An Introduction to Mathematica

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The Notebook

In the notebook, press Shift + Enter to run a command.

Commands are labelled by `In[`n`]:=` and their output by `Out[`n`]=`.

Special Characters

- Palettes > Writing Assistant
- Esc LaTeX Esc

Functions

$$\sin\left(\frac{\pi}{4}\right) \rightarrow \texttt{Sin[Pi/4]}$$

Notice:

- Uppercase `S` (CamelCase),
- square brackets for the argument
- and uppercase `P` in `Pi`.
  Similarly, `E` for e and `I` for i.

Variables

= is assignment.

```
In[2]:= pi = 3.14
Out[2]= 3.14
```

== is equality.

```
In[3]:= Pi == pi
Out[3]= False
```

```
In[4]:= Pi == 2 * ArcSin[1]
Out[4]= True
```

```
In[5]:= a = 2
Out[5]= 2
```

Mathematica often remembers variable assignment when you don't. It even remembers across notebooks.

Use `Clear[···]` to make it forget.

```
In[6]:= Log[a]
Out[6]= Log[2]
```

Mathematica didn't evaluate $\log(a) = \log(2)$ because it doesn't know I want a number. But I do.

```
In[7]:= N[%]
Out[7]= 0.693147
```

`N` evaluates expressions numerically and `%` references the previous output.

`%%` evaluates the second previous output, etc. I.e. `%`$\cdots$`%` (k times) is the k^{th} previous output.

`%`n evaluates `Out[`n`]`.

Functions like N

N is treated as a function just like Sin, Sqrt, etc.

```
In[8]:= Simplify[1 - Cos[2 x]]
Out[8]= 2 Sin[x]^2

In[9]:= Expand[(x + y)^8]
Out[9]= x^8 + 8 x^7 y + 28 x^6 y^2 + 56 x^5 y^3 + 70 x^4 y^4
        + 56 x^3 y^5 + 28 x^2 y^6 + 8 x y^7 + y^8

In[10]:= Factor[%]
Out[10]= (x + y)^8
```

Functions can also be applied after an expression. For example, `N[Sqrt[2]]` and `Sqrt[2] // N` are equivalent.

Getting Help

The Mathematica documentation is *excellent*.

Click on an object (like a function) and hit F1. Or mouseover and click ⓘ (for documentation) or ⪷ (for a brief summary).

Building Functions

```
In[11]:= f[x_, y_] := x - y
```

$$f(x, y) = x - y$$

What does `:=` ("`SetDelayed`") and `_` ("blank") mean?

lhs `:=` *rhs*

assigns rhs to be the delayed value of lhs. rhs is maintained in an unevaluated form. When lhs appears, it is replaced by rhs, evaluated afresh each time.

The `_` is a pattern-matching symbol. `x_` stands for anything, but names it `x` so it can be referred to on the right hand side.

```
In[12]:= f[1, 1]
Out[12]= 0
```

/.

`/.` is `ReplaceAll`.

expr `/.` *rules*
applies a rule or list of rules in an attempt to transform each subpart of an expression expr.

