Odd Problems

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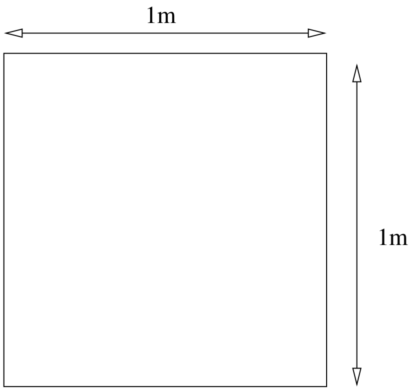
Odd Problems

Problems where solution doesn't match problem.

1. Ponds,
2. Rainbows,
3. Shortest roads,
4. Regular Solids.

Ponds



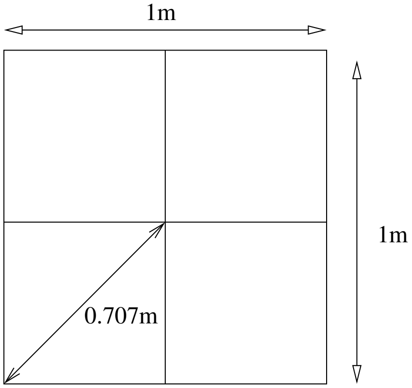
5 ducks are in this pond. Show that there are at least two of them closer than $1/\sqrt{2}m$.

Pigeon Hole Principle

With n pigeon holes and $n+1$ pigeons, two pigeons live in same hole.

https://en.wikipedia.org/wiki/File:TooManyPigeons.jpg

Pigeon Hole the Ducks



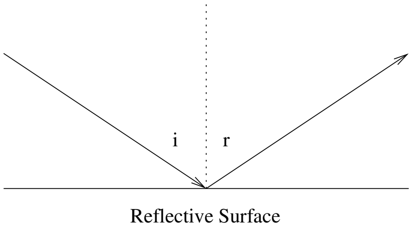
Rainbows

Try asking a physicist where rainbows come from.

Rainbow Angle: $\approx 42^o$.

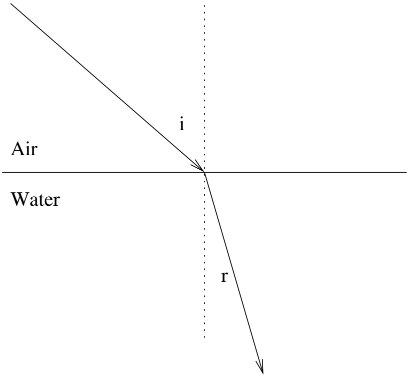
https://www.flickr.com/photos/bbusschots/32026575784/

Reflection



$$i = r$$

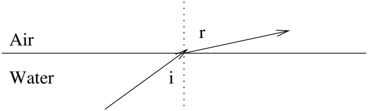
Refraction

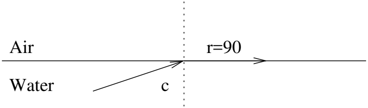


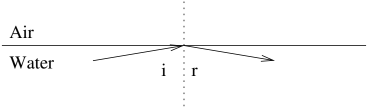
$$\sin(r) = \frac{n_a}{n_w} \sin(i)$$

For water going to air, n_a/n_w is about $3/4$.

Total Internal Reflection



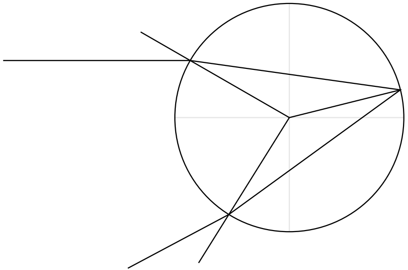




$$\frac{\sin(c)}{\sin(90)} = \frac{\sin(i)}{\sin(r)} = \frac{n_a}{n_w} \approx \frac{3}{4}$$

So $c \approx 48.6^o$.

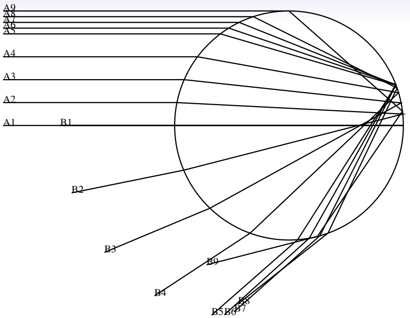
Rainbows not related to TIR!



