MA22S3 Tutorial Sheet 2

12-13 October 2016

Formulas:
The Fourier series expansion of a function $f(t)$ of fundamental period L can be written as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nt}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{L}\right),$$

where the coefficients are given by the Euler formulas:

$$\begin{aligned} a_0 &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\ dt \\ a_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\cos\left(\frac{2\pi nt}{L}\right)\ dt \\ b_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\sin\left(\frac{2\pi nt}{L}\right)\ dt \end{aligned}$$

Questions:
Define the function $f(t)$ as follows:

$$\begin{aligned} f(t) &= t^2 \quad \text{for } |t| < 1\,, \\ f(t) &= f(t+2). \end{aligned}$$

1. What is the fundamental period of $f(t)$?
2. Sketch $f(t)$ on the interval $-4 \le t \le 4$.
3. Is $f(t)$ even, odd, or neither?
4. Find the first four terms of the real Fourier series expansion of $f(t)$.
5. Without doing any further integration, sketch the following functions and write the first four terms of their real Fourier series.

$$\begin{aligned} g(t) &= 1 - t^2 \quad \text{for } |t| < 1\,, \\ g(t) &= g(t+2). \end{aligned}$$

$$\begin{aligned} h(u) &= \frac{u^2}{\pi^2} \quad \text{for } |u| < \pi\,, \\ h(u) &= h(u+2\pi). \end{aligned}$$