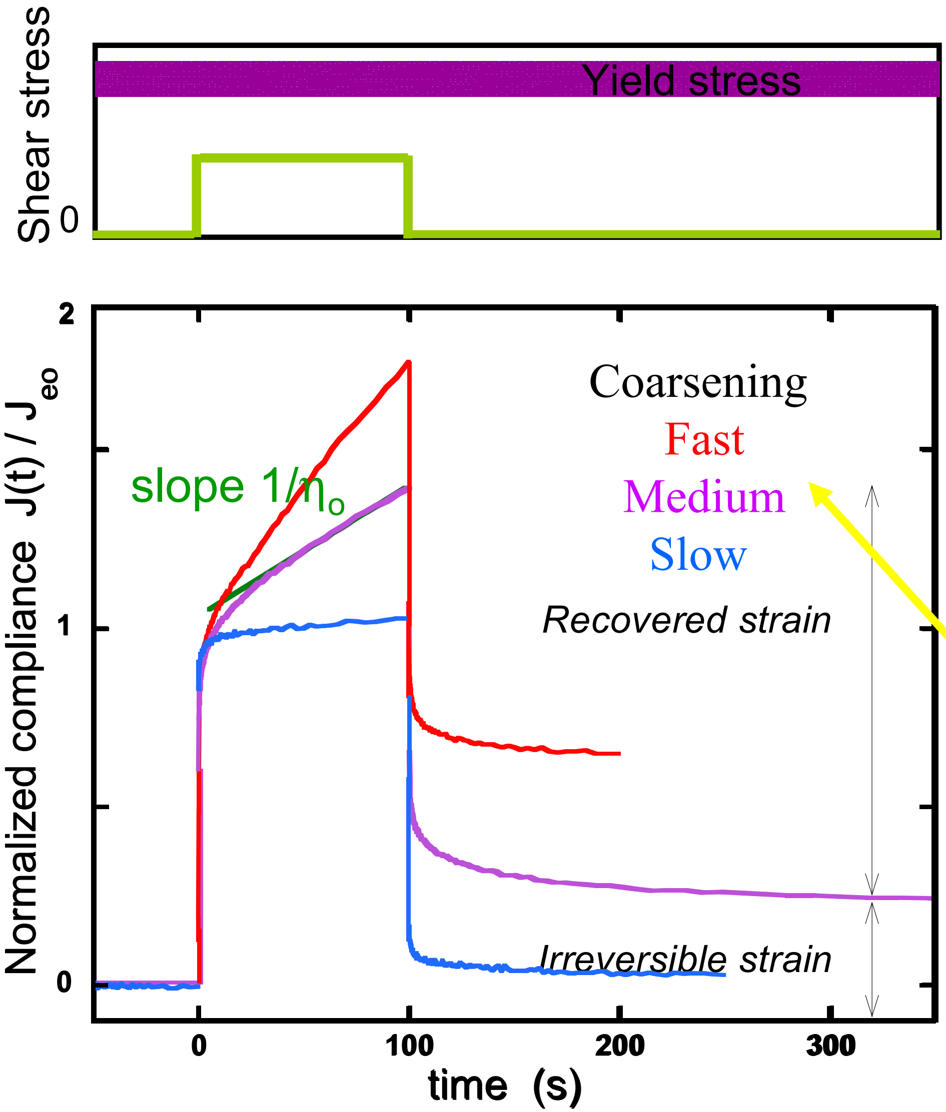
# *A simple mesoscopic model of relaxation due to coarsening-induced rearrangements*



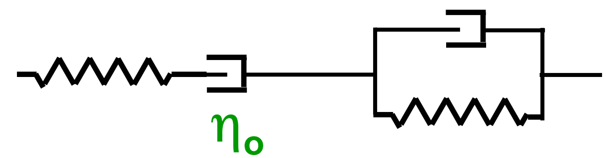

Surface Evolver simulations
S. Vincent-Bonnieu

# *Creep, DWS and DTS experiment*




Only two relaxation processes !

$$J(t) = J_o + \frac{t}{\eta_o} + J_1\left(1 - \exp\left(-\frac{t}{\eta_1 J_1}\right)\right)$$

