MA22S3 Tutorial Sheet 10

14–15 December 2016

1. Consider the following differential equation:

$$y'' + x^2 y = 0.$$

   (a) Are there any singular points of the differential equation? If so, are they regular singular points? Why or why not?

   (b) Write the recursive relation for coefficients in a series solution about the point $x = 0$.

2. Let λ be a constant complex number. Consider the following differential equation.

$$(x - x^2)y'' + (1 - x)y' + \lambda y = 0.$$

   (a) Are there any singular points of the differential equation? If so, are they regular singular points? Why or why not?

   (b) Find a general series solution about the point $x = 0$.

   (c) In general, what is the radius of convergence of this solution?

   (d) For which values of λ is the solution a polynomial?

   (e) Write three examples of polynomials satisfying the differential equation as well as the initial condition $y(0) = 1$, together with their required values of λ.