A Note About This Project

This project has no sources to quote from for two reasons

i) It is a piece of individual research.

ii) I had intended comparing my work to other similar work but I was denied access to the university libraries and other libraries were not up to date or extensive enough to even mention the topic of this project.

For the purposes of record the following equipment was used in the research.

i) Lots of paper , pens and manuscript.
ii) A piano.
iii) 2 computers an Amiga and a Spectrum.
iv) A 'C' compiler and a word processor.

A disk with the programs written is included. This is not an essential part of the project but may make parts of the project clearer. This disk will run on any Amiga computer.

A tape is also included. The sounds on the tape should be approached with both an open mind and a view to them being played on instruments rather than computer.

The musical extracts within the project are largely a demonstration of the results of a method rather than for show them selves. Ideally the methods outlined should be experimented with by the reader to enhance understanding.

David Malone March 1992

Fractal , Mathematical and Algorithm Generated Music

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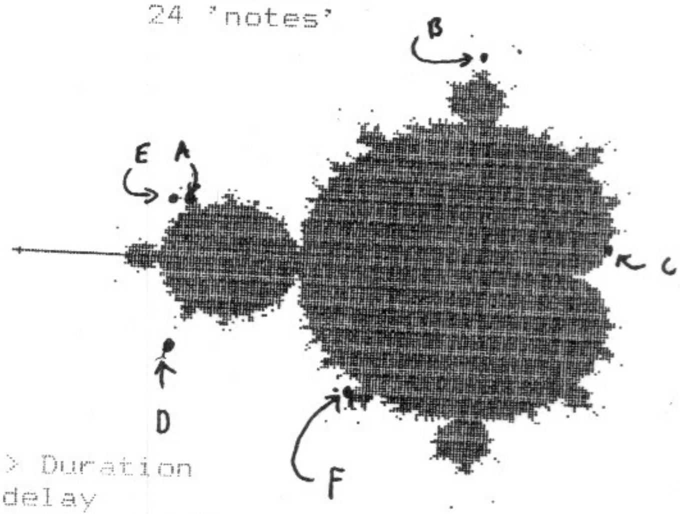
Tape Index:

Fig Tree

r = 3.1	8 'notes'
r = 3.5	16 'notes'
r = 3.59	16 'notes'
r = 4	24 'notes'

Mandelsound

From Point A
From Point B
From Point C
From Point D
From Point E
From Point F



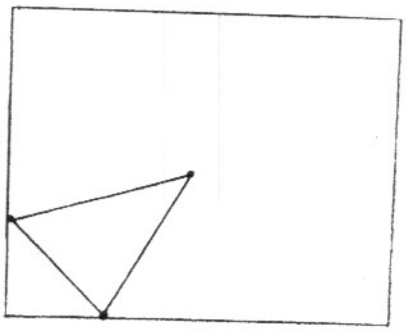
Lorentz

x -> Pitch	z -> Duration	
x -> Pitch	No delay	
x -> Pitch	.02 second delay	
x -> Pitch	y -> Pitch	z -> Pitch

Triangle

The corners were in position as shown

1)



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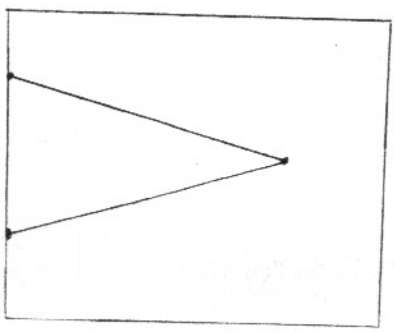

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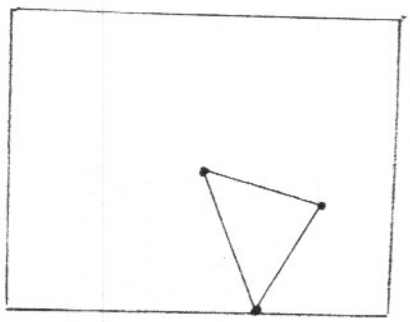

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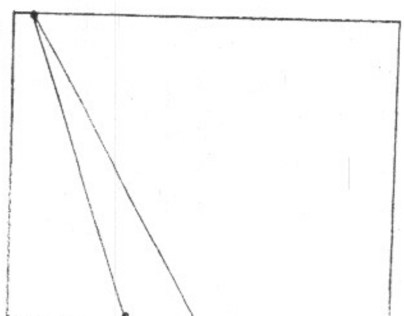


Introduction

Music has two parts to its being. This split is even displayed in the make up of the Leaving Cert. These two parts are composing and performing. Composing is the aspect which I would like to examine in an unusual light. Composing is the art of taking something and representing it with sound. That something is often an emotion, place or thing (eg. anger, a castle or the birth of Christ). The things I would like to represent with sound are shapes , numbers and processes. This project is really an experiment or piece of research to see if the sounds in question are musical. If no music is found the project is still a success as the aim is to search not to find.

So how do I aim to go about this. Well the idea is not as far fetched as it seems. Mozart composed a framework called 'Musical Dice Game' where by the rolling of two dice, waltzes could be created. The minuet and trio has a form similar to 'Fracatal' patterns where each large piece is similar to smaller pieces (see later for more detail). Such methods as inversion , augmentation or reversal of a line of music are used to expand a piece. Without a doubt these are similar to the recipical , multiplication and negation (making a minus a plus) of maths.

So what things am I to examine in the process of completing this project.First I will look at the significance of the number 2 in music. Then I will make my first attempt to create music from a shape known as the 'population tree' or 'fig tree'. Some famous shapes in mathematics - the 'Mandelbrot' and 'Julia' sets come under examination next. These two are fractals. Another shape which actually represents weather patterns will be looked at. It arises when you solve some differential equations known as lorentz's equations. A family of shapes with a special relation to polar equations are changed to sound next. This family includes spirals and circles. Two more fractals are now considered. These are a fractal triangle and a fractal snowflake. Finally a method for producing potentially fractal counterpoint is examined.

Please note that there are appendices dealing with some of the technical topics of the project.

Appendix	Subject
1	Fractals
2	Complex Numbers
3	Differential Equations
4	Polar Co-ordinates

Music and the Number 2

Two is one of the numbers you can't get away from. In music it crops up in the two most basic elements of musical sound - pitch and duration.

Pitch: As I'm sure you know if you multiply the length of a string by two the pitch goes down an octave. If you divide by 2 it goes up an octave. As it emerges - in the modern system of tuning an instrument to move up a semi-tone you divide the number by the twelveth root of 2. This is the number that when multiplied by itself 12 times gives 2. This number is about 1.059463094. This means that if you have a string and you want to move up n semi-tones you divide by the twelveth root of 2 n times or if you would like a formula.

$$L_n = L_0 \div 2^{n/12}$$

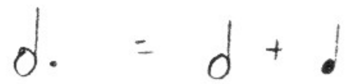
L_0 Original length
L_n New length

Duration: I think 2 simple charts will show you the relationship here.

Symbol	Value (in ♩)	n	2^n
𝅜	8	3	8
𝅝	4	2	4
𝅗𝅥	2	1	2
♩	1	0	1
♪	$\frac{1}{2}$	-1	$\frac{1}{2}$
𝅘𝅥𝅯	$\frac{1}{4}$	-2	$\frac{1}{4}$
𝅘𝅥𝅰	$\frac{1}{8}$	-3	$\frac{1}{8}$
𝅘𝅥𝅱	$\frac{1}{16}$	-4	$\frac{1}{16}$

Quite a strong link there I think.

It is interesting to note we have a special symbol for adding half the length to a note.

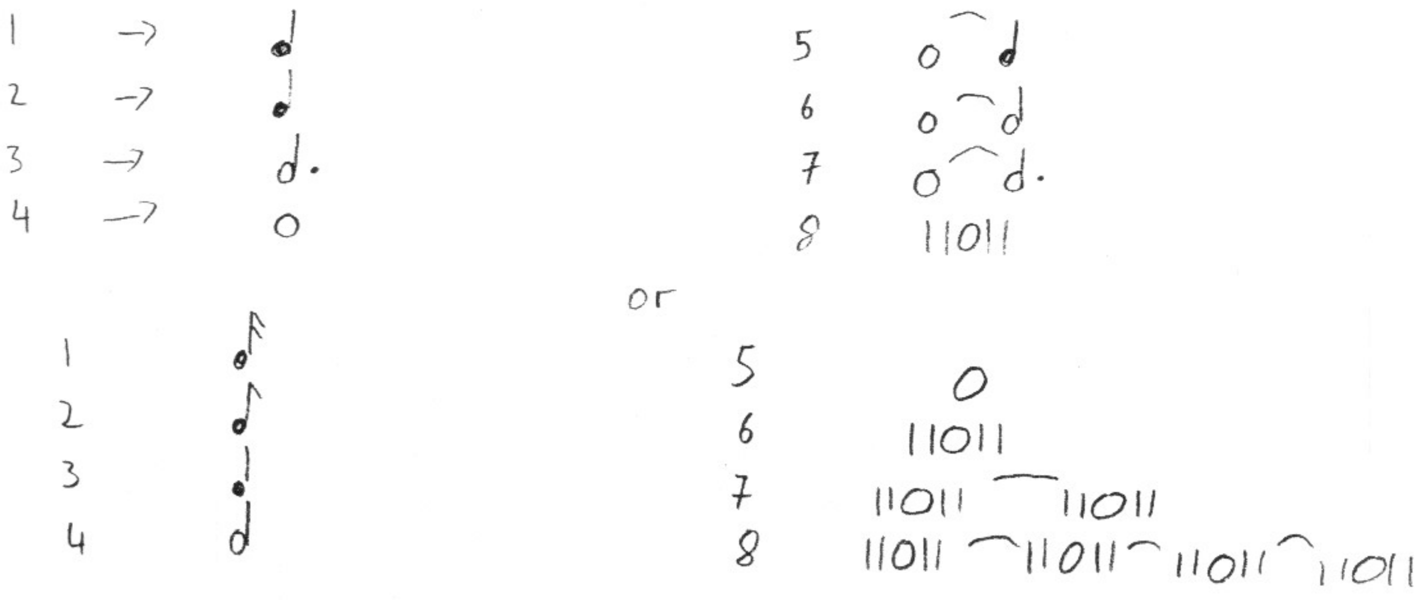


As a result of this link to the number 2 we need special notations to show note values linked to other numbers like triplets. A triplet may be written as infinitely many shortening notes tied together.

Why talk about all this. Well this relationship between numbers and notes gives me a way to interpret numbers as notes. Taking some numbers and turning them into notes I will call 'mapping'. For example maybe I want to map the numbers from 1 -> 8 onto a major scale. I might create a map which looked like this.

1 -> C
2 -> D
3 -> E
4 -> F
5 -> G
6 -> A
7 -> B
8 -> C'

Or I could map them onto duration in two ways (or maybe more).



Now I need some numbers to try creating mappings for. Unfortunately not all numbers are whole numbers (infact a majority are not whole numbers). I have considered two ways around this.

i) Round off the numbers to the nearest whole number.

ii) Create a mapping that makes notes of less than a semi-tone apart and get a computer to play them.

This idea of mapping is central to the first section of the project. I know what music sounds like and I know how to create these shapes the problem is to search for a good mapping which will take the shapes and turn them into music.

The Population Beep

What is the population tree ?

The population tree is an interesting graph of population vs. ability of the population to change. Imagine a large number of rabbits in a field. If there are too many rabbits they will begin to die due to lack of food. If there are only a few rabits then population will grow due to the abundance of food. A simple model of this population was constructed as follows.

P(n) = the population in year n
P(n+1) = the population in year n+1

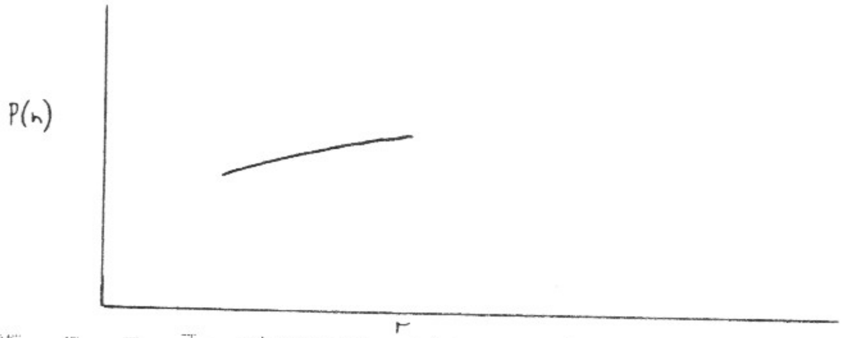
P(1) = the population in year 1 = .1 thousand rabits

and then you use the rule

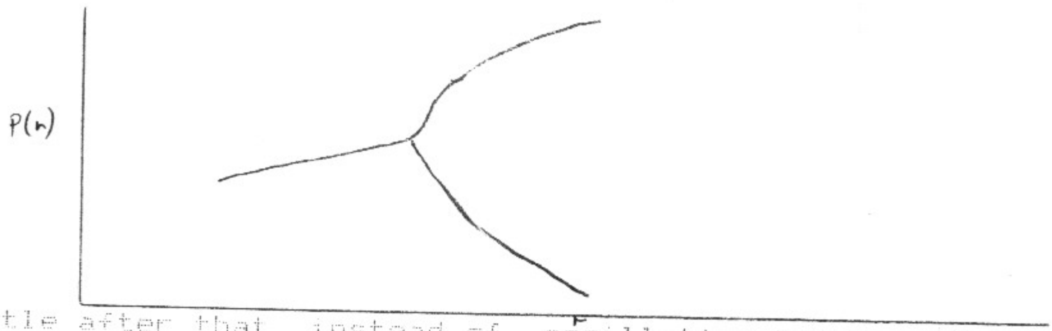
P(n+1) = r * P(n) * (1 - P(n))

where r is the ability of the population to change itself. Then you get your computer to see what happens after the first 100 years.