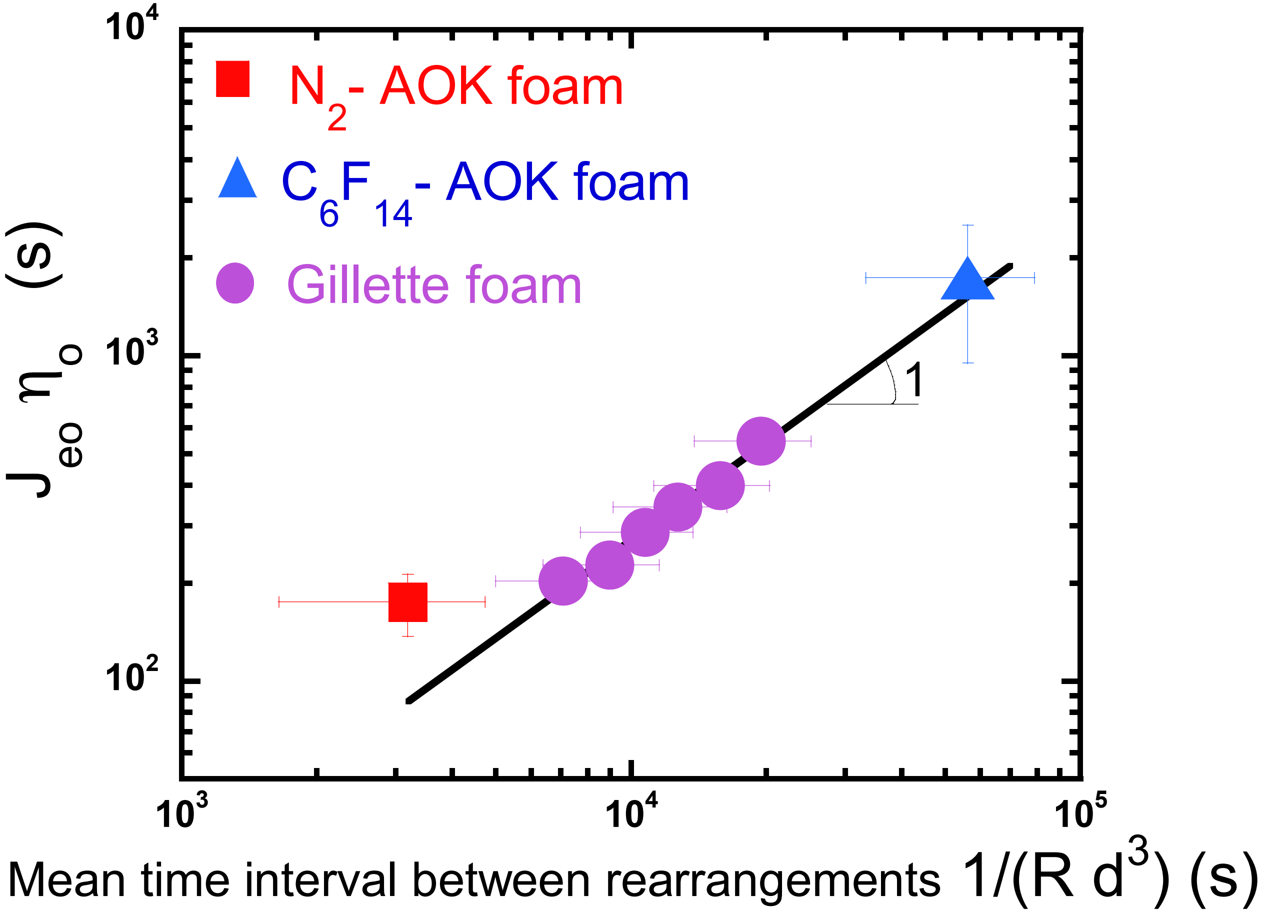


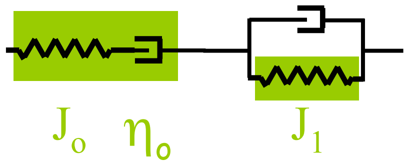

*In agreement with*
*Marze, Saint-Jalmes, Langevin 2005*

Tunable by the physico-chemistry

*C-A, Höhler, Khidas 2004*

# *Coarsening induced rearrangements are at the origin of the steady creep*



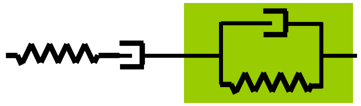

$$J_{eo} = J_o + J_1$$

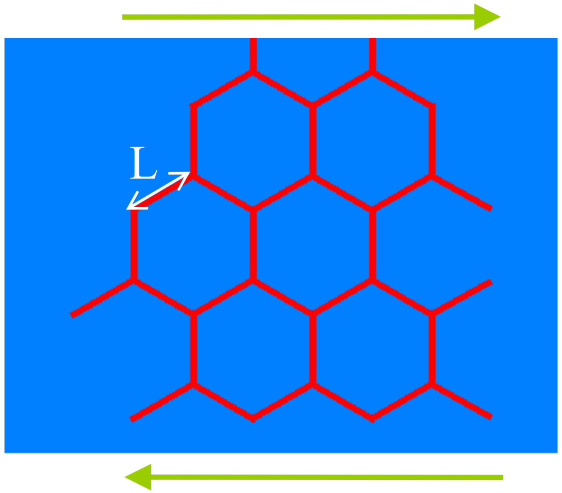
$$J_{eo}\,\eta_o \cong \frac{1}{R\,V}$$

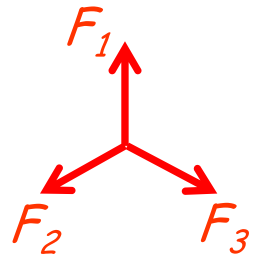
$$V \cong (3\ d)^3$$

# *Foams and other solids can flow on long time scales*

# *Interfacial relaxation*






Forces acting at the film junctions:
*Edwards, Brenner, Wasan 1991*

$$F_i = 2\left(T + \kappa \frac{1}{L}\frac{\partial L}{\partial t}\right)$$

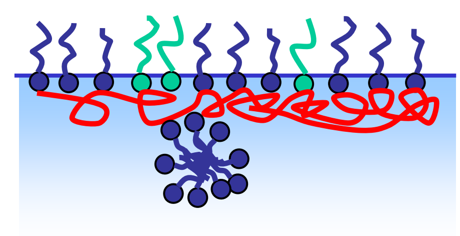
Surface tension (T)

