MA22S3 Tutorial Sheet 3

19-20 October 2016

Formulas:

• Definition of hyperbolic cosine and hyperbolic sine:

$$\cosh x \equiv \frac{e^x + e^{-x}}{2}, \qquad \sinh x \equiv \frac{e^x - e^{-x}}{2}.$$

• The complex Fourier series expansion of a function f(t) of period L can be written as

$$f(t) = \sum_{n = -\infty}^{\infty} c_n e^{in\frac{2\pi}{L}t},$$

where the coefficients are given by

$$c_n = \frac{1}{L} \int_{t_0}^{t_0+L} f(t) e^{-in\frac{2\pi}{L}t} dt.$$

Questions:

- 1. Write formulas for (a) the simple periodic extension, (b) the half-range even expansion, and (c) the half-range odd expansion of the following function.
 - h(t)



2. Compute the complex Fourier series of the following periodic function, and write all the terms with $|n| \leq 3$ explicitly.

$$f(t) = \cosh t \quad \text{for } -\frac{\pi}{2} \le t < \frac{\pi}{2}, \qquad \text{and } f(t+\pi) = f(t)$$

3. Convert the complex Fourier series found in the previous problem to a real Fourier series. Check that the coefficients are actually real-valued.