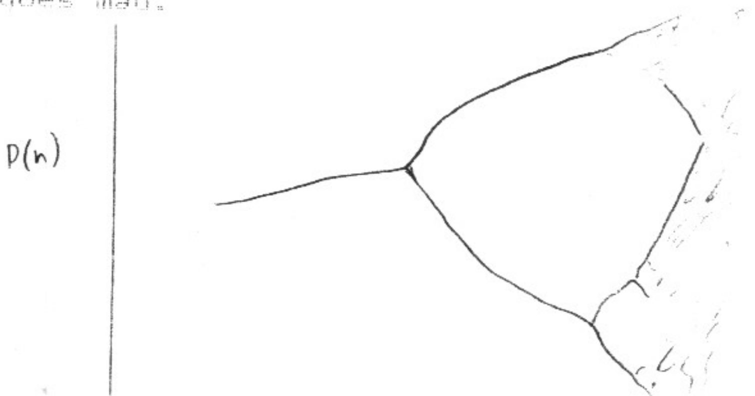
What does happen? Well if is up to about 3 the population settles down.



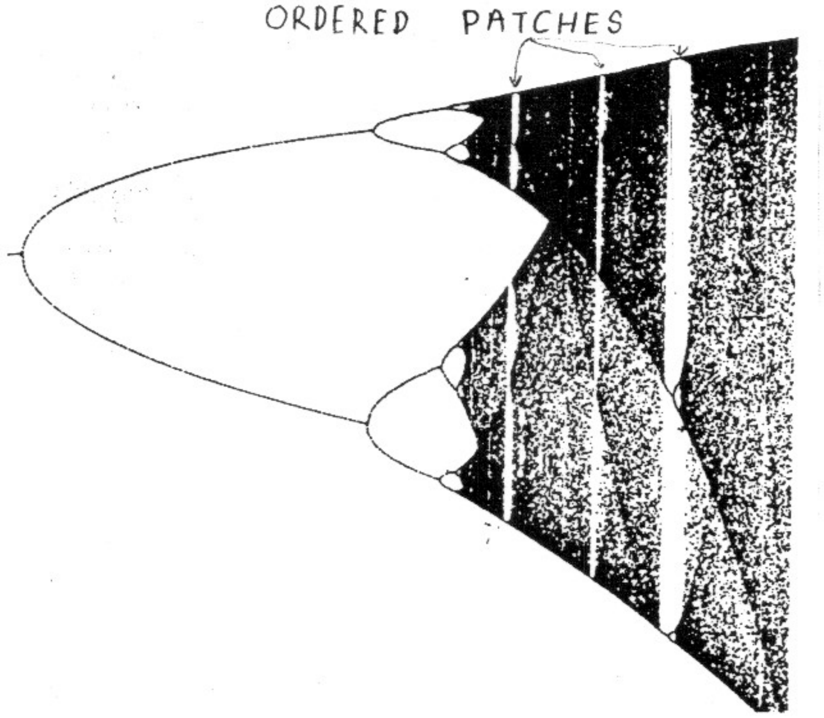
After r = 3 strange things begin to happen. The population oscillates from year to year.



A little after that instead of oscillating between 2 levels it wobbles between 4 , then 8 , then 16 and so on and so on until it goes mad.



Surprisingly though in the middle of this madness there is patches of order similar to the original order.



Why did I pick this shape for examination ? Well it produces a list of numbers of varying degrees of randomness. Some of them loop round and round. I hoped to find a repeated 'theme' or recognizable tune in some of these numbers.

So what map did I use. I felt I had two choices.

i) population -> frequency (no rounding to whole numbers)

ii)population -> chromatic scale (rounding to whole numbers)

After some experimentation I decided the second produced the better results. The pieces produced all sounded random and also the lack of rhythm made them monotonous. As my teacher reminded me rhythm is an essential part of music.

<u>Problems encountered:</u> Theywere largely technical - getting the computer to produce the desired sounds. This problem was over come by examining programs written by others to produce similar beepy noises.

The main failing was the lack of rhythm. I now had to think about obtaining 2 numbers one for pitch and the other for duration. I decided to look to complex numbers for inspiration.

Mandelbrot and Julia Sets

NB. Appendix 2 is useful in the reading of this chapter.

Julia was a french mathematician who now has a family of shapes named after him.These shapes arise from the following process.

i) Take a comolex number call it c.

ii) Take a point on the Argand diagram - call it Z(Ø).

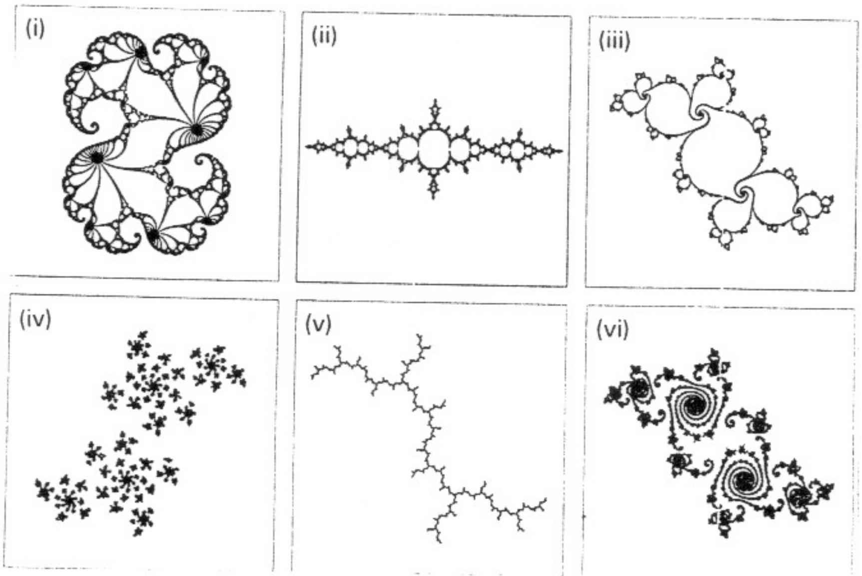
iii) Repeat the process

$$Z(n+1) = Z(n)^2 + c$$

If |Z(n)| gets bigger and bigger as time goes on colour the point Z(Ø) white. Otherwise color it black.

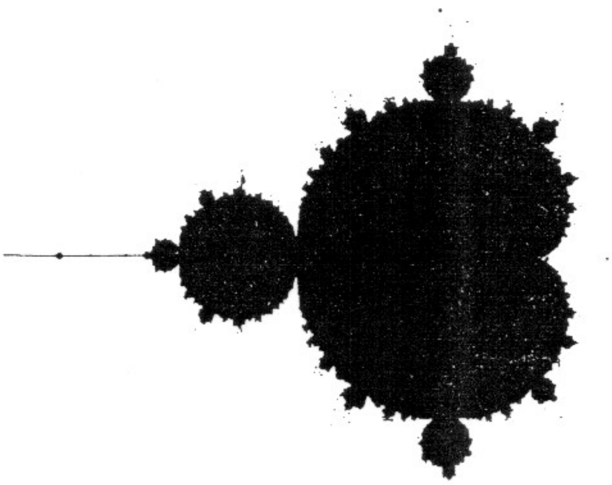
iv) Pick another point for Z(Ø) and go back to step iii.

As more and more points are 'checked' in this fashion a picture begins to build up. Some of these figures are solid and some like dust. They are often very pretty verging on beautiful.A second mathematician called Mandelbrot made a map of which of these were dusty and which were solid. this is the beetle like mandelbrot set.



ASORTED JULIA SETS ↑

↓ THE MANDELBROT SET



I decided to use these sets as because the numbers(Z(1), Z(2), Z(3), etc.) generated by them had two parts and one could be used as pitch and one as duration. I wrote a program which draws a Mandelbrot set and allows you to select a point. Values for the set of numbers were calculated and then mapped onto notes. The Real part of Z(n) was mapped onto a chromatic scale and the imaginary part onto several note lengths. I had to experiment to get a suitable set of note lengths.

I found after some time that points close to the edge produced the best results. Some points I picked actually seemed to fit into bars and would have made good bass lines for 'Hardcore' dance music.

This section was more successful than the previous producing varied results. There is room for expansion of this idea as there are whole families of Mandelbrot and Julia sets to be exploited in this manner.

Lorentz Attractor

NB; Appendix 3 is useful in the reading of this chapter.

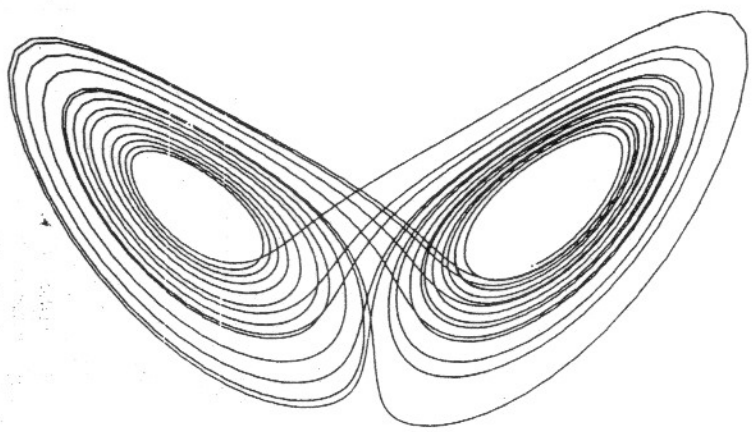
Edward Lorentz was a mathematician who, in the early 60's was doing some work on modeling weather systems. He stripped the weather of all its bells and whistles and came up with three differential equations which looked something like this.

$$\frac{dx}{dt} = -10x + 10y$$

$$\frac{dy}{dt} = 28x - y - xz$$

$$\frac{dz}{dt} = -\frac{8}{3}z + xy$$

If the xy and xy weren't in these equations they would be pretty easy to solve but the xy and xz leave only one option open. brute force - use a computer. The solving of these equations led to the discovery of the 'Butterfly Effect' - which is quite nice as the computed solution to the equations looks quite like a butterfly.



So what did I do with this butterfly ? Well I took several steps in mapping it before I settled for a finished product....

i) I took two from x,y,z and mapped them onto pitch and duration. the effect was rather dull.

ii) Then I thought - "I'm solving these equations with respect to time (the 'dt' bit of the equation means with respect to time). So why don't I just change the pitch of the note at regular intervals according to the numbers I get". I tried this and got a noise that sounded like a special effect from a cheap film.

iii) "Its changing too fast!". This indeed was the problem. I experimented until I found that a gap of slightly more than .02 of a second between each change of pitch. This created a wandering tune. I was still using x as pitch at this stage.

iv) "I have three variables here , why don't I use all three and have a three part 'harmony'." So I tried this. The result sounded like the wind. It was quite enchanting in its own way. The coincidence of getting the sound of wind from equations which are derived from the weather added to the effect.

This I think demonstrates what can be produced from these shapes if the correct mapping is used. If you like the sound or not you can not deny the charm of the sound of wind from these equations. They lead to the discovery of the butterfly effect and they look like a butterfly. They are a model of the weather and they sound like the wind. Is God a musician and mathematician ?

Music From Some Polar Shapes

NB: Appendix 4 useful in the reading of this chapter.

Shapes which are in polar form are from a technical point of view Ideal for the type of mapping I have being considering. Any given point has two values (x,y) calculated from a third (θ). It is simpler to get a point on the curve than it was for the fractal ones.

In polar co-ordinates

$x = f(\theta) * Cos(\theta)$

$y = f(\theta) * Sin(\theta)$

Where Sin and Cos are the Trigonometrical functions and $f(\theta)$ is some simple function of θ such as

$f(\theta) = 5$ (A circle)

$f(\theta) = \theta$ (An Archimedean Spiral)

$f(\theta) = 1 - Cos(\theta)$ (A cardioid).

So what did I try ? Well what I did was I took points on shapes at 30^0 intervals and mapped the x value onto duration and the y value onto a major or chromatic scale.

If you look at the circle example included two should see a point marked 'A'. If you follow point 'A' up you should see it lies under the box with a semi-breve in it. So the note this point represents is a semi-breve. Then read across , you should find it is level with the box marked 3. If you look this up on the chromatic scale you will find the chromatic equivalent of this point is a semi-breve A. Likewise the major equivalent is a semi-breve F.

The second example is an archimedean spiral.

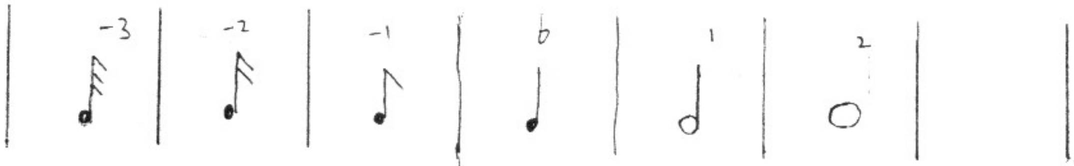
The third example is a cardioid.(It's supposed to look like a heart , so maybe these pieces should be called love).