Dilatational surface viscosity ($\kappa$)

Scaling law for the relaxation time, independent of bubble size : $\tau_1 \approx \kappa / T$

$\tau_1 \cong 3 - 5$ s $\rightarrow$ $\kappa \approx 0.05 - 0.15$ kg s$^{-1}$

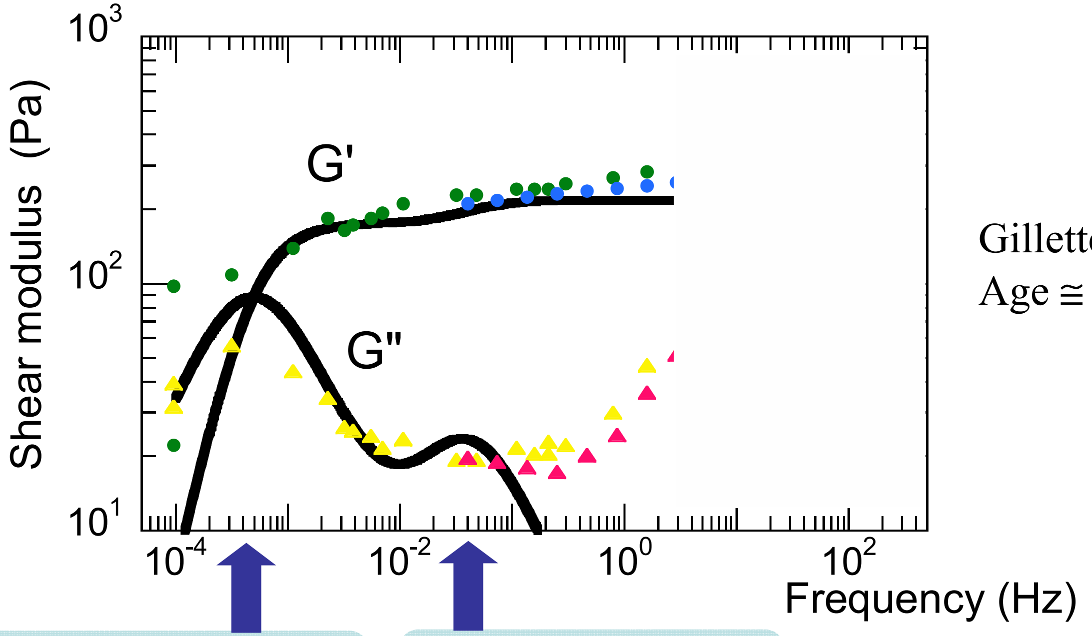
Surfactants + Dodecanol + Polymer



# *Back to the frequency response*

Complex shear modulus : $$G^*(\omega) = \frac{1}{i\omega \mathcal{L}[J(t)][i\omega]}$$

