MA22S3 Tutorial Sheet 4

26–27 October 2016

Formulas:

- The real Fourier series expansion of a function $f(t)$ of fundamental period L can be written as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n t}{L}\right),$$

  where the coefficients are given by the Euler formulas:

$$\begin{aligned} a_0 &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\, dt \\ a_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\cos\left(\frac{2\pi n t}{L}\right)\, dt \\ b_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\sin\left(\frac{2\pi n t}{L}\right)\, dt \end{aligned}$$

- Parseval's Theorem: For a function of period L whose real Fourier series expansion is written in the form above, the following equation is true:

$$\frac{1}{L}\int_{t_0}^{t_0+L} f(t)^2\, dt = \left(\frac{a_0}{2}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right).$$

- Fourier Transform:

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt\,.$$

Questions:

1. In Tutorial Sheet 2, we computed the Fourier series of the following function:

$$\begin{aligned} f(t) &= t^2 \quad \text{for } |t| < 1\,, \\ f(t) &= f(t+2). \end{aligned}$$

   The following Fourier coefficients were obtained:

$$a_0 = \frac{2}{3}, \qquad a_n = \frac{4(-1)^n}{\pi^2 n^2}, \qquad b_n = 0\,.$$

   Use Parseval's Theorem to evaluate the sum of the infinite series

$$\sum_{n=1}^{\infty}\frac{1}{n^4}\,.$$

2. Using the definition given above, compute the Fourier transform of $f(t) = te^{-2|t|}$.