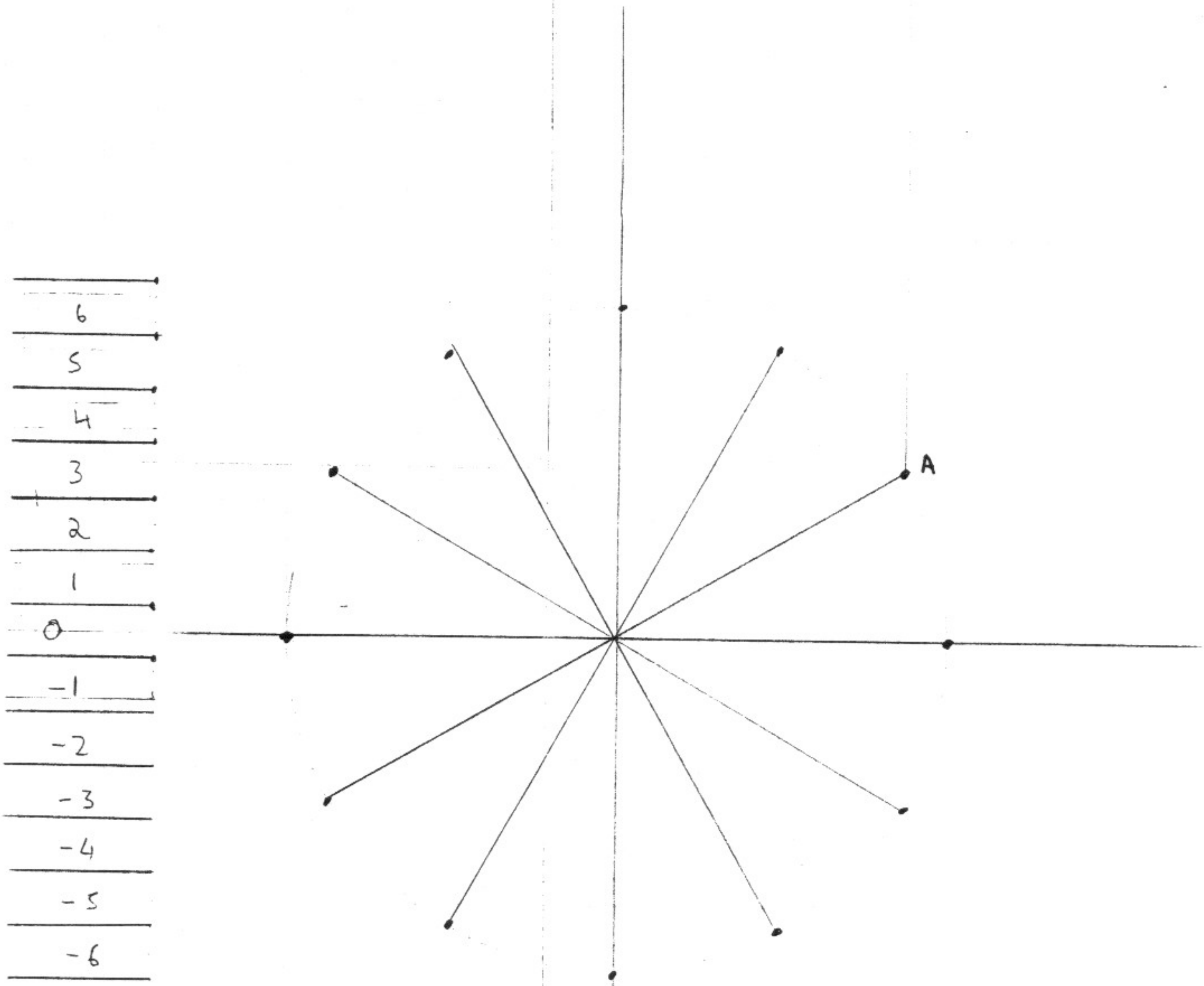
The forth example is a flower like object.The reason I took 4 pieces from this one was because they possess a very serialist quality. Watch for the inversions and reversals.

The fifth example is the same shape as in example 4 but twisted through 45^0. The inversions and reversals remain but not as clearly as above.

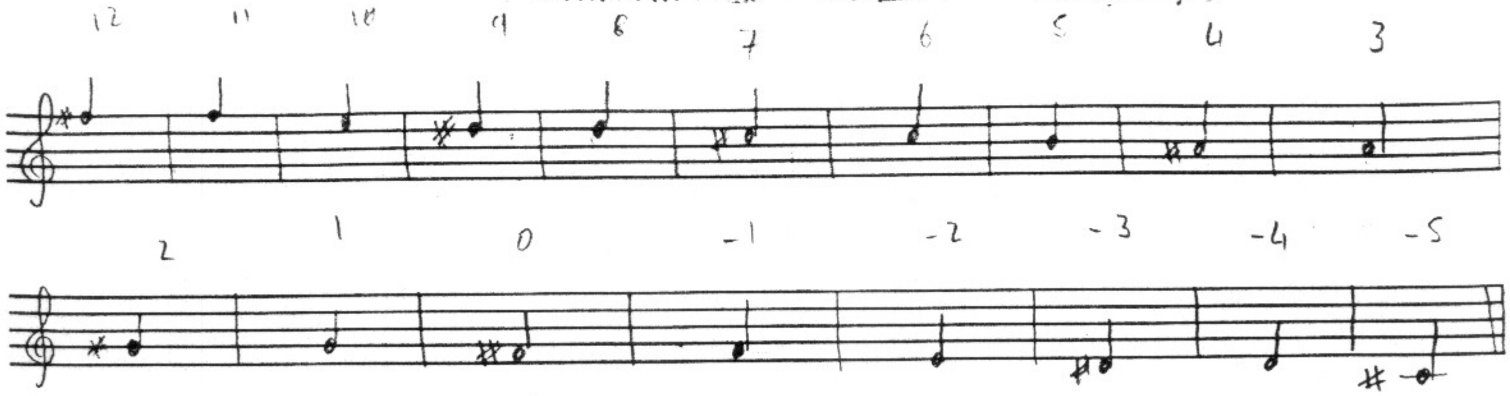
The sixth example is also in possession of some symmetries. If you turn this one upside down and back to front you are left with the original tune.

The final example was my attempt to find the most exotic shape with a simple function. Play it and make up your own mind.


-3
-2
-1
0
1
2


6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
A

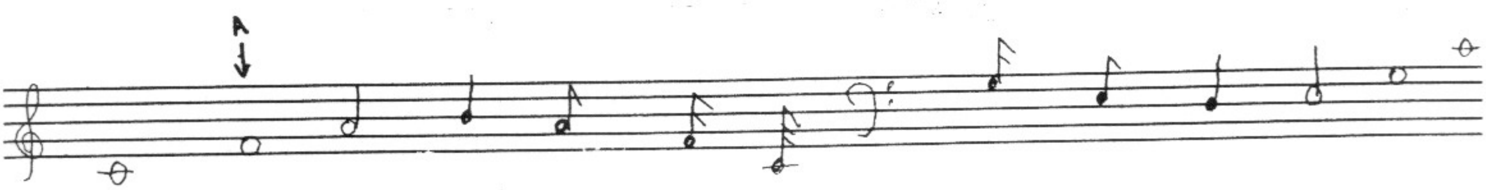
CHROMATIC KEY



MAJOR 'KEY'

THE CIRCLE - CHROMATIC

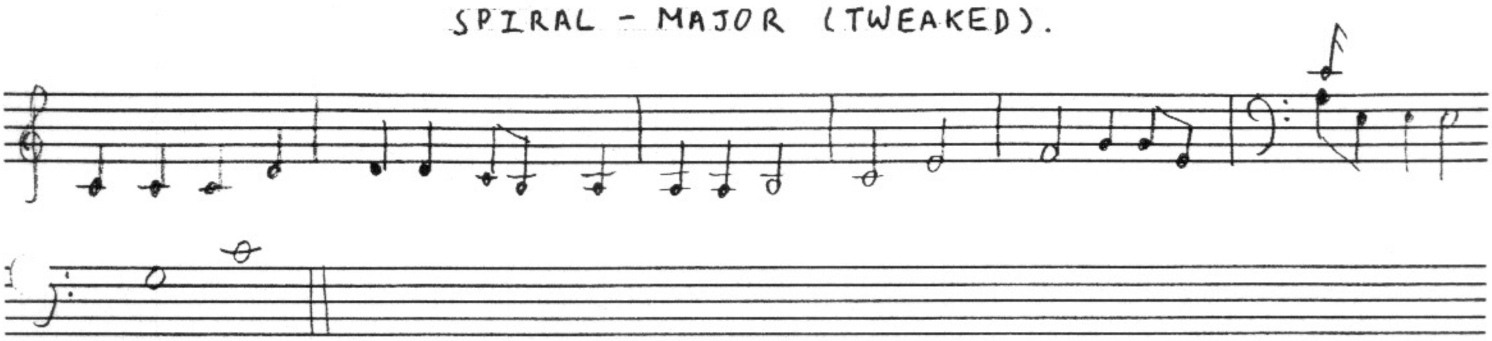
THE CIRCLE - MAJOR



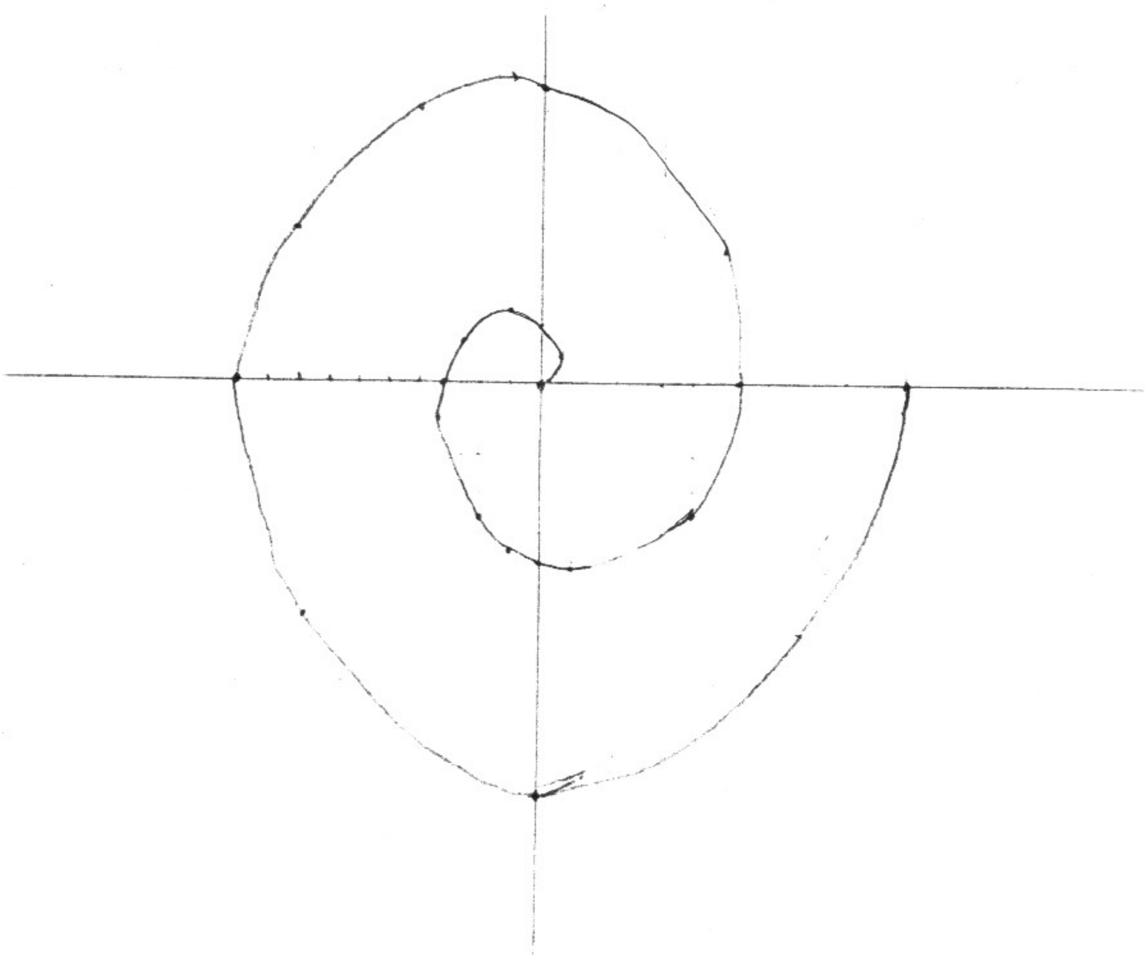
THE CIRCLE - TWEAKED.

SPIRAL - CHROMATIC

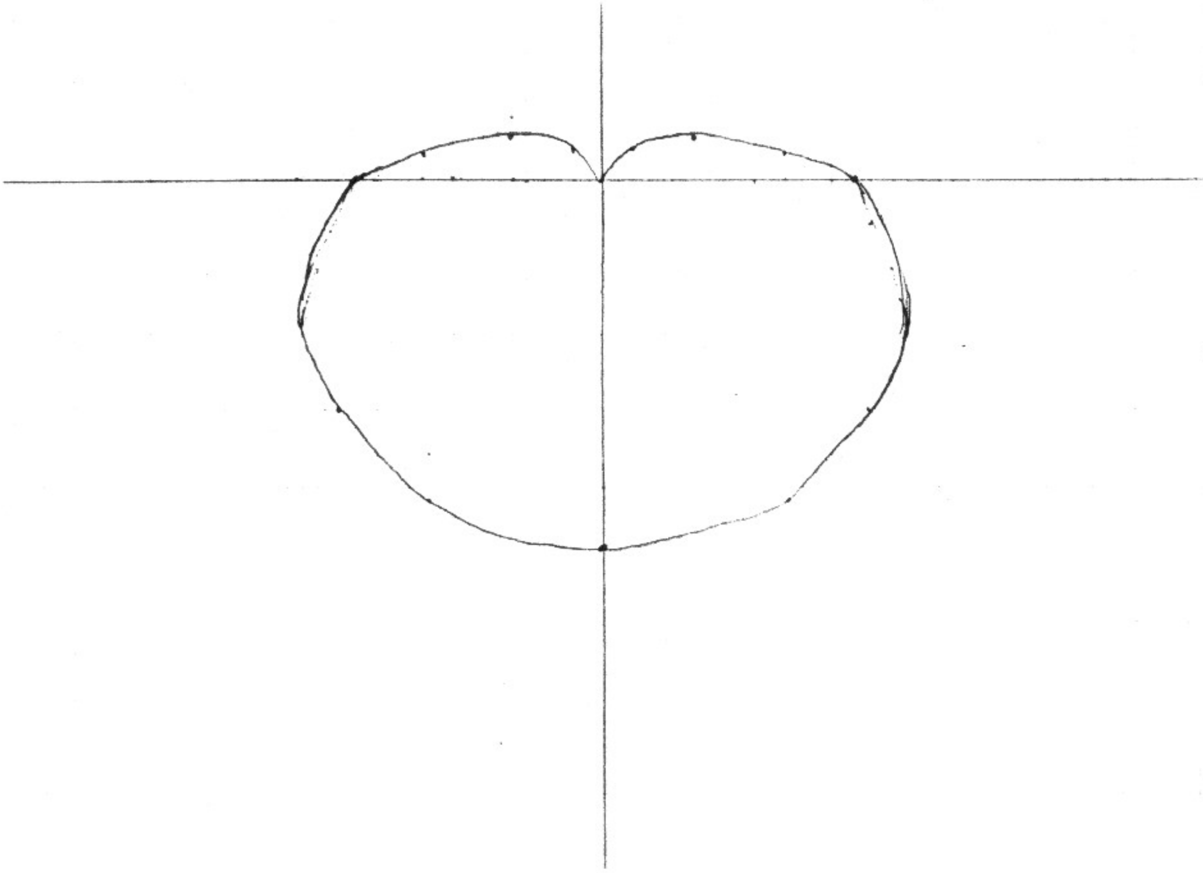
SPIRAL - MAJOR (TWEAKED).