$J_o$ $\eta_o$ $J_1$ $\eta_1$

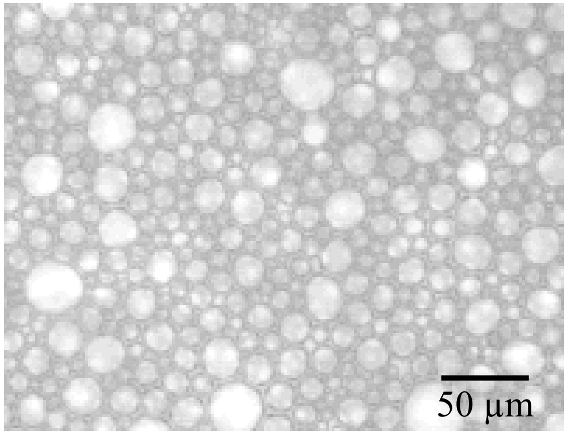



Gillette foam
Age ≅ 100 min

**Coarsening induced rearrangements**

**Surface viscosity**

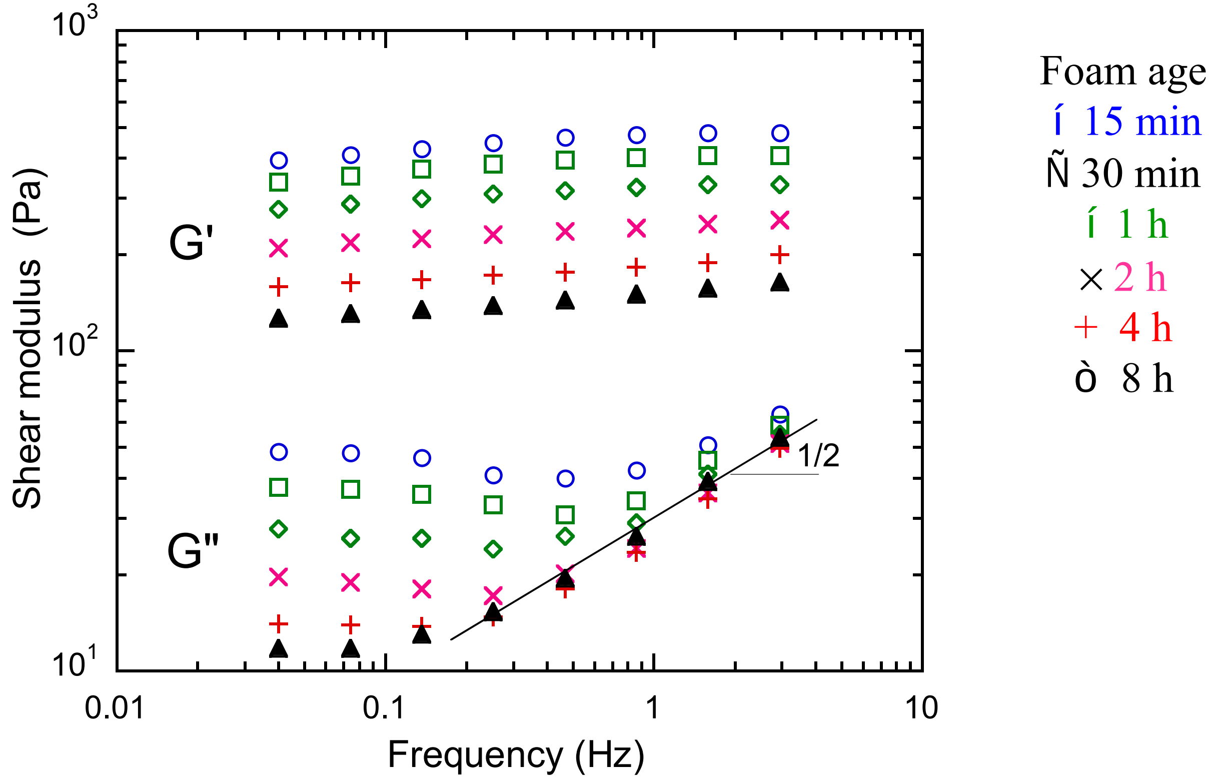
# *Evolution upon coarsening*



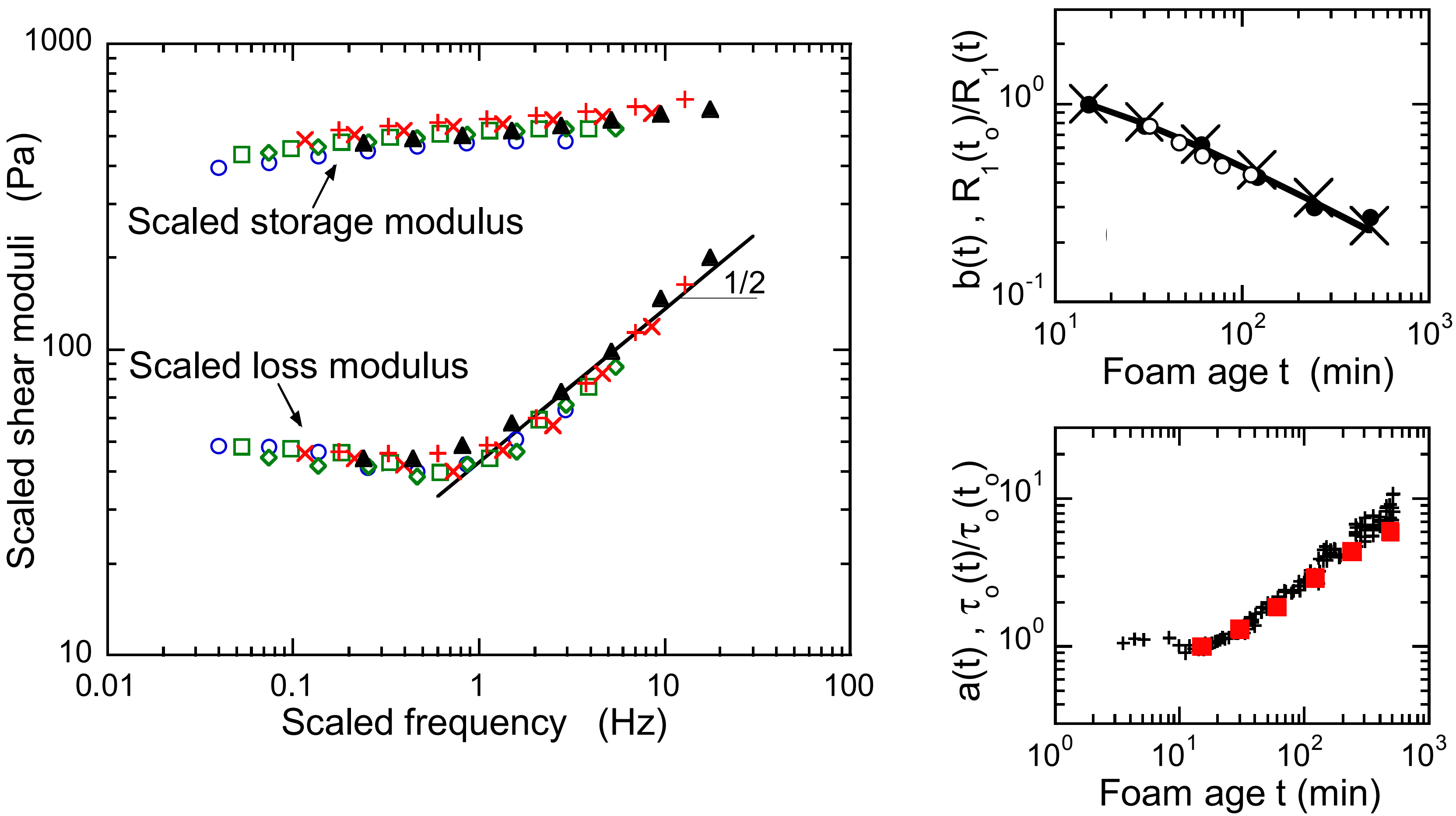

Growth law of mean bubble diameter due to Laplace pressure differences

$$d^2 = d_o^{\,2} + K\,(t-t_o)$$

*Mullins 1986*



# *Scaling law for G*(ω, age)*



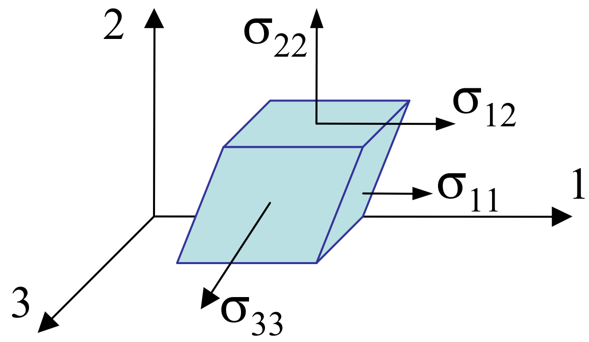