For example,

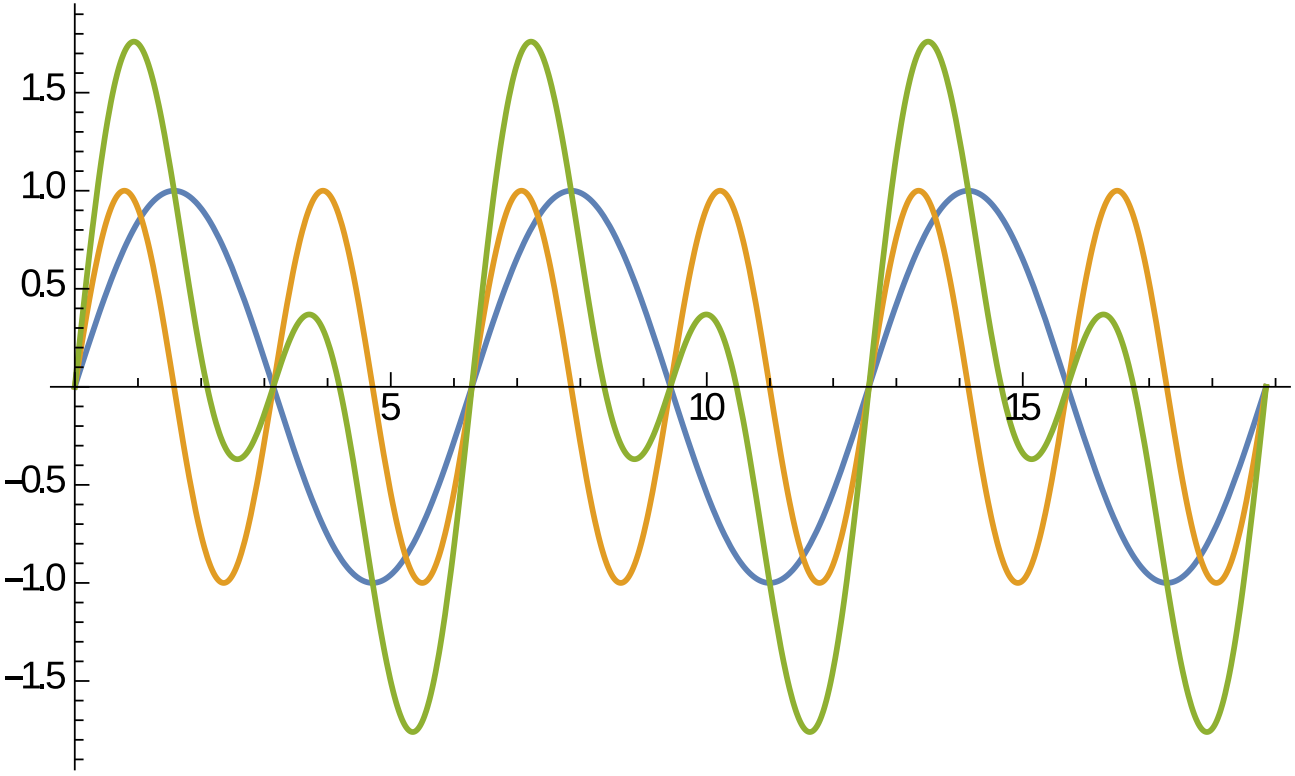
```
In[13]:= f[x, y] /. x → 0
Out[13]= -y
```

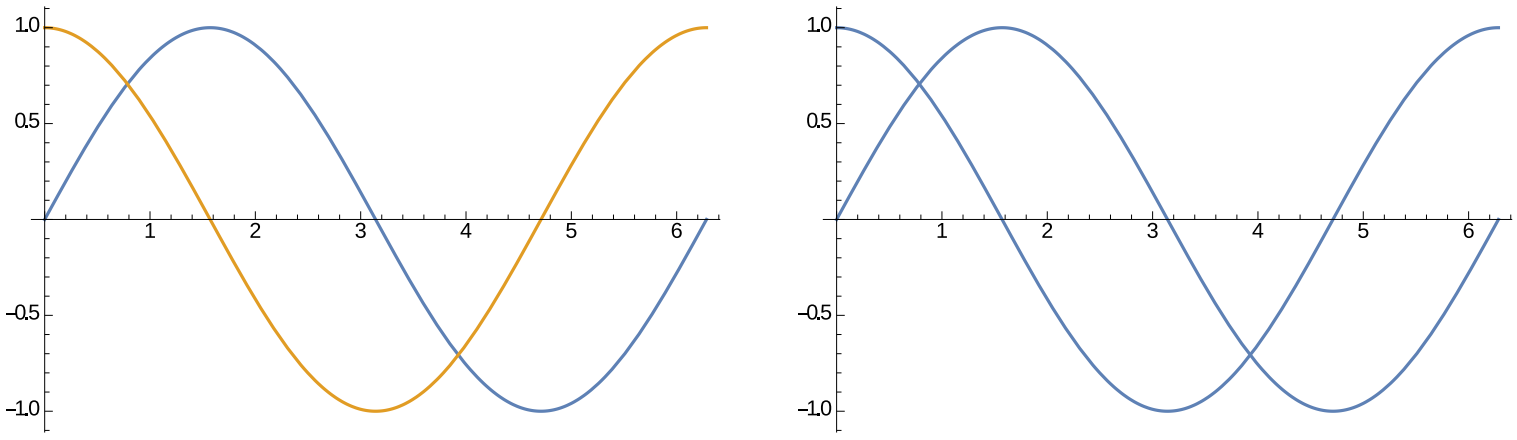
or even

```
In[14]:= Sin[x] /. Sin → Cos
Out[14]= Cos[x].
```