SPIRAL - R=Θ

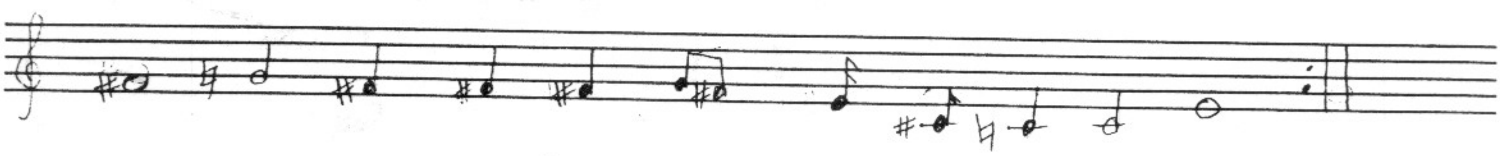


Cardioid $R = (1 - \sin\theta)$

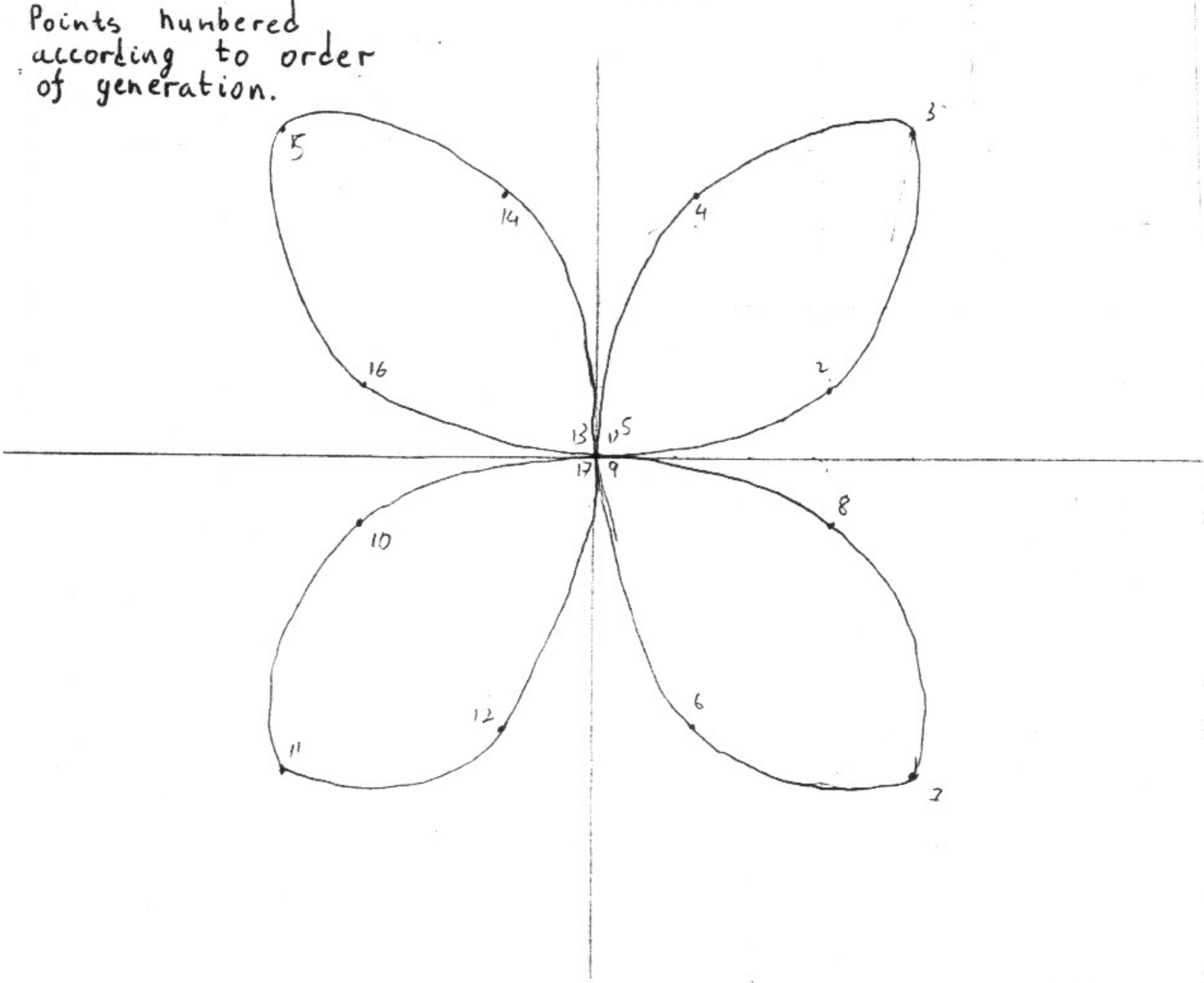


THE CARDIOID - MAJOR

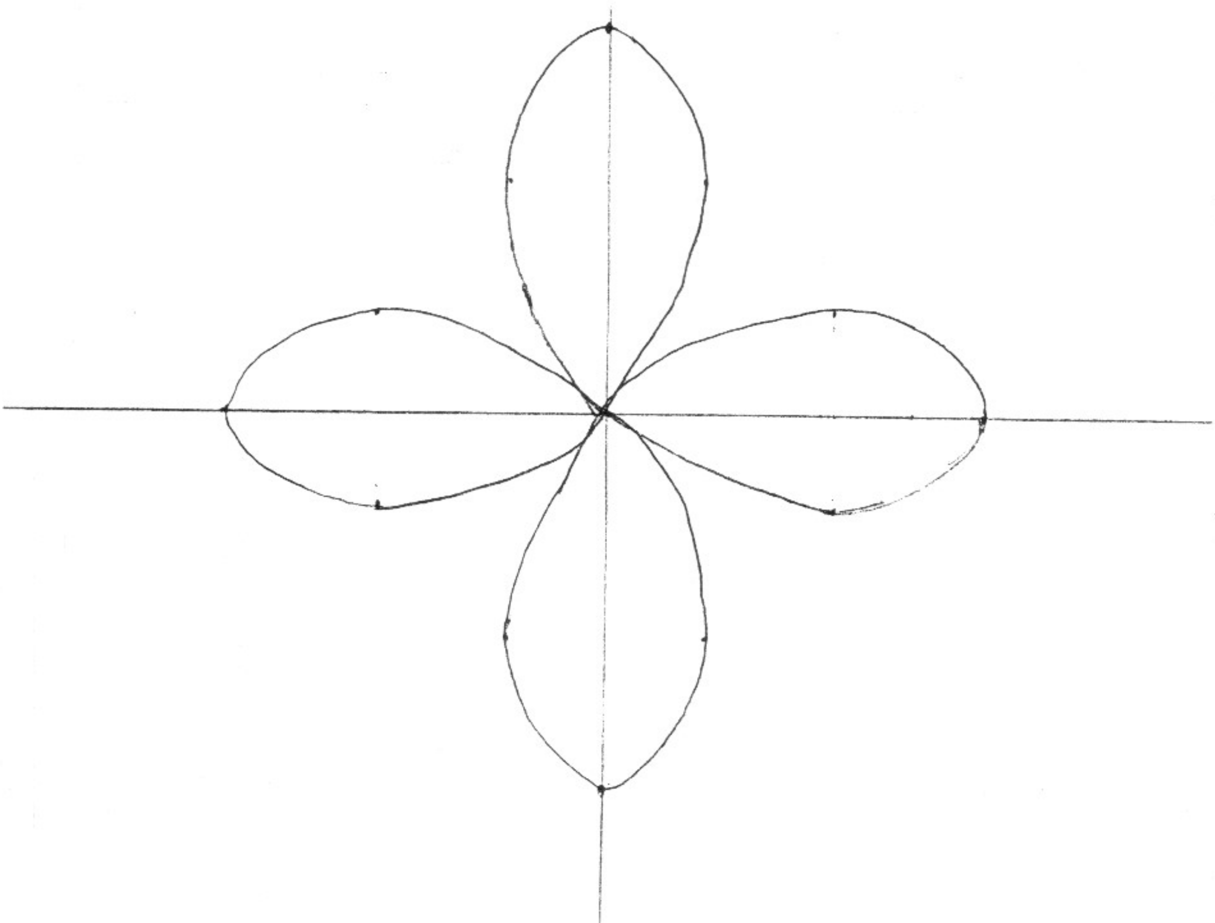
THE CARDIOD - CHROMATIC.



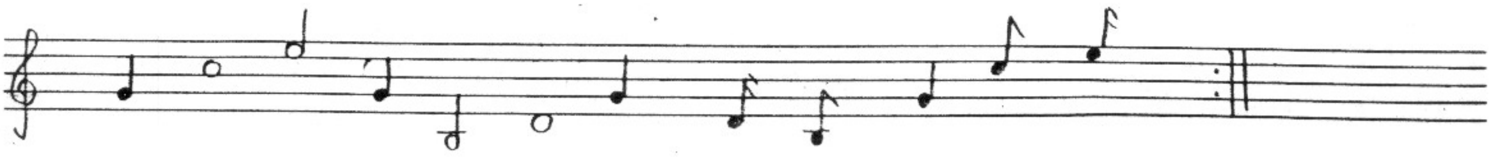
4 PETTLE ROSETTE
R = Sin(2θ)



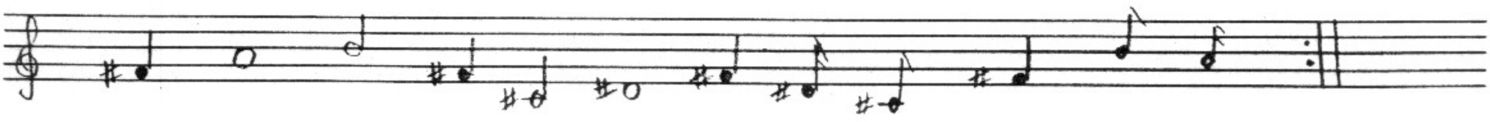
4 Pettle Rosette II (R= Cos 2θ)



4 PETTLE ROSETTE - MAJOR

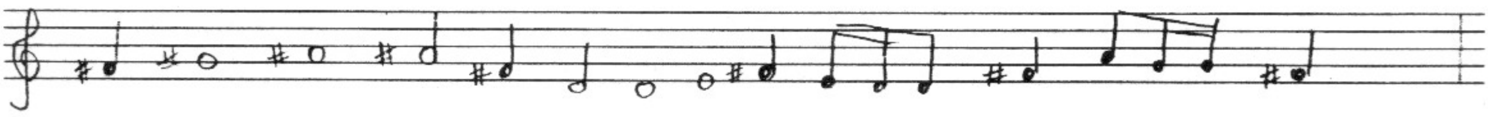


4 PETTLE ROSETTE - CHROMATIC



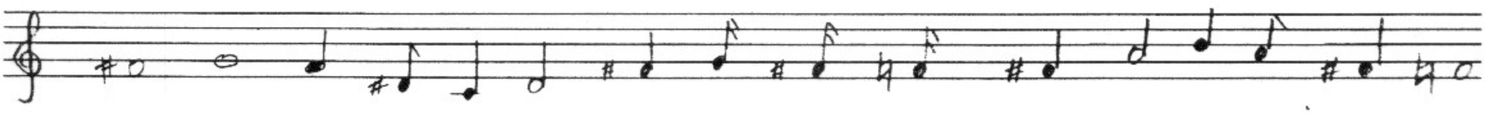
4. PETTLE ROSETT - MAJOR (ANGLE CHANGE = 22.5°)

AS ABOVE - CHROMATIC



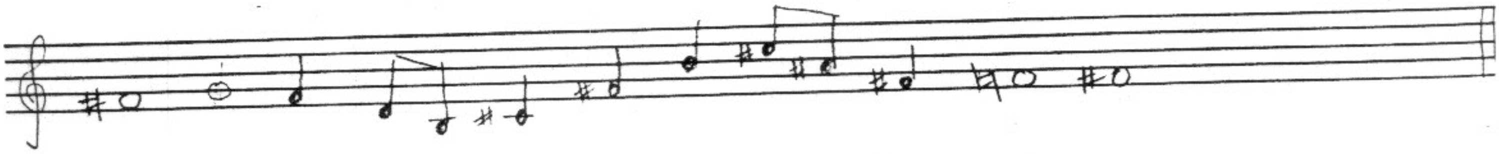
4 PETTLE ROSETTE II - MAJOR.