How much does the angle change?

$$\delta = (i - r) + r + r + (i - r) = 4r - 2i$$

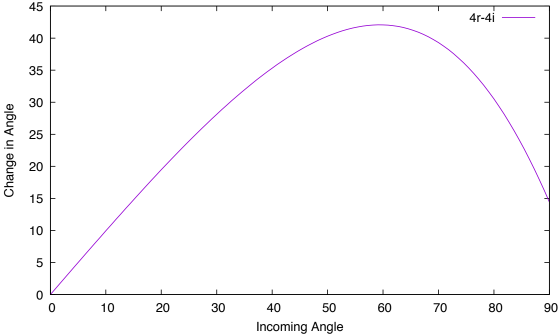
Remember we know $\sin(r) = 3\sin(i)/4$.



$$\begin{aligned}\delta &= 4r - 2i \\ &= 4\sin^{-1}\left(\frac{3\sin(i)}{4}\right) - 2i\end{aligned}$$

Because $\sin(r) = 3\sin(i)/4$.

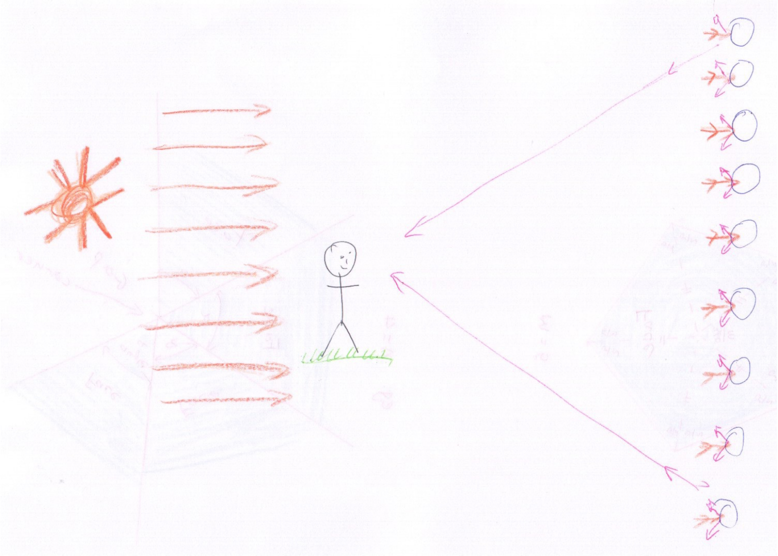
For water:



You can use differentiation

$$\sin(i) = \sqrt{\frac{4 - \left(\frac{n_a}{n_w}\right)^2}{3}}.$$

If you figure out the turn $4r - 2i \approx 42.3^o$



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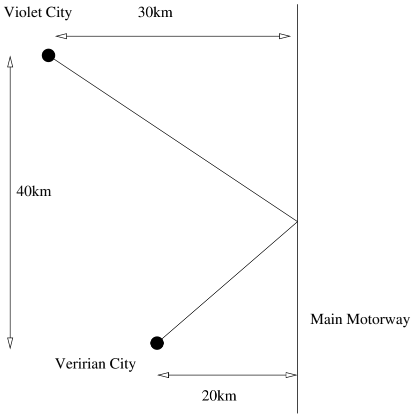
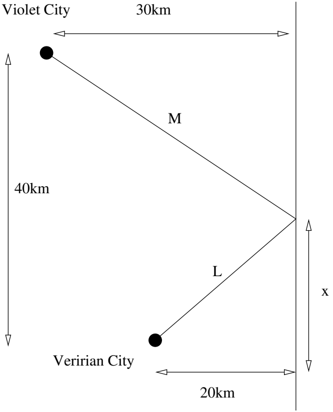
Road Building