$$G^*(\omega, t) = b(t)\, G^*(\omega a(t), t_o)$$

*C-A, Hoballah, Höhler 1998*

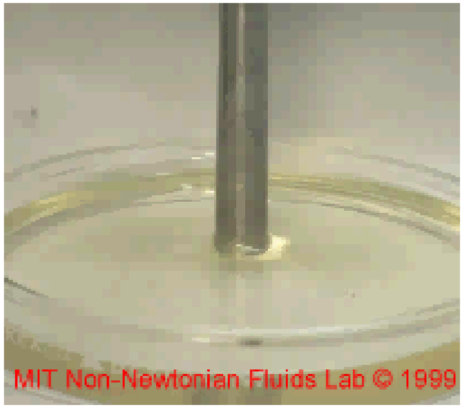
# *Shear induced normal stresses*

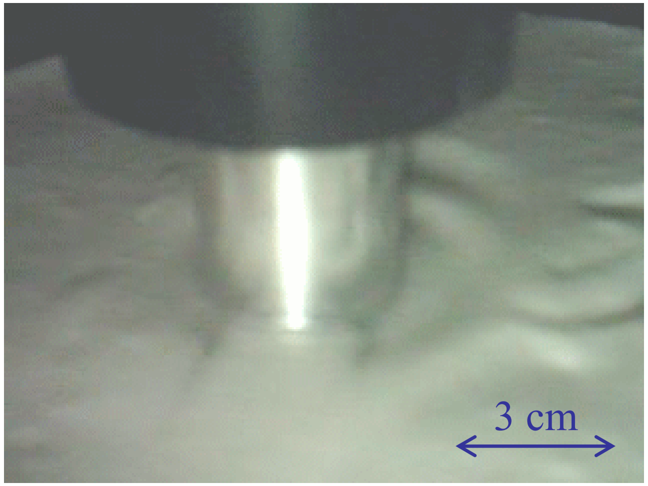
$$N_1 = \sigma_{11} - \sigma_{22}$$

$$N_2 = \sigma_{22} - \sigma_{33}$$




- Stationary flow : Weissenberg effect



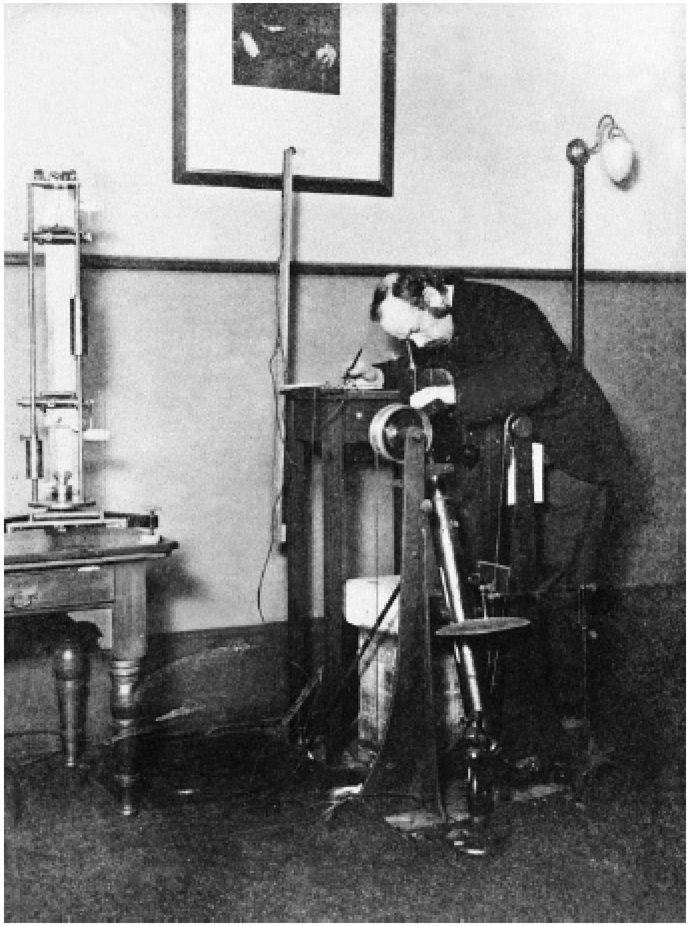




*Labiausse*

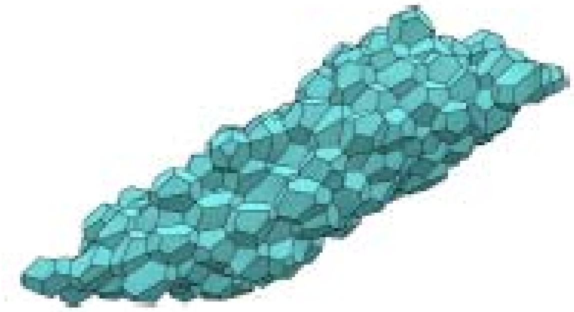
- Elastic regime : Poynting effect *1909*