4 PETTLE ROSETTE II - CHROMATIC

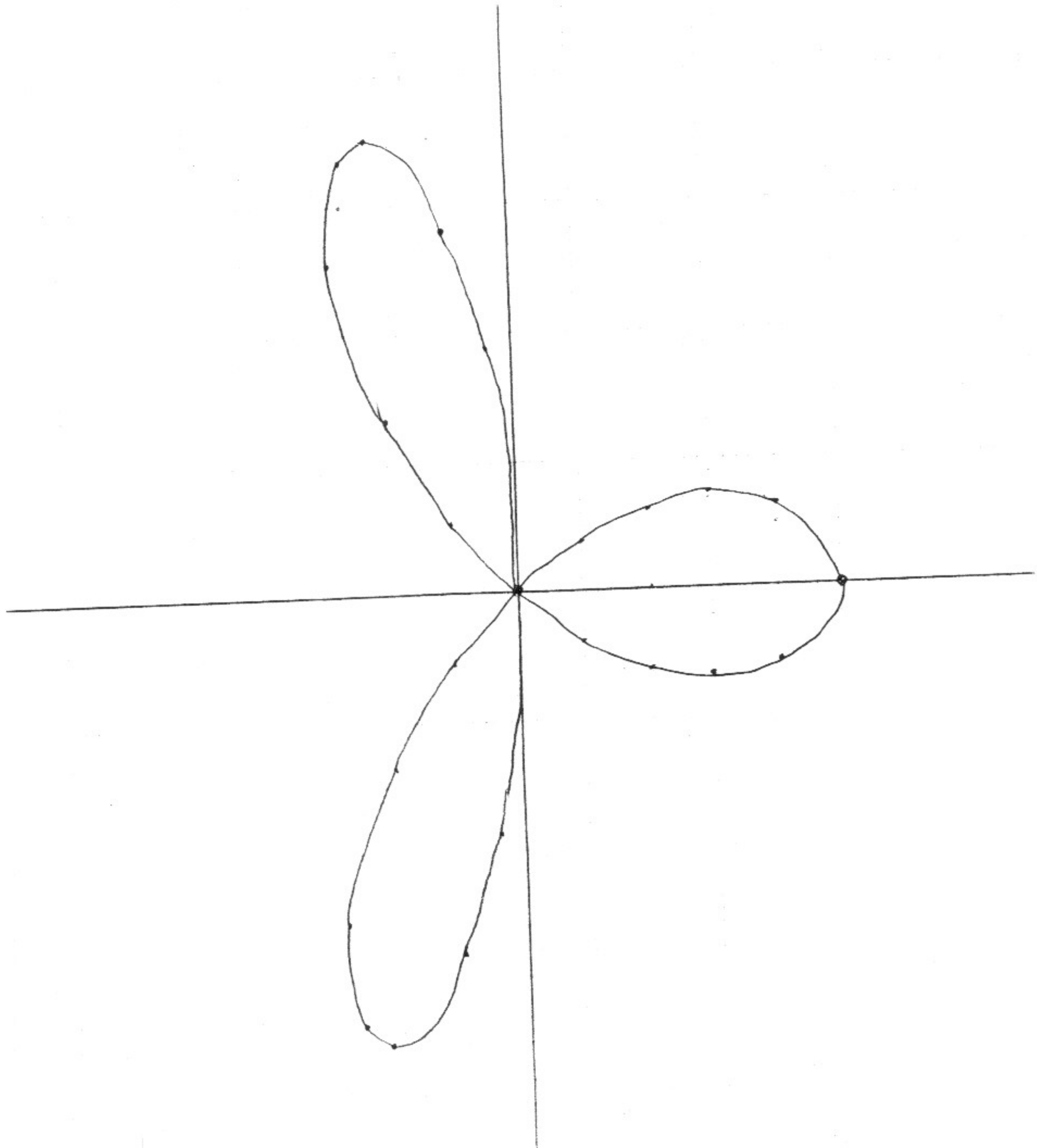


3 PETTLE ROSETTE - MAJOR.

3 PETTLE ROSETTE - CHROMATIC



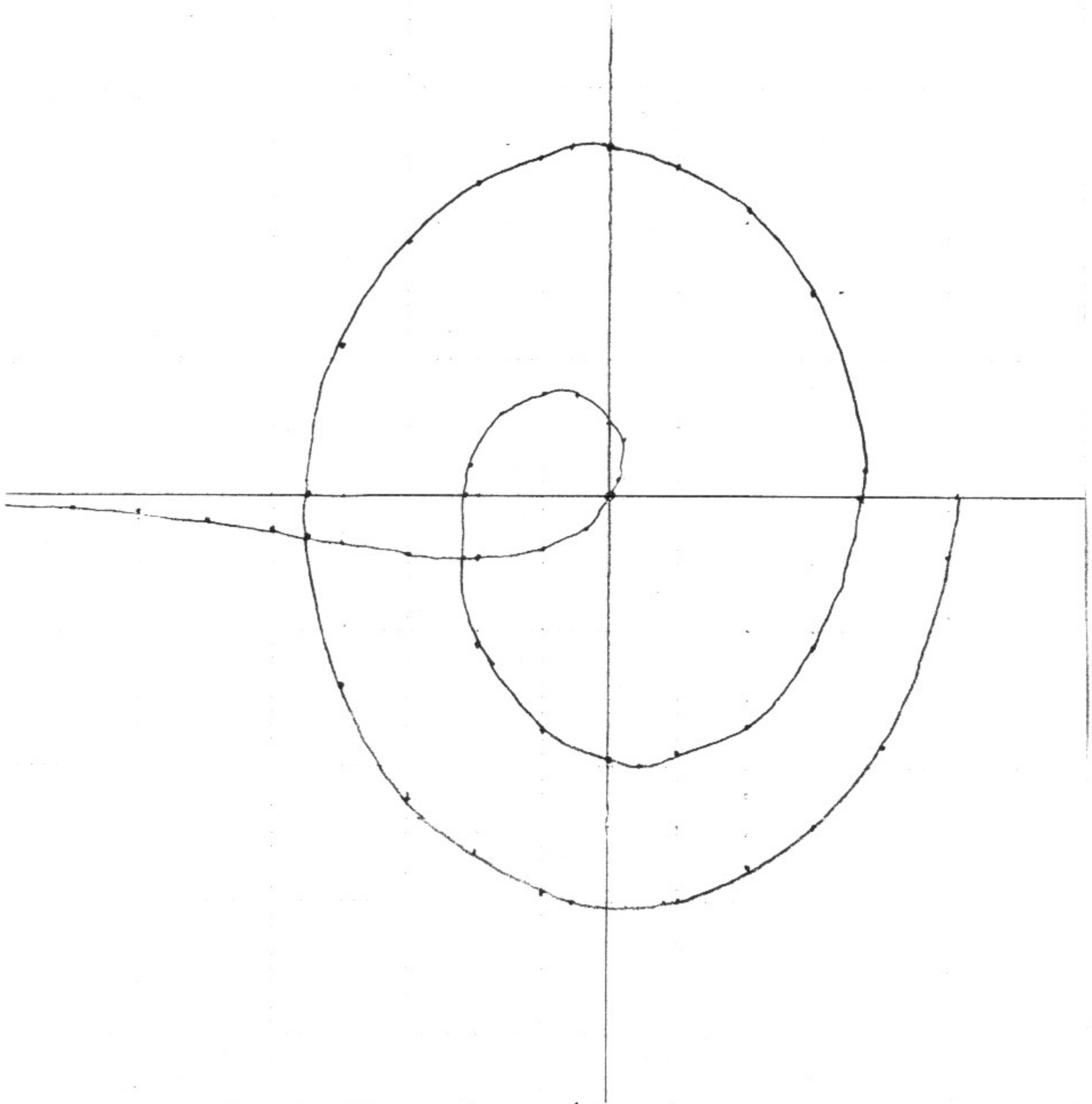
3 Pettle Rosette $R = \cos(3\theta)$



The only problem I encountered with this method was having to stretch the shapes so they would fit nicely onto the map. If you look at the three pettle rosette you will see how it is distorted.

After so many examples I am sure you are capable of assessing the success of this method. I personally like the sound of the circle.

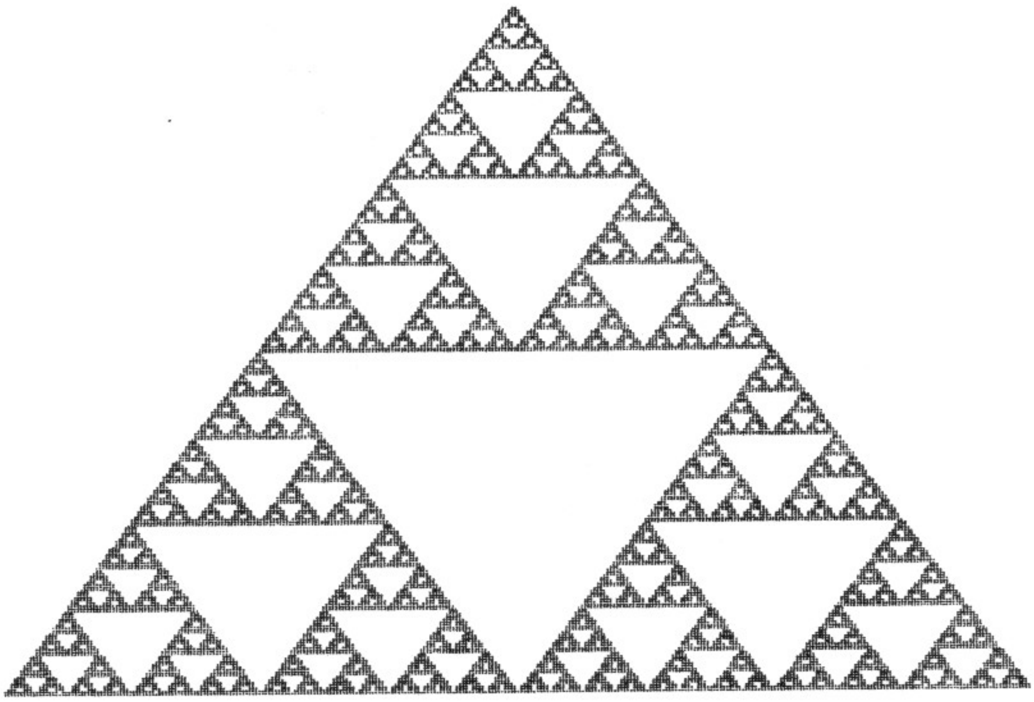
SPIRAL II $R = Ln(\theta)$



Spiral II chromatic.

Triangles

The next shape I considered was a much more simple shape. It is a process where if you are given the corners of a triangle you can fill them with the following pattern.



What you do is :

i) Pick a corner, call it P..
ii) Pick a corner at random. Move P half way toward this corner . Colour P in black.
iii) Repeat from step ii.

Surprisingly the shape that emerges as you repeat this process is the one above. This time I worked with the points generated by this process. The program allowed you to pick the corners of the triangle and the played and drew the points.

I tried mapping the co-ordinates in the following ways.

i) x --> Duration (in fiftieths of a second)
y --> Period

ii) x --> Duration (𝅝 𝅗𝅥 ♩ ♪ 𝅘𝅥𝅯)
y --> Period

iii) x --> Duration (𝅝 𝅗𝅥 ♩ ♪ 𝅘𝅥𝅯)
y --> Chromatic scale

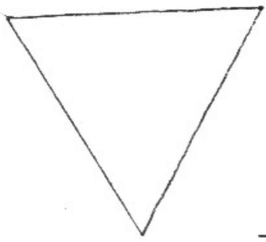
iv) x -> Chromatic scale
y -> Chromatic scale
Timing was in a manner similar to the final product of the Lorentz sound.

The results were singly unimpressive and dull. The process is just too random and the sound made tended to sound like back round noises from early Dr. Who.....
The simplicity of this shape is its beauty geometrically but is its death musically.

The von Koch's Snowflake

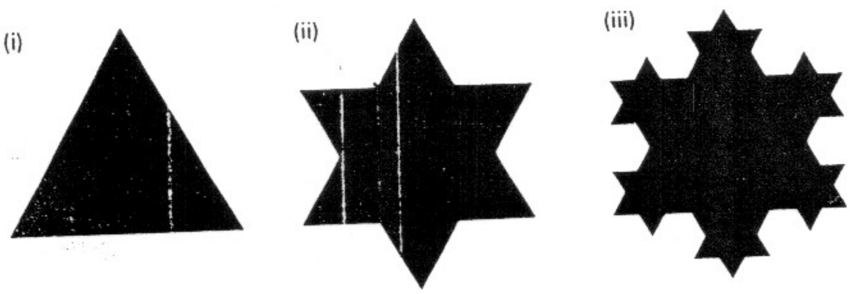
This object is quite a pretty figure with some surprising properties.

To draw it you start with an equilateral triangle



and use the rule

So every time you find a --- replace it with -^- .The picture below shows what happens after the first couple of times.



The figure you would eventually end up with (if you repeated this processes indefinitely) has an infinite perimeter but a finite area. The idea of replacement and self similarity was what appealed to me. I wanted to try a musically similar Idea where you start with a musical fragment and work up. This is similar to several Ideas already used in music. The first is the minuet and trio which is of the form

```
            A          B          A
```

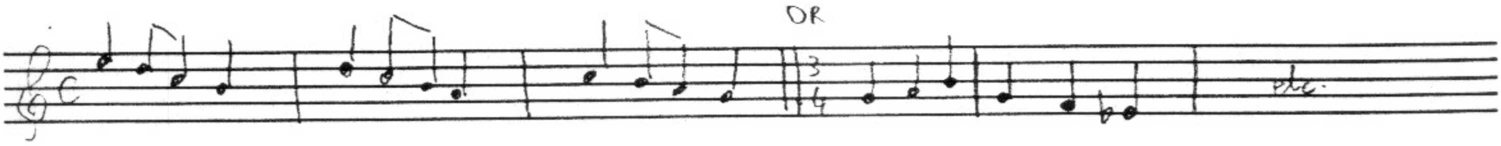
But each can be replaced by a further ABA

```
            A          B          A
           /|\        /|\        /|\
          / | \      / | \      / | \
        A   B   A  A   B   A  A   B   A
```

This is the same idea on a macroscopic scale.

The next Idea which this is similar to is augmentation where the value of a note may be doubled or even quadruple.