Plotting

```
In[15]:= Plot[{Sin[x], Sin[2 x], Sin[x] + Sin[2 x]},
          {x, 0, 6 Pi}]
Out[15]=
```

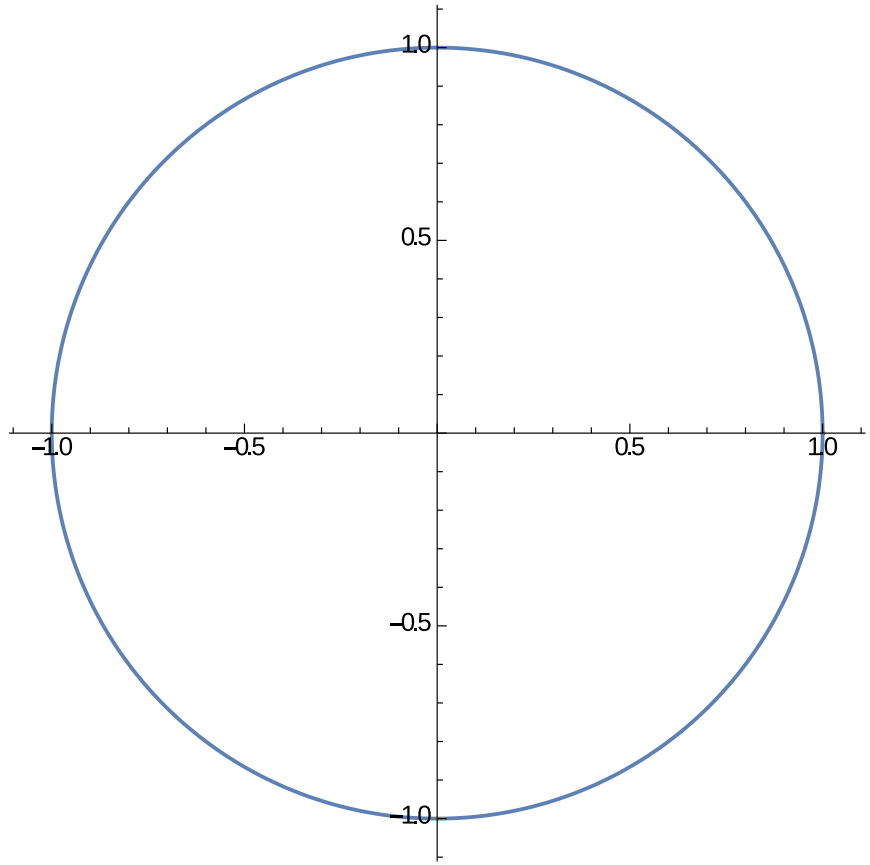




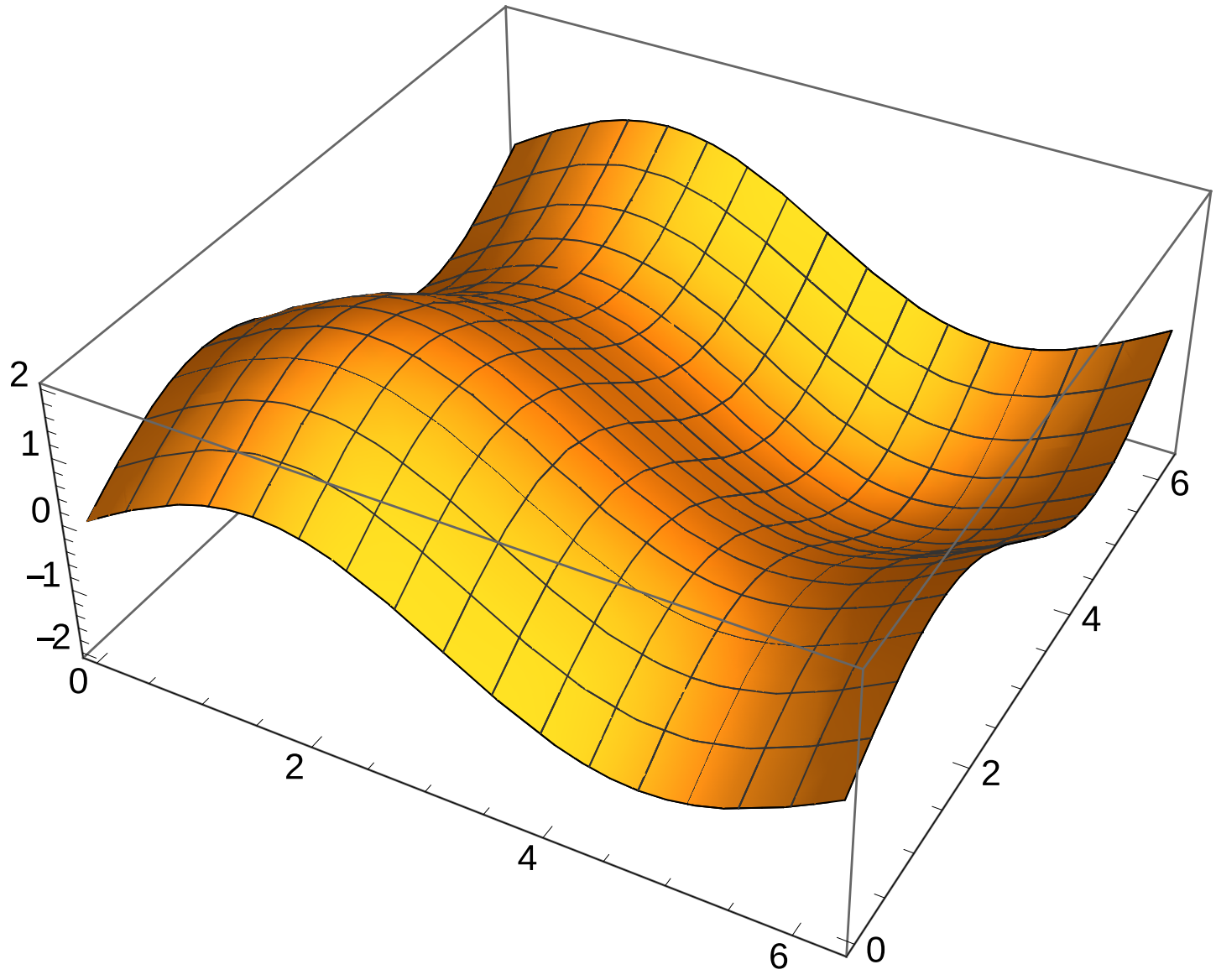
`Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}]` on the left and `Show[Plot[Sin[x], {x, 0, 2 Pi}], Plot[Cos[x], {x, 0, 2 Pi}]]` on the right.

Generally, you should use `AxesLabel` $\rightarrow$ `{"x", "y"}`.