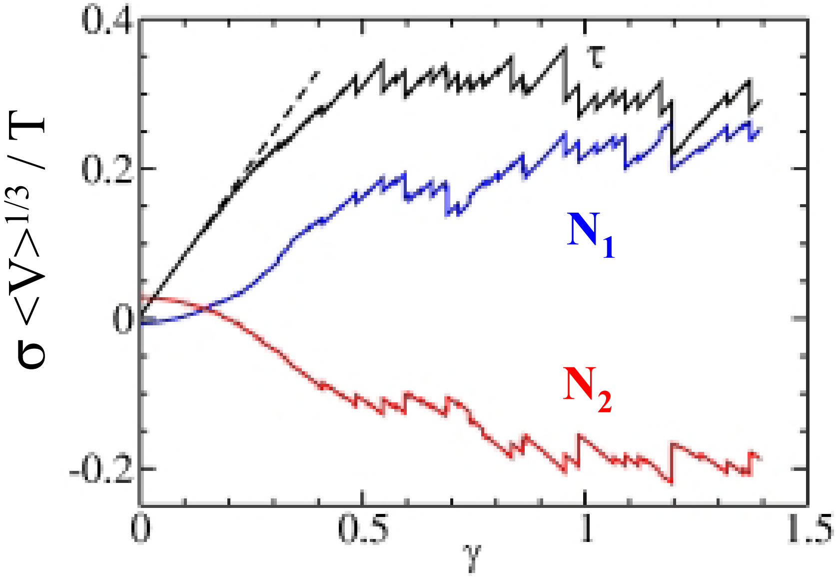
$$N_1 = \sigma_{12}\,\gamma = G\,\gamma^2$$

# *Poynting 1852-1914*



# *Surface evolver simulations*

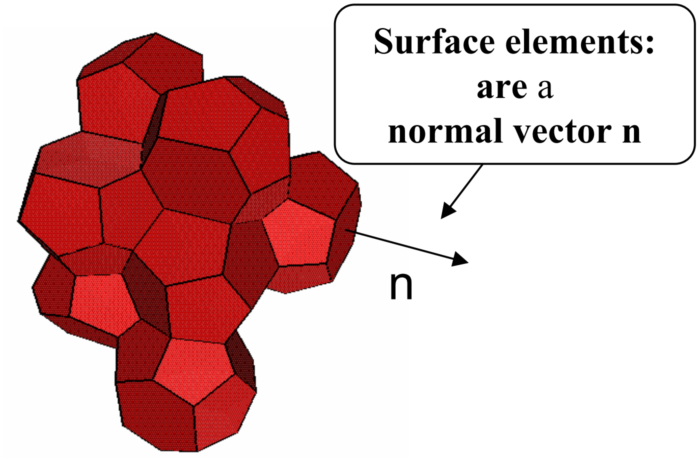





$$N_1 \cong G\,\gamma^2 \qquad\qquad N_2 \cong -0.85\,N_1$$

*Kraynik, Reinelt 2004*

# *A nonlinear analytical constitutive law for foam*




**Application of a homogeneous affine strain** :

**Modified area a'**: $a'^2 = a^2 \ \mathbf{n}^T \mathbf{C}^{-1} \mathbf{n}$

**Energy density :** $W(\mathbf{C}) = \frac{Ta}{V} \sum_j \sqrt{\mathbf{n}_j^T \mathbf{C}^{-1} \mathbf{n}_j}$

Surface tension T, sample volume V

Right Cauchy Green tensor C

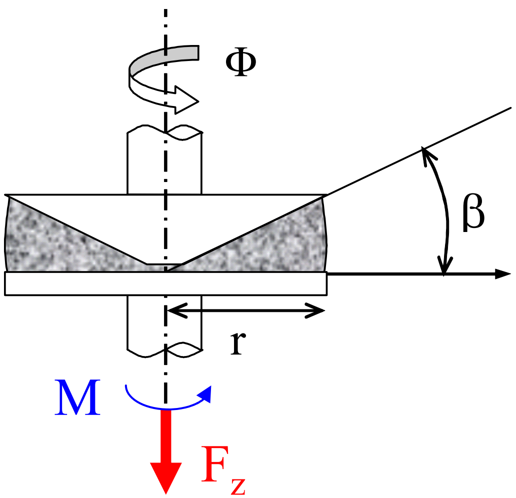
Express W in terms of the invariants of C , denoted $I_c$ and $II_c$: $W = \frac{G}{14}\left((I_c - 3) + 6(II_c - 3)\right)$

