Triangulation and Education
or
Where is Dunsink?

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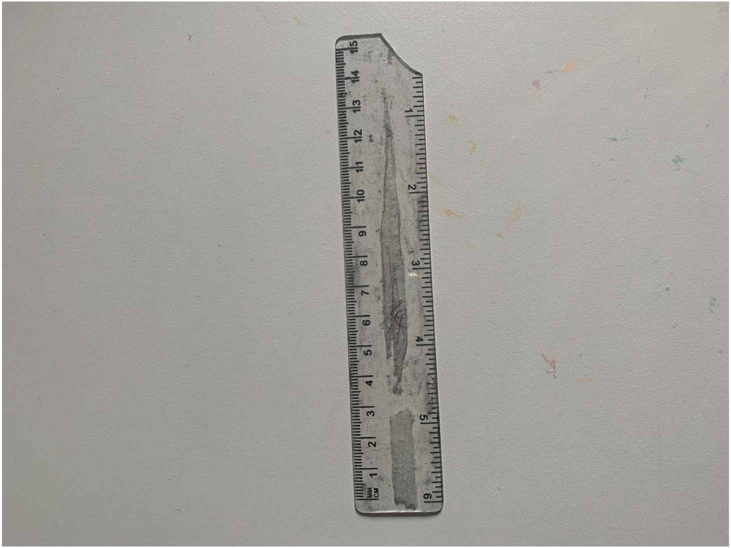
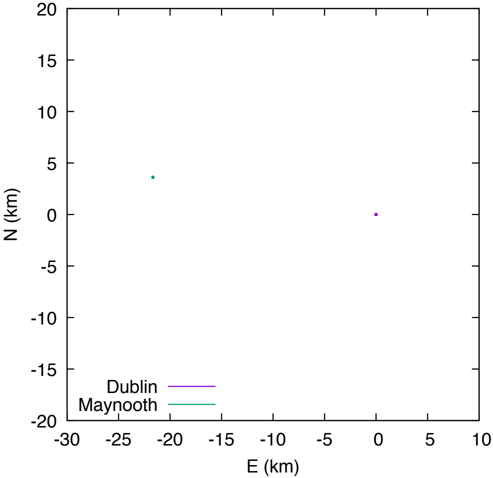
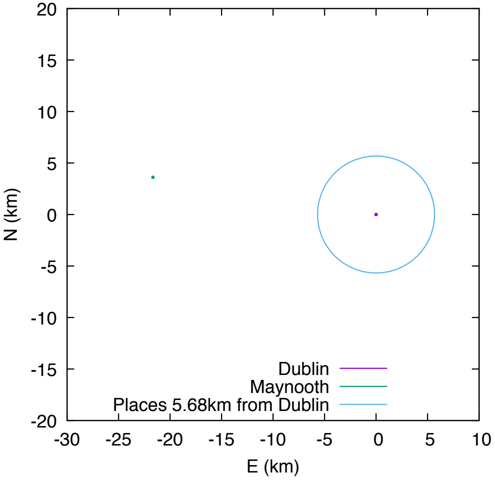
Measuring Space



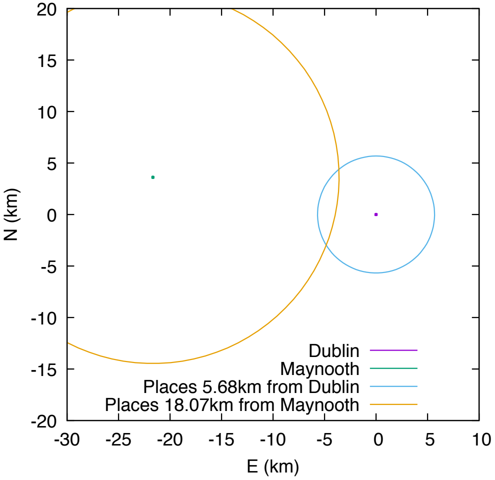
Trilateration: Find Dunsink



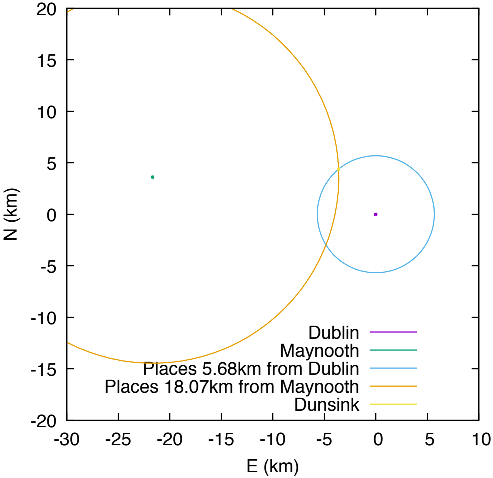
Trilateration: Find Dunsink



Trilateration: Find Dunsink



Trilateration: Find Dunsink



As an equation, a circle of radius R with centre (x_c, y_c) is

$$R^2 = (x - x_c)^2 + (y - y_c)^2$$

We can use twice to get our circles and solve for Maynooth.