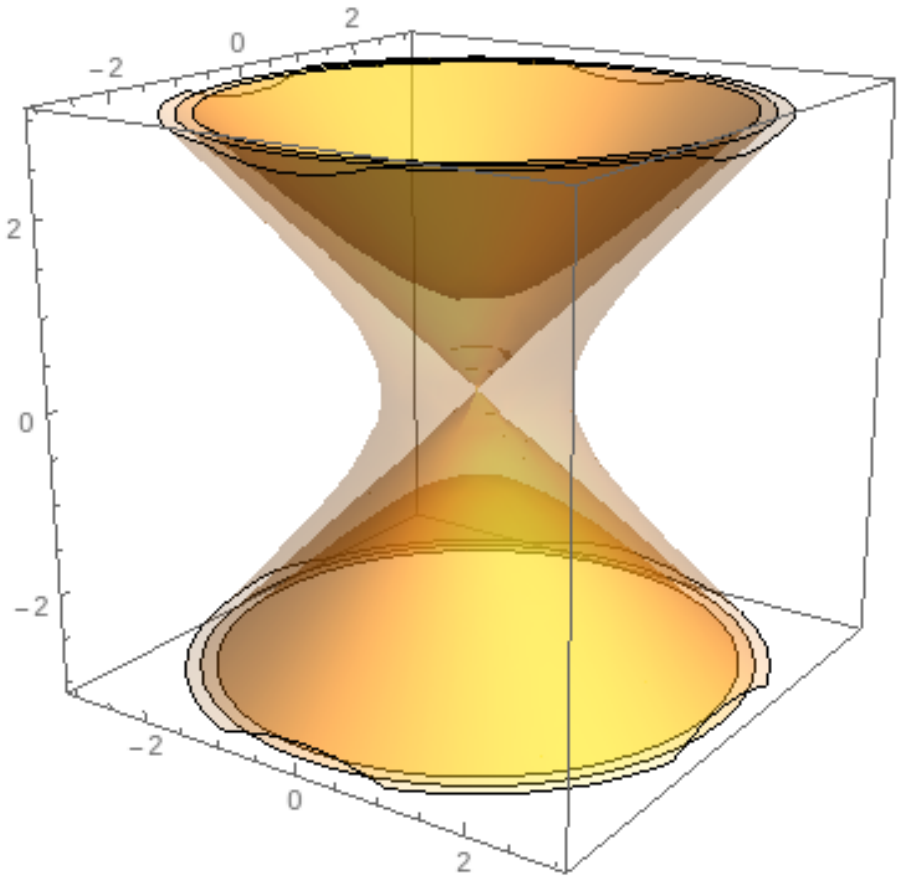
```
ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi}]
```



```
Plot3D[Sin[x] + Sin[y], {x, 0, 2 Pi}, {y, 0, 2 Pi}]
```



```
Show[{ContourPlot3D[ x^2 + y^2 - z^2 == 1, {x, -3, 3}, {y,
-3, 3}, {z, -3, 3}, Mesh → None, ContourStyle →
 Opacity[0.2]]}, {ContourPlot3D[ x^2 + y^2 - z^2 == 0, {x,
-3, 3}, {y, -3, 3}, {z, -3, 3}, Mesh → None, ContourStyle
 → Opacity[0.3]]}, {ContourPlot3D[ x^2 + y^2 - z^2 == -1,
{x, -3, 3}, {y, -3, 3}, {z, -3, 3}, Mesh → None,
 ContourStyle → Opacity[0.4]]}]
```



Differentiating

The differential operator is `D`.

$$\frac{\partial}{\partial x} f(x, y) \rightarrow \texttt{D[f[x, y], x]}.$$

```
In[16]:= D[x^3, x]
Out[16]= 3 x^2

In[17]:= g[x_, y_] := 2 (x y)^2

In[18]:= D[g[x, y], x, y]
Out[18]= 8 x y

In[19]:= D[g[x, y], x, y] == D[g[x, y], y, x]
Out[19]= True

In[20]:= D[D[g[x, y], x], x]
Out[20]= 4 y^2
```

Integrating

$$\int f(x)\,\mathrm{d}x \rightarrow \texttt{Integrate[f[x], x]}.$$

```
In[21]:= Integrate[Log[x], x]
Out[21]= -x + x Log[x]
```

Notice that the constant is omitted.

With limits,

```
In[22]:= Integrate[(1/x)^2, {x, 1, Infinity}]
Out[22]= 1.
```

Limits and Sums

```
In[23]:= Limit[Sin[x]/x, x → 0]
Out[23]= 1
```

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

```
In[24]:= Sum[r^n, {n, 0, Infinity}]
Out[24]= 1/(1 - r)
```

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

Series Expansion

`Series` generates a power series.

$$\sum_{n=0}^{k} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n \rightarrow \texttt{Series[}f\texttt{, \{}x, x_0, k\texttt{\}]}$$

Solving Equations

- `Solve` for polynomials, systems of equations, e.g.
  `Solve[x^2 - x - 1 == 0, x]`.
- `Dsolve` for differential equations (ordinary or partial) or systems of differential equations, e.g.
  `DSolve[y'[x] == Cos[x], y[x], x]`.
- ...

Lists, Tables, Matrices

`v = {a, b, c}` is equivalent to `v = List[a, b, c]`.

Pick element n from a list `v` using `Part[v, n]`. This is the same as `v[[n]]`.

List indices start at 1!

Matrices look like `{{a, b}, {c, d}}`.

Making Assumptions

```
Integrate[x^a, {x, 0, 1}, Assumptions → a > -1]
Assuming[a > -1, {Integrate[x^a, {x, 0, 1}]}][[1]]
```

Other Functions

- `Dot`
- `Cross`
- `FindRoot`
- `Do`
- `If`
- `Manipulate`
- `...`

Nobody knows how many functions there are. Use the documentation and the internet to find the ones you need.

Getting Help (Again)

Use the documentation.

- `mathematica.stackexchange.com`,
- the internet,
- &c.