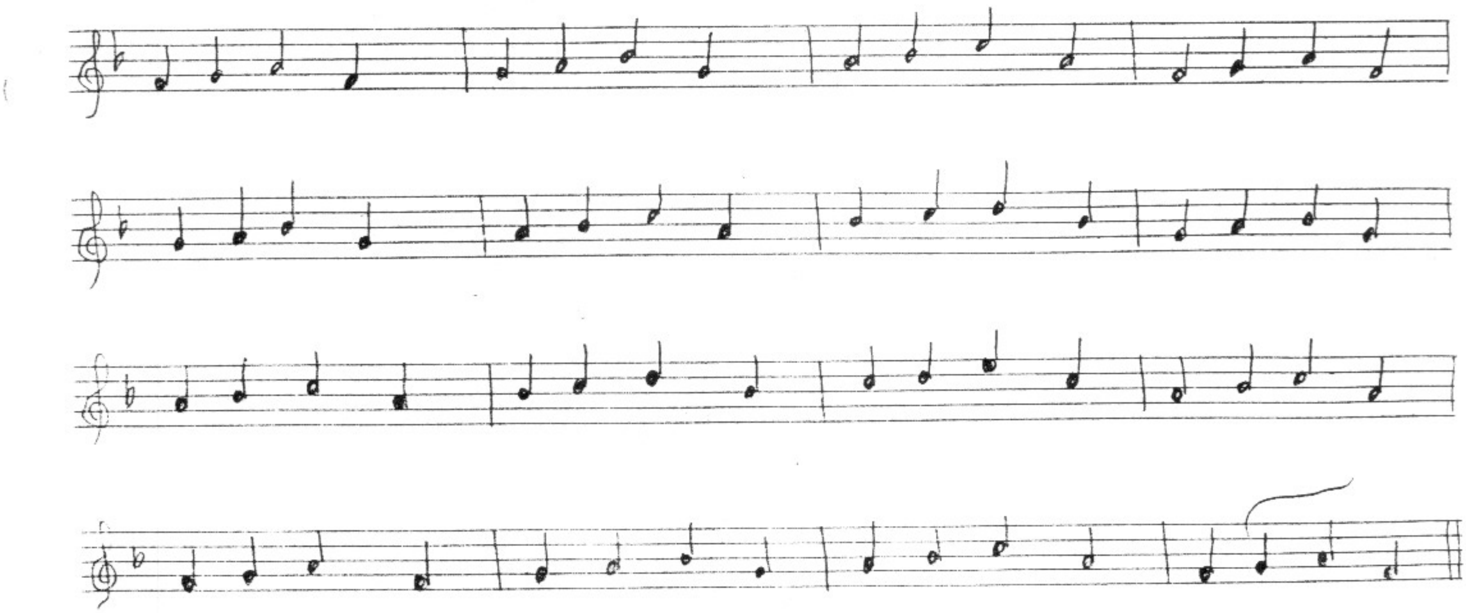
The final similarity is with serialism and when a sequence on notes occurs. In both of these things are repeated in different forms to expand a piece.



I'll begin to demonstrate my Idea with something simple. You begin with one simple bar.

Then you use it to replace itself. Each note in the first becomes one bar of the second.

And repeat the process again.



Not very exciting true but there are always more exciting phrases to start with. The following example shows a slightly more exciting initial phrase , but highlights the problems of following this process exactly (Tied semi-breves leave a little to be desired in most cases).

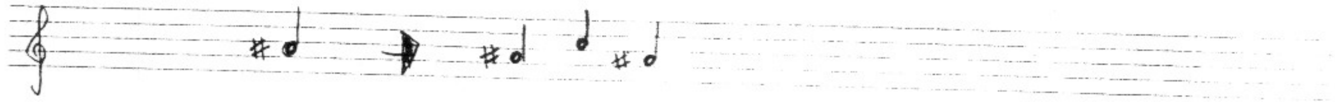
The example on the next page was written with key changes in mind. They are not the most fluid or elegant but they exists none the less. I relaxed the rules and made changes where I felt appropriate.

For the next piece the original and the replacement were different. The replacement was

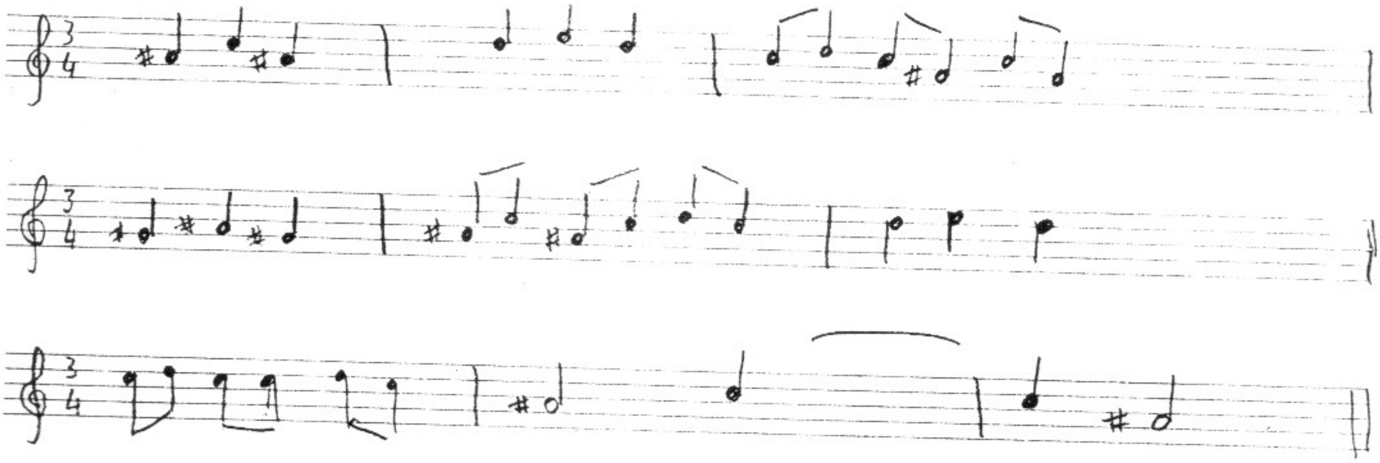
where the middle crotchet was a raising of pitch by one tone. For the sake of variety I based this one on the whole tone scale and in 3/4 time.

ORIGINAL

Replacement

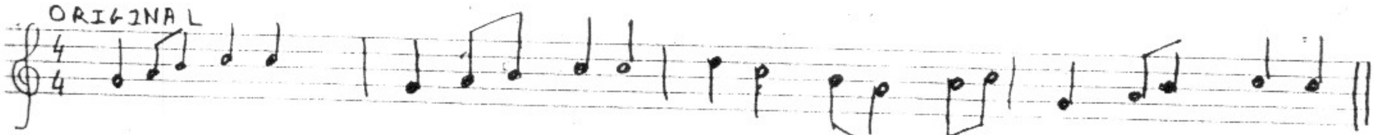


Expansion



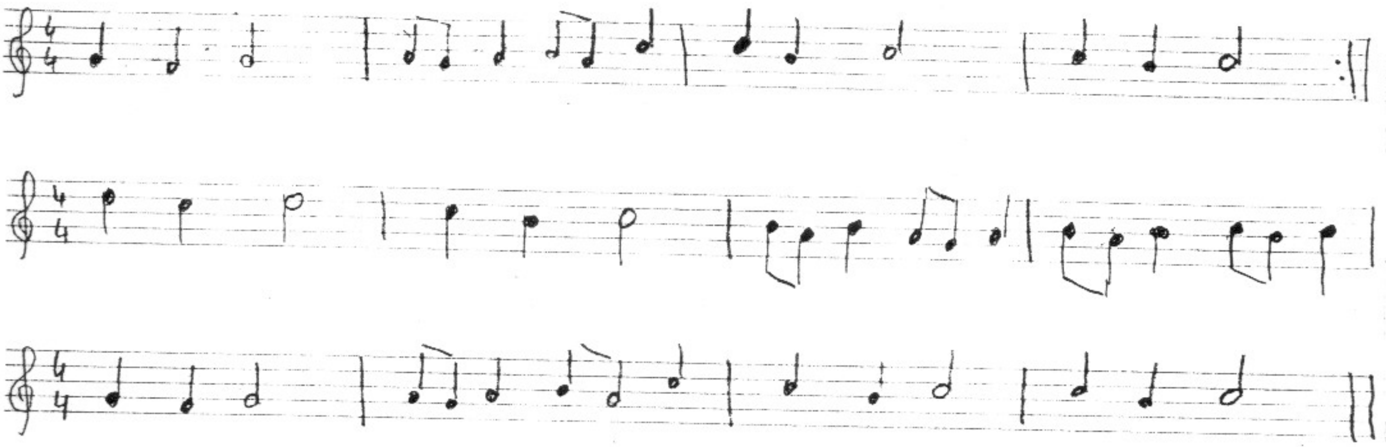
In the next piece I went back to a major key and 4/4 time. I also wrote the original so as to have the resulting expansion in the form AABA. The piece on the bottom of the next page Is my extravagant - go out with a bang - end of the chapter piece. It could just be possible that simple pleasures are the best.

ORIGINAL



Replacement

Expansion



Replacement

ORIGINAL

Expansion

A Counterpoint Generator

After further thought I decided to continue in the style of the last chapter and use methods based around more conventional music theory. Counterpoint was a lightly target. Fractals tend to be generated by two series of numbers running in parallel and I thought counterpoint could capture this as it is two series of notes running in parallel. So I created some rules.

1) Take a bar of counterpoint (in 4/4)

2) To write the upper line of the next bar use the following system:

i) Copy the rhythm of the lower first bar.
ii) Change the pitch of the notes by looking up the pitch of the note they were first harmonized with above. Here is the table of modifications. You may move things up or down an octive as you desire.

Upper Note	Pitch Modification
D	-3
R	-2
M	-1
F	0
S	+1
L	+2
T	+3

3) To find the rhythm of the lower bar look the rhythms which were played against one another up on the rhythm chart. To harmonize this rhythm look the pitches of the lower first bar up on the following chart and harmonize the upper second bar accordingly.

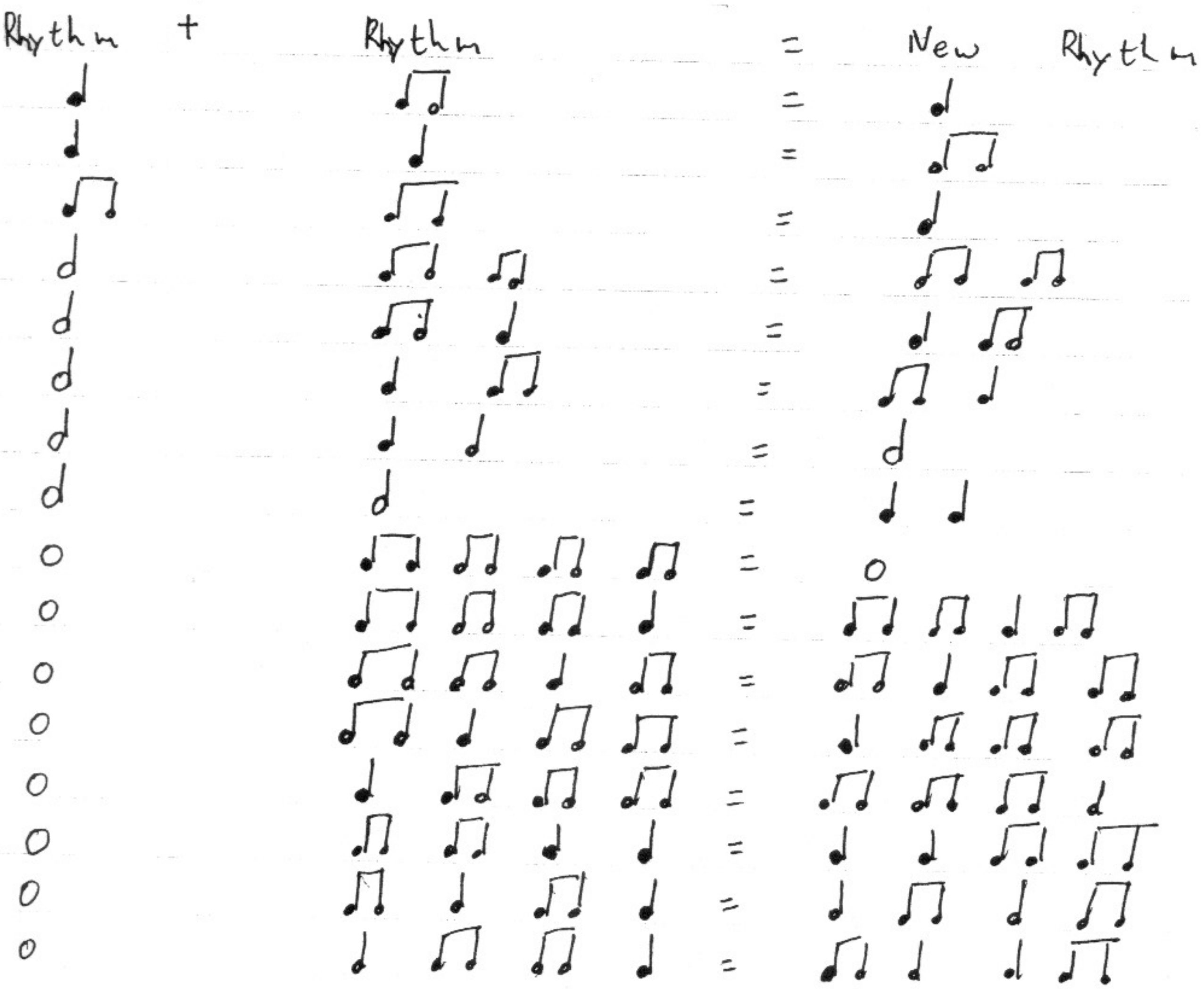
Note in first lower bar	Interval below
D	6th
R	1st
M	3rd
F	4th
S	5th
L	6th
T	3rd