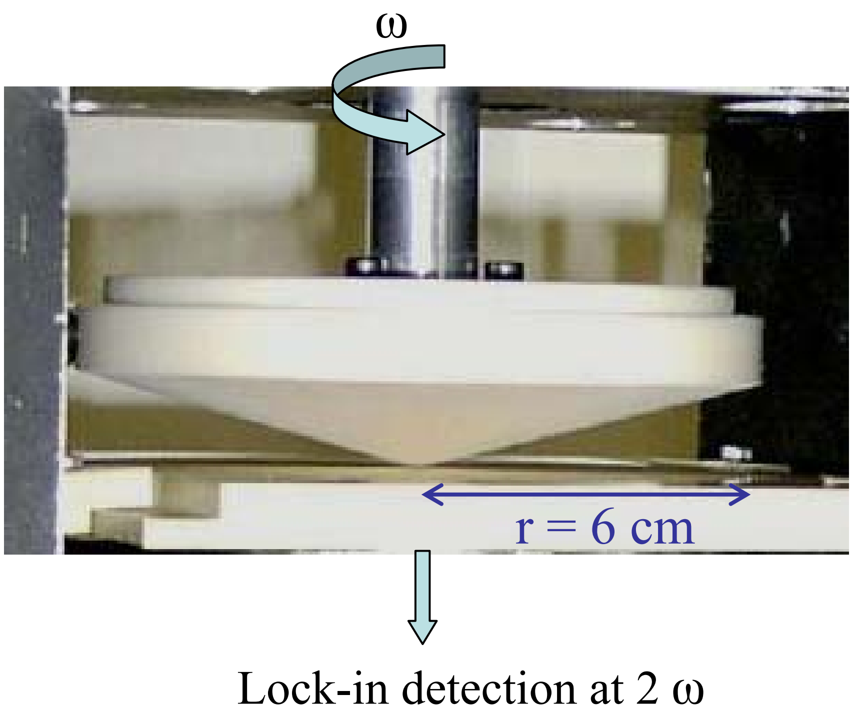
Finger tensor B

$$\boldsymbol{\sigma} = -p\,\mathbf{I} + \frac{G}{7}\left(\mathbf{B} - 6\,\mathbf{B}^{-1}\right) \quad \rightarrow \quad N_1 = \sigma_{12}\,\gamma \quad N_2 = -\frac{6}{7} N_1$$

