MA22S3 Tutorial Sheet 1

5–6 October 2016

1. *Kronecker delta.* Evaluate the following.

   (a) δ_{02}

   (b) $\sum_{i=1}^{\infty} \delta_{ik} a_i$

   (c) $\sum_{i=1}^{\infty} \sum_{j=-\infty}^{\infty} \delta_{ik}\delta_{jl}(-1)^j b_{ii}$

   (d) $\sum_{i=2}^{4} \sum_{j=-1}^{1} \delta_{ij}$

   (e) $\sum_{i=2}^{4} \sum_{j=2}^{3} \delta_{ij}$

   (f) $\sum_{i=2}^{4} \sum_{j=2}^{3} \delta_{ij}(i+j)^2$

2. Let V be the vector space of all polynomials in x of degree 2 with real coefficients, with an inner product defined by

$$\langle p(x)|q(x)\rangle = \int_0^1 p(x)q(x)\ dx.$$

   (a) What is the dimension of V?

   (b) Is there a real value of c such that the polynomials $x^2 - cx$ and x are orthogonal?

3. Consider the following three vectors in $\mathbb{R}^3$:

$$v_1 = (1, 0, 1), \qquad v_2 = (1, \sqrt{6}, -1), \qquad v_3 = (\sqrt{3}, -\sqrt{2}, -\sqrt{3}).$$

   (a) Show that $\{v_1, v_2, v_3\}$ is an orthogonal set with respect to the usual dot product.

   (b) Because the number of vectors in the set above is equal to the dimension of the vector space, they form a basis. Expand the vector $v = (1, 2, 1)$ in terms of this basis: find real coefficients a_1, a_2, a_3 such that

$$v = a_1 v_1 + a_2 v_2 + a_3 v_3.$$

4. Define the following operation on $\mathbb{R}^2$:

$$\langle (x_1, x_2)|(y_1, y_2)\rangle = 2x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$$

   (a) Show that this operation is symmetric, bilinear, and positive definite. Therefore it defines an inner product on $\mathbb{R}^2$.

   (b) With this definition, compute the inner products of the following pairs of vectors:

   - $(1, 0)$ and $(0, 1)$
   - $(1, 0)$ and $(-2, 1)$
   - $(0, 1)$ and $(-2, 1)$

   (c) With respect to this same inner product, write an orthonormal set with 2 elements.