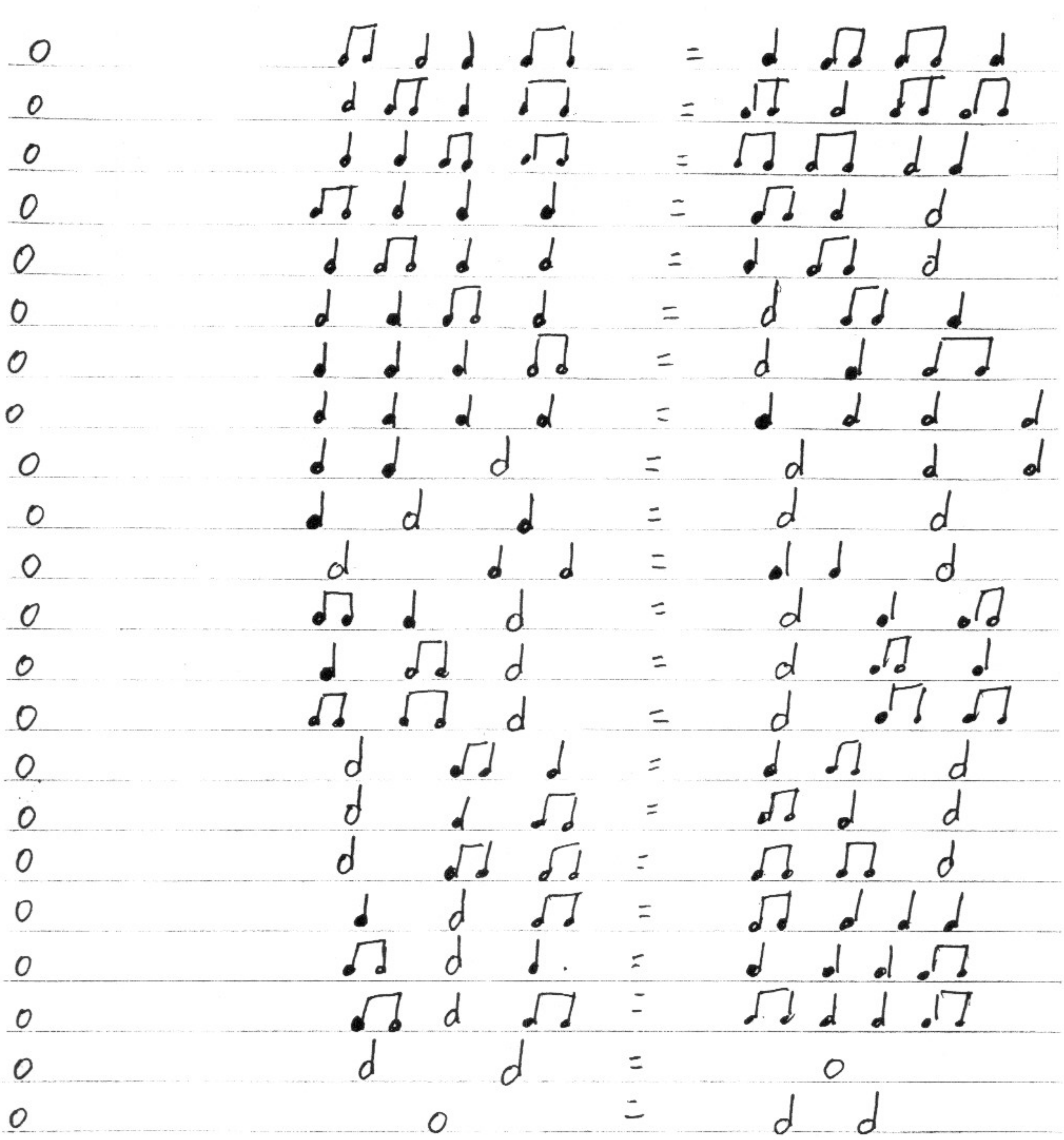
If there are more notes in the lower second bar repeat the interval again (eg. If you have a G in the upper second bar and two notes below it and the interval is a third , then the notes below the G will be a E and a C .). Again the octive is left to the discretion of the person using the rules.

If you are confused by these instructions there is a worked example at the end of this chapter.

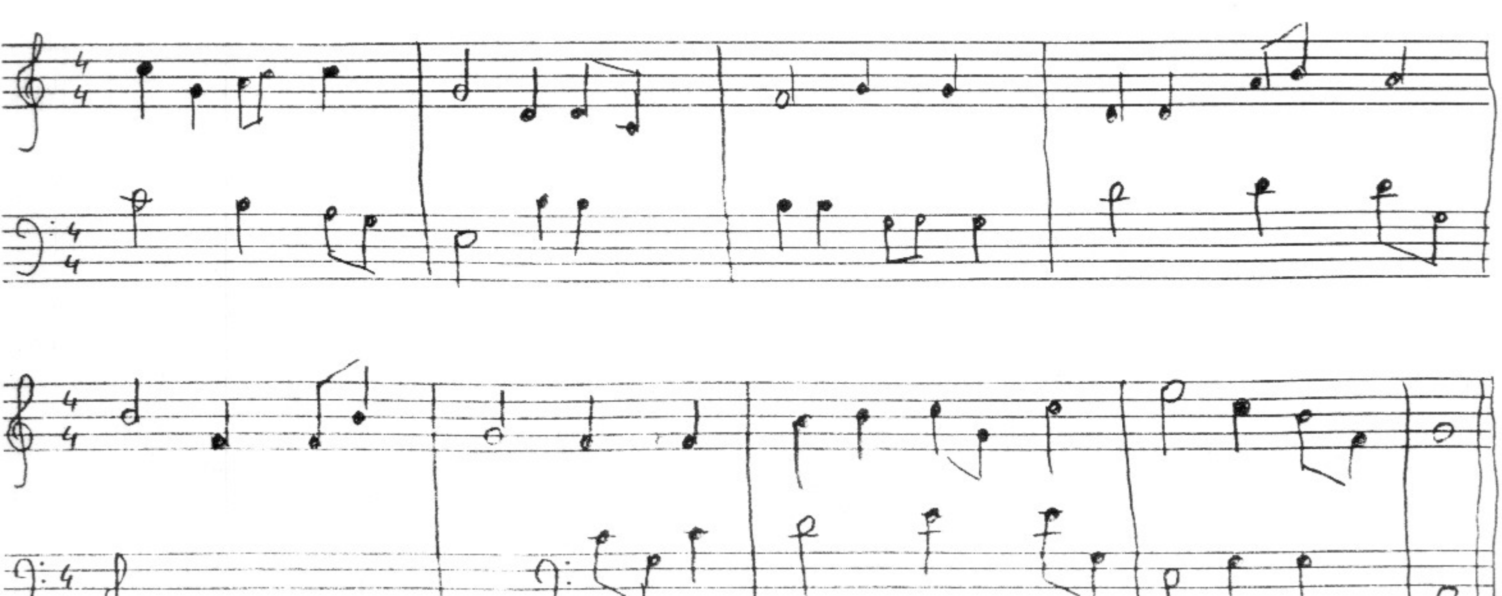
I was quite pleased with the results of this method. It produce reasonable counterpoint (It should in that is contains the most basic rules of counterpoint). Unfortunately the music produced (as usual) doesn't seem to be going anywhere in particular. I suspect that a skilled composer could improve these rules and then 'fine tune' the results by hand to produce a good phrase or two.

The rhythm table follows. As you will notice some rhythms are not catered for. You can make your own rules for these or avoid them if I were to cater for them the rhythm table could be about 30 times longer.



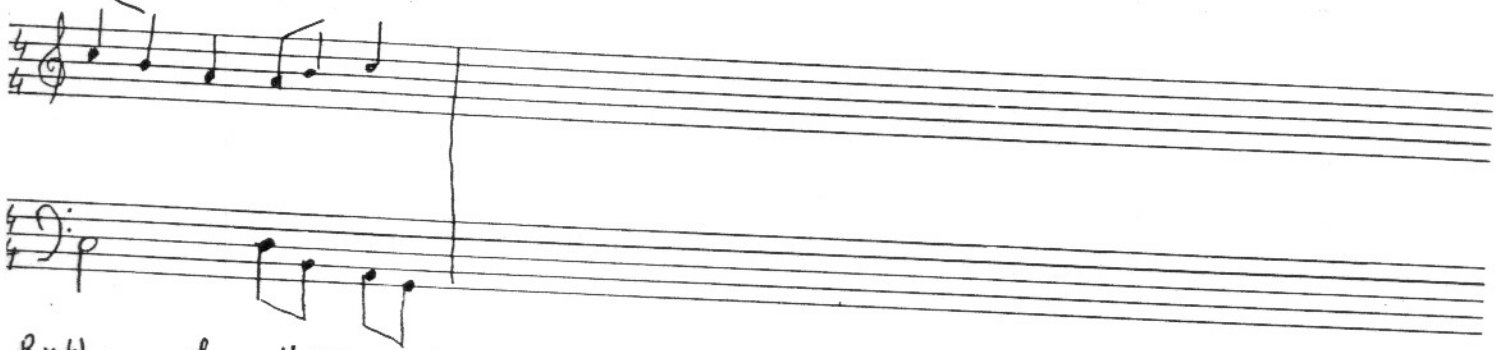


BELOW: A PIECE FROM THE DEVELOPMENT OF THIS CHAPTER
NOTE THE 'TWEAKED' IC, V, I



WORKED EXAMPLE.

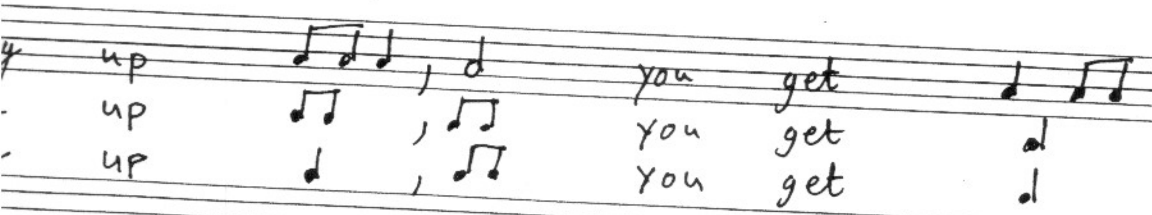
ORIGINAL TWO Bars



Rythm of Next top Bar is + Modify Pitch

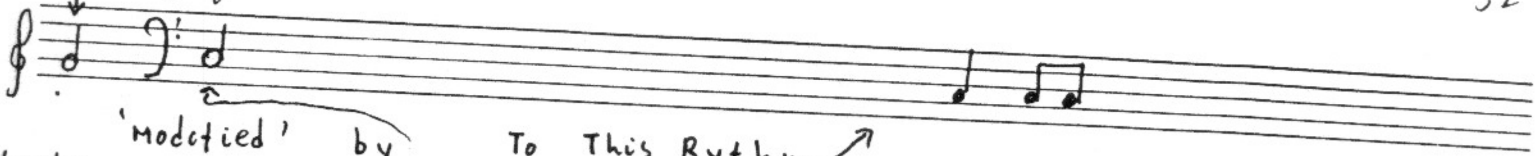
odified by Doh = G
Modified by Lah = C
Modified by Te = E
odified by Doh = E
Modified by Doh = D

So the piece is Now



e Rythm of the Lower Second bar is.

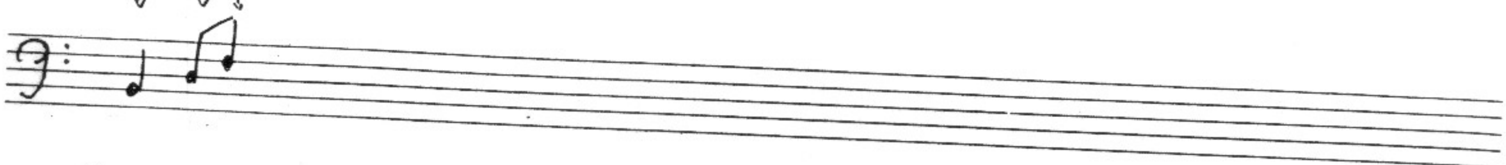
Harmonising



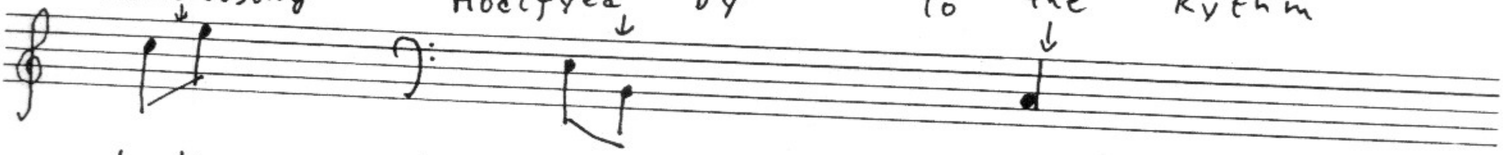
'Modified' by To This Rythm

Looking up Doh we Find the interval is a 6th

6th Below G 6th Below B 6th Below D.

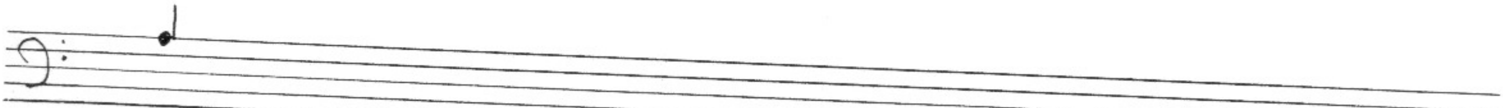


Harmonising modifyed By To the Rythm

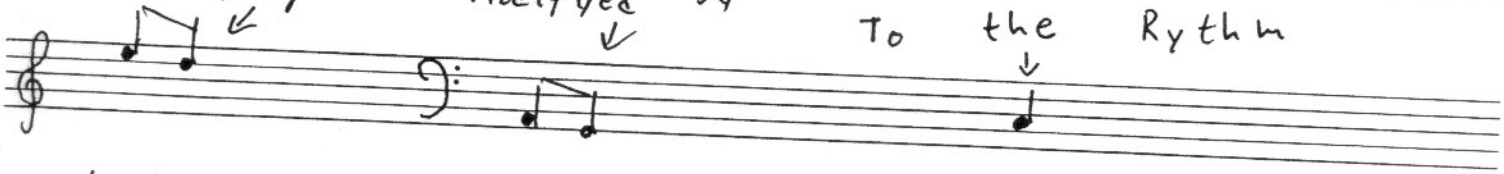


Look up Mee we Find the interval is 3rd.