*Höhler, C-A, Labiausse 2004*

# *Measuring $N_1$*



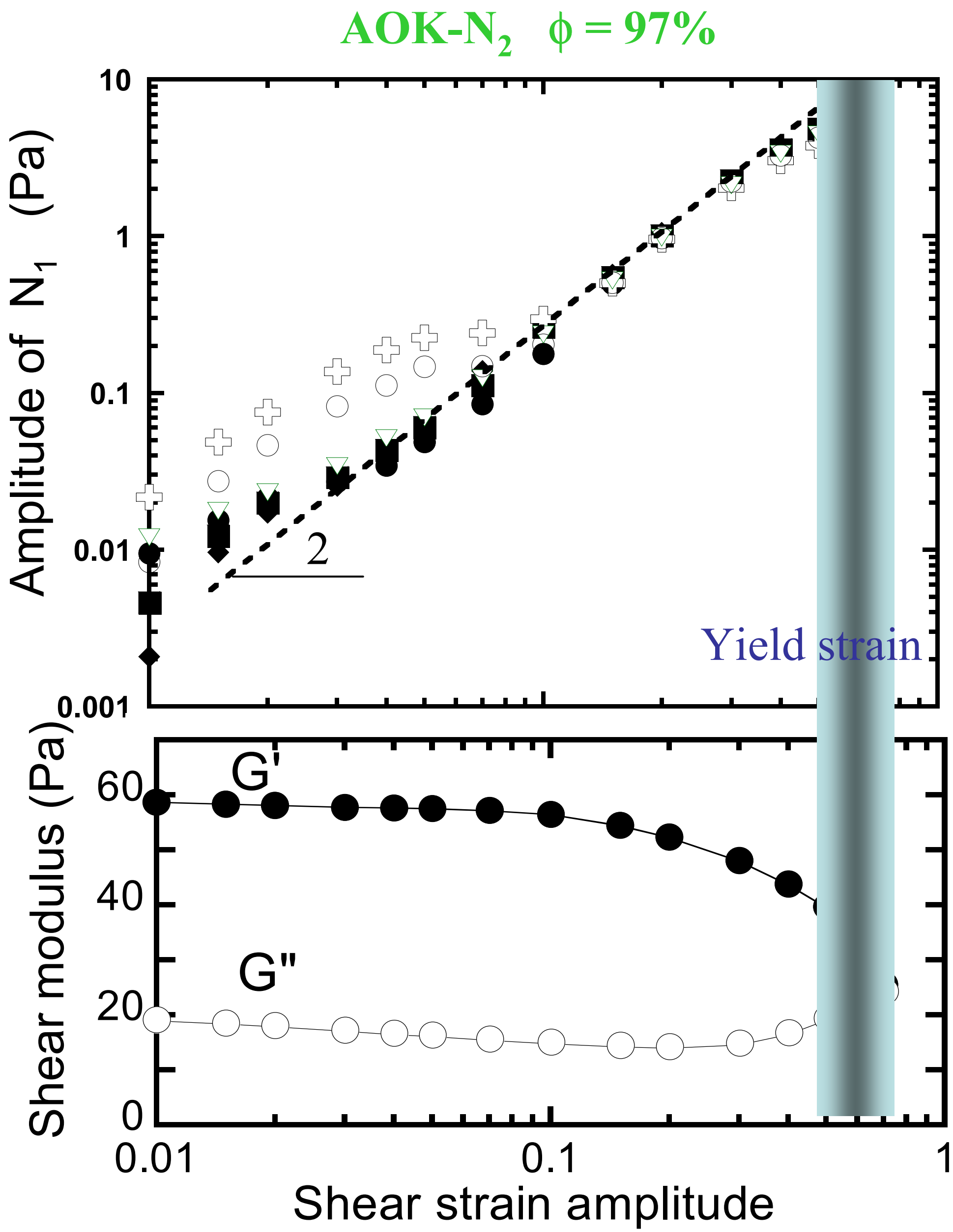

$$\gamma = \frac{\phi}{\beta}$$

$$\sigma_{\phi\theta} = \frac{3M}{2\pi r^3}$$

$$N_1 = \frac{2F_Z}{\pi\, r^2}$$

*Macosko 1994*

# *Good agreement with Poynting's law for dry foams*

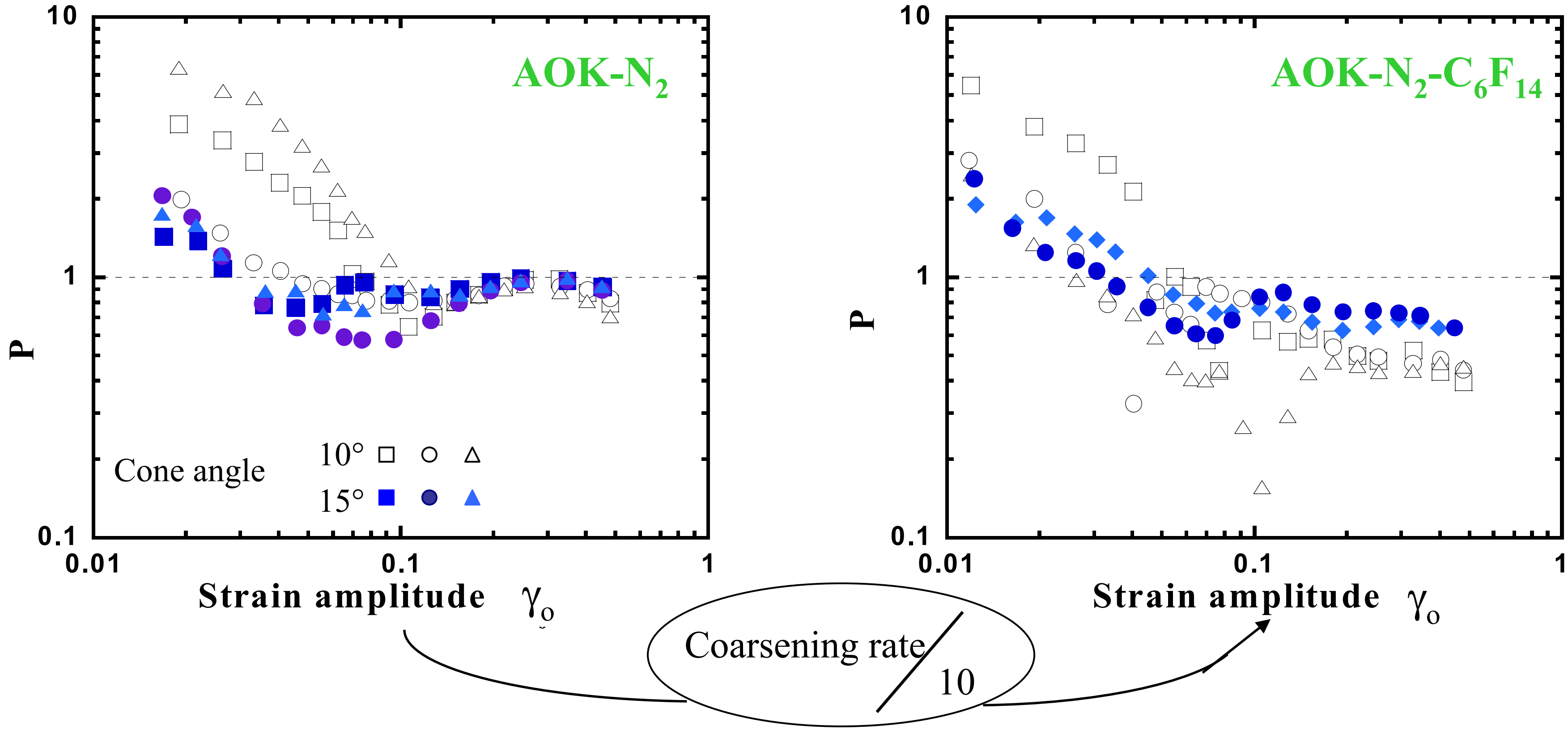



| | Cone angle |
|---|---|
| Empty symbols | 10° |
| Filled symbols | 15° |

**frequency 1 Hz**

# *Strain history effect*

$$P \equiv \frac{2N_1}{G\gamma_o^2}$$




Coarsening releases part of the stresses trapped due to the strain history.
=> more isotropic structure