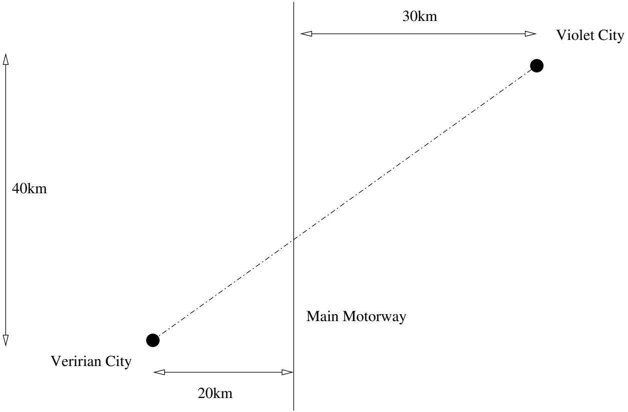
Want to connect two cities to a motorway.

Road Building



Wrong way: $L^2 = x^2 + 20^2$ and $M^2 = (40 - x)^2 + 30^2$ and Algebra.

Road Building



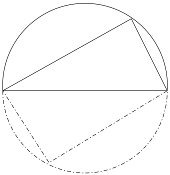
Now it's obvious!

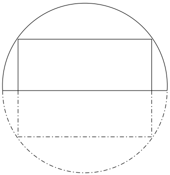
Another Example

Show the biggest triangle you can inscribe has the same area as biggest rectangle.

Can do trig and algebra and . . .

Another Example

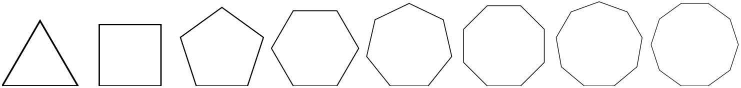




Complete the circle.

Regular Platonic Solids

Regular polygons in 2 dimensions:



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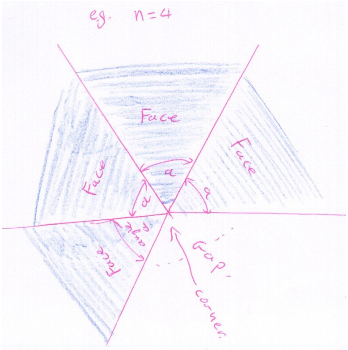
In three dimensions the situation is very different. There are only 5!

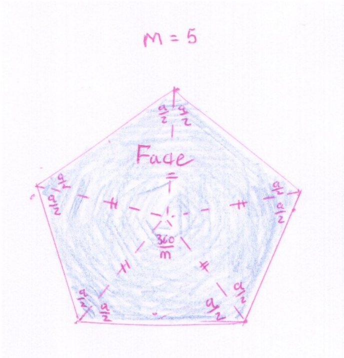
How could we count them?

Counting

Count n faces at a corner and m edges for each face.



$$na < 360.$$



$$a = 180 - \frac{360}{m}$$

We need:

$$n\left(180 - \frac{360}{m}\right) < 360$$

So,

$$180 - \frac{360}{m} < \frac{360}{n}$$

with $n \geq 3$ and $m \geq 3$.

$n = 3$	$\Rightarrow \quad 180 - \frac{360}{m} < 120$	
	$\Rightarrow \quad 60 < \frac{360}{m}$	
	$\Rightarrow \quad m < 6$	$m = 3 \Rightarrow$ tetrahedron
		$m = 4 \Rightarrow$ cube
		$m = 5 \Rightarrow$ dodecahedron
$n = 4$	$\Rightarrow \quad 180 - \frac{360}{m} < 90$	
	$\Rightarrow \quad 90 < \frac{360}{m}$	
	$\Rightarrow \quad m < 4$	$m = 3 \Rightarrow$ octahedron
$n = 5$	$\Rightarrow \quad 180 - \frac{360}{m} < 72$	
	$\Rightarrow \quad 108 < \frac{360}{m}$	
	$\Rightarrow \quad m < 3.333\ldots$	$m = 3 \Rightarrow$ icosahedron
$n \geq 6$	$\Rightarrow \quad 180 - \frac{360}{m} < 60$	
	$\Rightarrow \quad 120 < \frac{360}{m}$	
	$\Rightarrow \quad m < 3$	Nothing!

Odd Problems

- You can pigeon hole ducks.
- Rainbows are really a mathematical thing.
- Sometimes reflecting makes things easier.
- You can count the Platonic solids.