Question 15

Question 0.1 *Determine which of the following subsets of* $\mathcal{R}^2$ *are open and which are closed:*

$$\{f(x,y) \in \mathcal{R}^2 : (x-2)^2 + y^2 < 9\}$$

This subset is open, because firstly it does't contain its limit points, secondly because its complement $\mathcal{R}^2 \backslash f(x,y)$ is closed.

$$\{f(x,y) \in \mathcal{R}^2 : (x-2)^2 + y^2 \geq 9\}$$

This subset is closed, because it contains its limit points and its complement is open.

$$\{f(x,y) \in \mathcal{R}^2 : (x-2)^2 + y^2 < 9 \; and \; x \leq \; 0\}$$

This subset is neither open or closed. This is because the $(x-2)^2 + y^2 < 9$ would make the set open but the $x \geq \; 0$ would make it closed.

$$\{f(x,y) \in \mathcal{R}^2 : (x-2)^2 + y^2 \geq 9 \; or \; x \leq \; 0\}$$

This subset is closed because both sections of the set. ie the $(x-2)^2 + y^2 \geq 9$ part is closed and the $x \geq \; 0$ would also make the subset closed.