

MA22S3 Tutorial Sheet 5

2-3 November 2016

Formulas:

- Fourier Transform:

$$\mathcal{F}[f(t)] = \tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt.$$

- We have computed the Fourier transforms of a few specific functions, including the following:

$$\mathcal{F}\left[e^{-a|t|}\right] = \frac{1}{\sqrt{2\pi}} \frac{2a}{a^2 + \omega^2}, \quad \text{for } a > 0$$

- Inverse Fourier Transform:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t} d\omega.$$

- Convolution:

$$(f \star g)(t) = \int_{-\infty}^{\infty} f(u) g(t - u) du.$$

Questions:

1. Evaluate the following integrals.

(a) $\int_{-\infty}^{\infty} \delta(3x)(x + 2) dx$

(b) $\int_0^3 \delta(x + 1) \cos(x) dx$

(c) $\int_{-1}^1 \left[\frac{d}{dx} \delta(x)\right] x dx$

2. Compute the convolution $f \star g$ of the following pair of functions.

$$f(t) = t^2 - 2t$$

$$g(t) = \delta(t - 3) + \delta(t - 4)$$

3. What are the Fourier transforms of the following functions?

(a)

$$\frac{1}{\sqrt{2\pi}} \frac{2a}{a^2 + t^2}, \quad \text{for } a > 0$$

(b)

$$\frac{1}{\sqrt{2\pi}} \frac{2a}{a^2 + (t - 1)^2}, \quad \text{for } a > 0$$

(c)

$$\frac{1}{t^2 - 2t + 5}$$

(d)

$$\frac{1}{t^2 - t + 1}$$