Furry’s Theorem

In QED,

\[ \langle \Omega | T[j^\mu(x_1)j^\nu(x_2)j^\lambda(x_3)]|\Omega \rangle = 0 \]

where the current is

\[ j^\mu = \bar{\psi}\gamma^\mu \psi. \]

This is true for any odd number of currents. In other words, the vacuum expectation value of any odd number of electromagnetic currents must vanish.

Charge conjugation $C$ is a unitary linear operator which is conventionally defined to bring a fermion into an antifermion with the same spin. It is a symmetry of QED, so

\[ C|\Omega\rangle = |\Omega\rangle, \quad \langle \Omega | C^\dagger = \langle \Omega |. \]

However $j^\mu(x)$ changes sign under $C$,

\[ j^\mu(x) \overset{C}{\rightarrow} C^\dagger j^\mu(x)C = -j^\mu(x), \]

so its vacuum expectation must vanish, as

\[
\begin{align*}
\langle \Omega | T j^\mu(x)|\Omega \rangle &= \langle \Omega | C^\dagger j^\mu(x)C^\dagger C|\Omega \rangle \\
&= -\langle \Omega | C^\dagger T j^\mu(x)C|\Omega \rangle \\
&= -\langle \Omega | T j^\mu(x)|\Omega \rangle \\
&= 0.
\end{align*}
\]

If we now consider an odd number of currents,

\[ \langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_{2n+1}}(x_{2n+1})]|\Omega \rangle , \]

and inserting $C^\dagger C = 1$ as before yields

\[
\begin{align*}
\langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_{2n+1}}(x_{2n+1})]|\Omega \rangle &= \langle \Omega | T[C^\dagger j^{\mu_1}(x_1)C^\dagger C \cdots C^\dagger C j^{\mu_{2n+1}}(x_{2n+1})C^\dagger C]|\Omega \rangle
\end{align*}
\]
= (-1)^2n+1 \langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_{2n+1}}(x_{2n+1})] | \Omega \rangle

= -\langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_{2n+1}}(x_{2n+1})] | \Omega \rangle

since 2n + 1 is odd. Therefore,

\langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_{2n+1}}(x_{2n+1})] | \Omega \rangle = 0

or

\langle \Omega | T[j^{\mu_1}(x_1) \cdots j^{\mu_n}(x_n)] | \Omega \rangle = 0

for any odd n.