Where distances are in km and

- Dublin is at $(0, 0)$,
- Maynooth is at $(-21.664772, 3.608656)$
- $R_{\text{Dunsink,Dublin}} = 5.679756$ and
- $R_{\text{Dunsink,Maynooth}} = 18.071940$.

Trick is called *trilateration*.

My Workings

(1)

$$R^2_{Dub} = (x - x_{Dub})^2 + (y - y_{Dub})^2 \quad (1)$$

$$R^2_{May} = (x - x_{May})^2 + (y - y_{May}) \quad (2)$$

Dublin $(0,0)$, Maynooth $(-11, 3.5)$

$R_{Dub} = 4.5$ $R_{May} = 11$

Fill in (1) + (2)

$$(1) \Rightarrow (4.5)^2 = (x-0)^2 + (y-0)^2 \Rightarrow 4.5^2 = x^2 + y^2$$

$$(2) \Rightarrow 11^2 = (x+11)^2 + (y-3.5)^2 \Rightarrow 11^2 = x^2 + 22x + 11^2 + y^2 - 7y + (3.5)^2$$

Note, both equations have x^2+y^2 in, so if we subtract we get

$$4.5^2 - 11^2 = x^2 + y^2 - x^2 - 22x - 11^2 - y^2 + 7y - (3.5)^2$$

$$\Rightarrow 4.5^2 - 11^2 = -22x - 11^2 + 7y - (3.5)^2$$

$$\Rightarrow (4.5)^2 - 11^2 + 22x + 11^2 - 7y + (3.5)^2 = 0$$

$$0 = 32.5 + 22x - 7y$$ ← this is a line.

← In fact, it is this line.

(2)

Now solve for y

$$7y = 32.5 + 22x \Rightarrow y = \frac{32.5 + 22x}{7} \quad (3)$$

Remember $x^2 + y^2 = 4.5^2$, so fill in for y.

$$x^2 + \left(\frac{32.5 + 22x}{7}\right)^2 = 4.5^2$$

$$x^2 + \frac{1056.25 + 1430x + 484x^2}{49} = 20.25$$

$$49x^2 + 484x^2 + 1430x + 1056.25 = 20.25 \times 49 = 992.25$$

quadratic!

$$\Rightarrow 533x^2 + 1430x + 64 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-1430 \pm \sqrt{(1430)^2 - 4(533)(64)}}{2 \times 533}$$

Fill into (3)

$x = -0.04592$	$x = -2.637$
$y = 4.4997$	$y = -3.6461$
⇓	⇓
check $x^2 + y^2 = 4.5^2$ ✓	check $x^2 + y^2 = 4.5$ ✓
check $(x+11)^2 + (x-3.5)^2 = 11^2$ ✓	check $(x+11)^2 + (x-3.5)^2 = 11^2$ ✓

↑ North side solution.

GPS

GPS is a bunch of flying atomic clocks.

It wants to know (x, y, z, t) and solves for it using 4 equations (satellites).

Meteors

How are the meteors located?

- Locations of each camera is known.
- Distances aren't, but the direction the camera points is.
- So, we can measure the angle and use *triangulation*.
- Uses a bit of trigonometry.
- With enough equagions (cameras) we get the location.

Not too hard, but best done by coding it up for a computer.

Citizen Science

Great to get *real* people involved in the science, . . .
. . . while hiding some of the boring stuff.

Why?

- In theory anyone can check the science themselves.
- Getting involved helps in lots of ways
  - Advancing the science,
  - Understanding limitations of data, analysis, . . .
  - Thinking about what science you/we should do,
  - Giving ownership of a bit of science,
  - Giving context for the science,
- And, examples of where all that maths is used.

Thanks!

Problem with midnight — it depends where you live.

Dunsink Observatory

Usher: 25m7–48s (1787).
Brinkley: 25m22s (1832).
Romney-Robinson: 25m21s (1838).
Elliott-(Ray-Drury-Malone): 25m21.02s (2017).

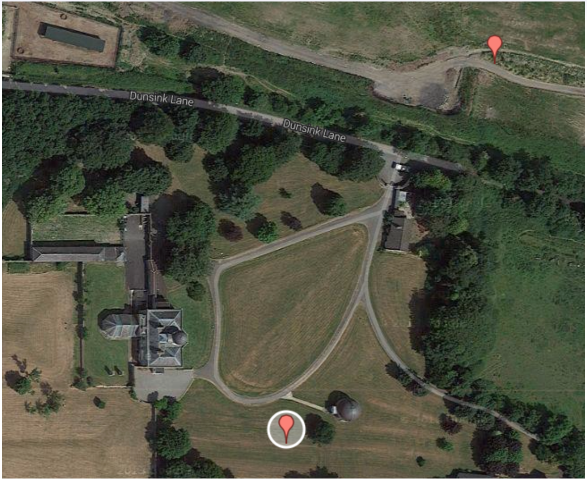


Image: Google Maps

$$6^\circ 20.3', 53^\circ 23.2' \text{ vs } 6^\circ 20.2', 53^\circ 23.3'$$

Used ET, based on Newcomb's measurements from 1750–1892.

Tidal forces (1.7ms/d/c) mean UT and SI seconds are different (by 2.5ms).

Coordinated Universal Time is a compromise. It ticks once per SI second, in sync with TAI.

If UTC is more than one second from UT1 then UTC is adjusted.

Doppler

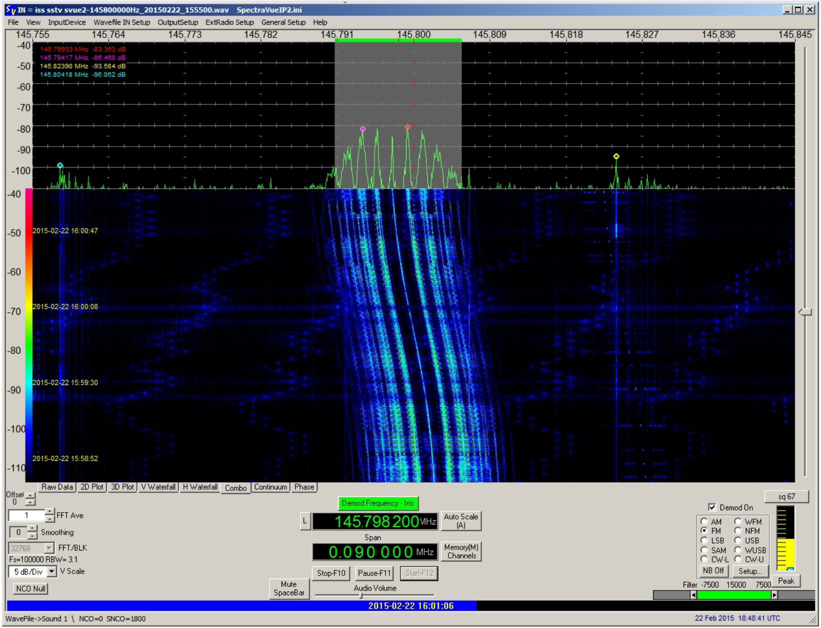
Waves pile up if you move towards source, or source moves towards you.

$$f = \frac{1 + \frac{v_r}{c}}{1 + \frac{v_s}{c}} f_0$$

If you aren't moving fast compared to the wave

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0$$

Where Δv is the relative speed of source and receiver.
There is also a version that takes relativity into account!



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145.79933 MHz -83.363 dB
145.79417 MHz -86.460 dB
145.82390 MHz -93.584 dB
145.80418 MHz -96.062 dB
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2015-02-22 15:58:52
Raw Data 2D Plot 3D Plot V Waterfall H Waterfall Combo Continuum Phase
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Auto Scale (A)
Span
0.090 000 MHz
Memory(M) Channels
FFT Ave
Smoothing
32768 FFT/BLK
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5 dB/Div V Scale
NCO Null
Mute SpaceBar
Stop-F10 Pause-F11 Start-F12
Audio Volume
Demod On
AM WFM FM NFM LSB USB SAM WUSB CW-L CW-U
NB Off Setup...
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Peak
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22 Feb 2015 18:48:41 UTC