3rd Below C

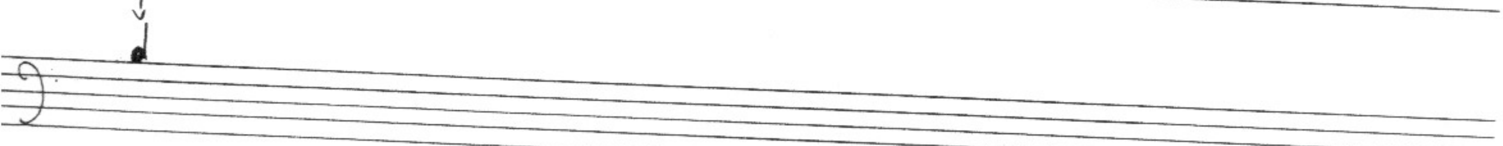


Harmonising modifyed By To the Rythm

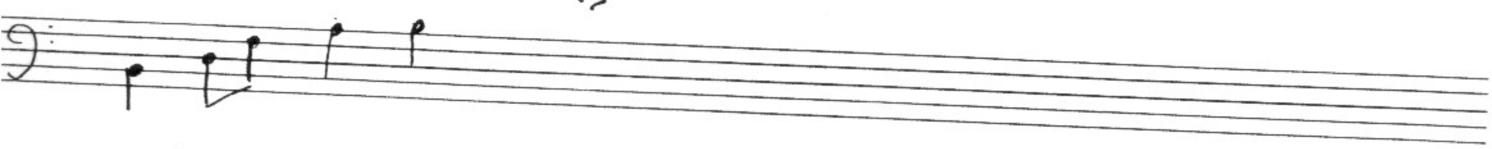


Looking up Fah we Find the interval is 4th.

4th Below



So the Lower bar is



SPOT THE MISTAKE.

Appendix 1 - Fractals

Fractals are a set of shapes recently discovered in mathematics. They are infinitely detail (you can keep enlarging them and as you do more and more detail appears). They are self similar (as you enlarge it smaller copies of the original appear). They are often beautiful an invariable intriguing. They arise in relation to an area of maths called 'non-linear' mathematics or more popularly 'Chaos'.

In this project I have included the two most well known shapes from this subject - the Lorentz attractor and the Mandelbrot set. Needless to say each of these figures has given rise to whole families of similar figures, all of which could be examined in a similar light musically (time and space providing).

To get a grip on the visual aspect of fractals I would suggest one of two paths :

i) Go to a bookshop and look through the computer and mathematics sections for books on Fractals and Chaos. Have a flick through these but don't buy them (they are always expensive).

ii) If you have an IBM compatible computer get a copy of the public domain program 'FractInt'. Any obliging mathematics, computers or physics student should be able to supply this. There are many Fractal programs for other computers (Macs, Amigas, STs, even BBC and Spectrums) so if you keep your eyes open maybe you could obtain some sort of fractal generator.

Appendix 2 - Complex Numbers

Some time ago some mathematicians hit a problem. They couldn't find solutions to the equation

$$x^2 + 1 = 0$$

The problem was that all numbers when squared were greater than zero. And when you add 1 to a number greater than zero you get another number greater than zero. (Remember minus by a minus gives a plus so $(-1)^2 = 1$, $(-5)^2 = 25$, $(-7)^2 = 49$ etc.)

So the problem was to find a number that when it was squired gave a negative answer. Well , why bother finding one when you can make one up ? They said

"

$$i = \sqrt{-1}$$

so $\qquad i^2 = -1$ ".

And so all there is to complex numbers is remembering i = -1. A complex number is an ordinary number with some i's added to it.

An example of some complex numbers

$3 + 2i$, $6 + \frac{3}{4}i$, $4.5 - 87.27i$, $-\frac{8}{13} + 16i$

If you are adding complex numbers you just add numbers to numbers and i's to i's.

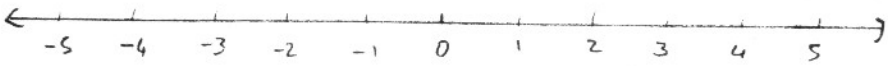
$(3 + 2i) + (4 - 8i) = 7 - 6i$

Note : this is known as the real part

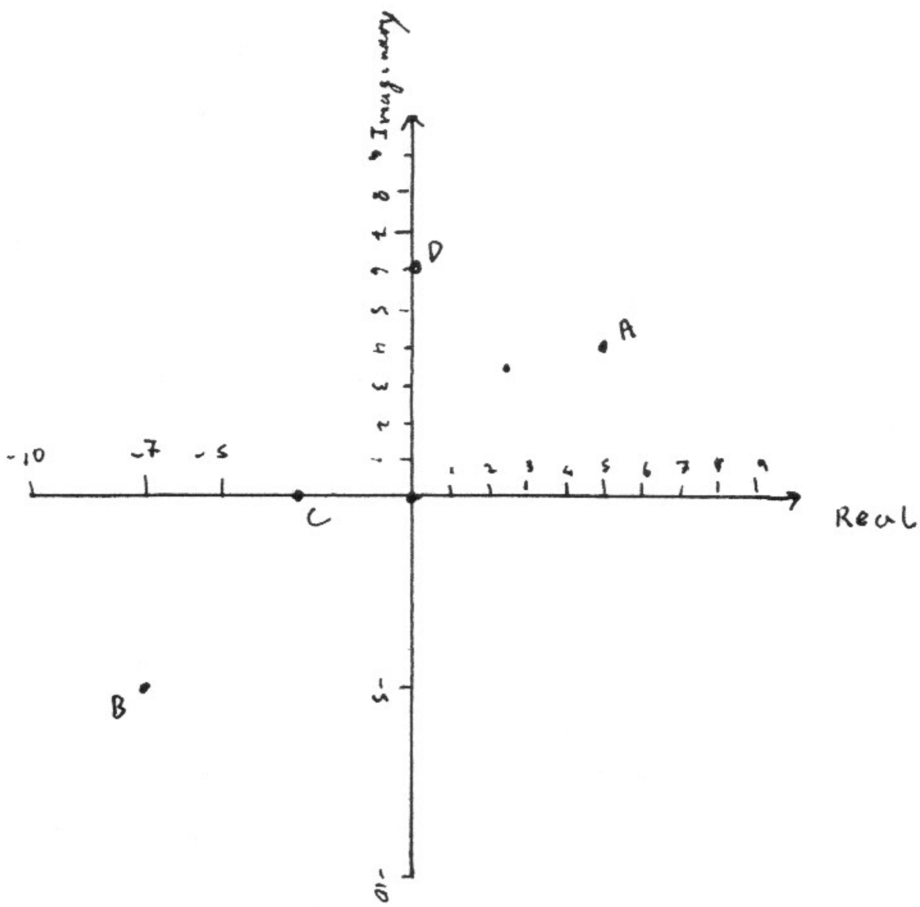
Note : this is known as the imaginary part

In a similar manner you can subtract , multiply , divide and do all the things you can do to with normal numbers to complex numbers.

Do you remember the number line ?



Well there's a similar thing for complex numbers called the Argand diagram. The thing is for complex numbers you need two lines since they have two parts. Like this:



Each complex number is represented by one point. Eg. The point A (see picture previous page) is 5 + 4i because it is 5 out and 4 up.

The point where the two lines cross represents (0 + 0i) or just 0. This point is also known as the origin. The distance from the origin to A would be written as |A|. This is true of any complex number Z the distance from Z to the origin is written as |Z|.

Some other points (see diagram - previous page)

B = -7 - 5i
C = -3
D = 6i
E = 2.5 + 3.5i

For more information on complex numbers see any Leaving Cert. Mathematics book.

Appendix 3 - Differential Equations

Differential equations deal with how things change. I could be how temperature changes 'with respect' to humidity. It might be how distance changes 'with respect' to time.

To give you a example:

Question : You are told if you drop a brick from the top of a skyscraper after t seconds it has fallen $10\ t^2$ meters. How fast is it moving after 5 seconds.

Answer: It you remember that

$$\text{Speed} = \frac{\text{Distance moved}}{\text{Time Taken}}$$

The problem is that as the brick falls it gets faster and faster.

One thing you could do is take approximations.

Time 4s to 5s

	Distance moved after 5 seconds = 10(5)(5)	= 250 m
-	Distance moved after 4 seconds = 10(4)(4)	= 160 m
	Distance moved in 4th second	= 90 m

So in the one second just before the time in question the brick moved 90 m so its average speed was 90 meters per second. If you repeat the approximation for .5 of a second you would find its average speed was 95 meters per second. As the number of seconds before gets smaller and smaller the speed gets closer to 100. (If you didn't follow that It doesn't really matter).

only thing you really need to know about differential
tions is piece of notation

" dy "
$\frac{}{dx}$ = the rate at which y is changing with respect to x

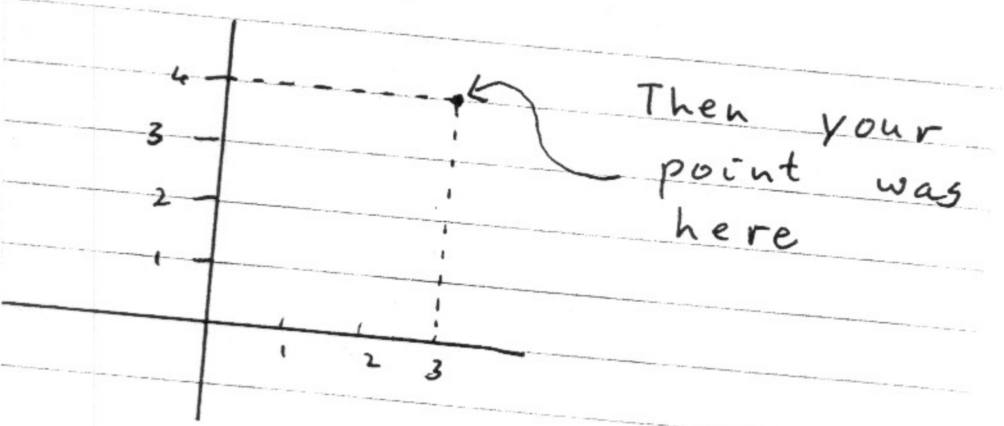
" dv "
$\frac{}{dt}$ = the rate at which velocity is changing with respect it time

mentioned example (the Lorentz attractor) Lorentz worked
w 3 characteristics of the weather changed with respect to
ence the $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dz}{dt}$

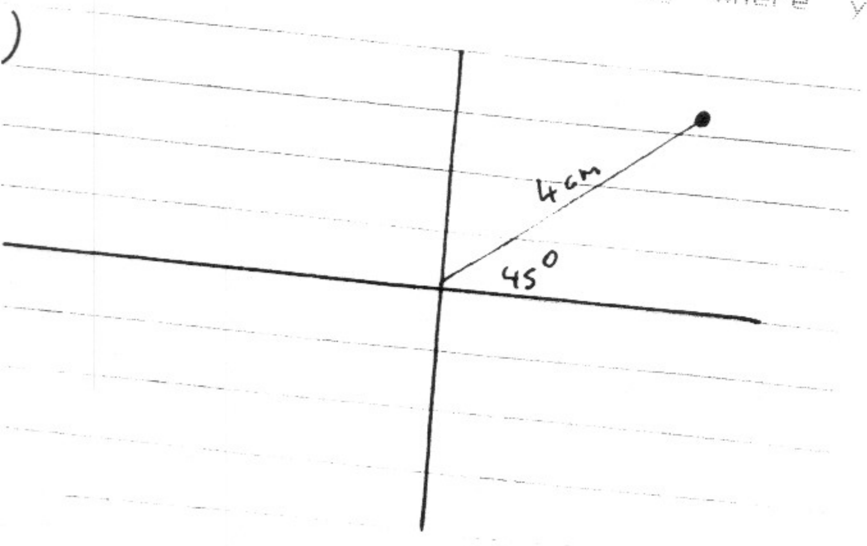
are books and books on Calculus and Differential equations.

Appendix 4 - Polar Form

remember co-ordinate geometry you may remember using two
to specify a point. These numbers were usual called x
for example if (3,4) was your point.



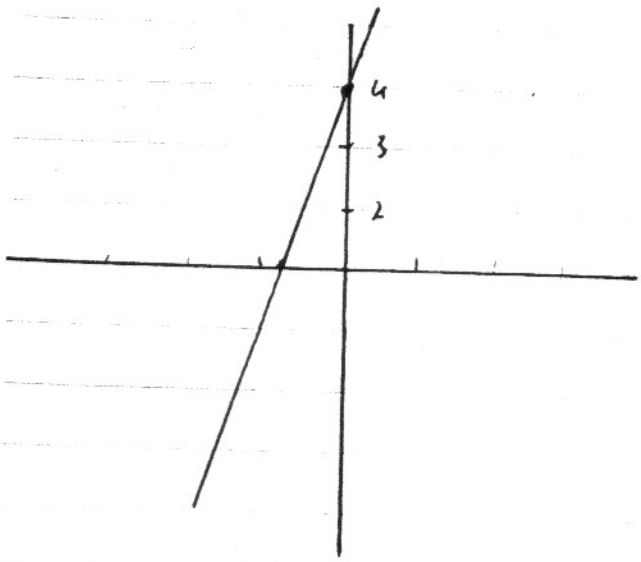
form is another way of measuring where you use an
and a radius (r)



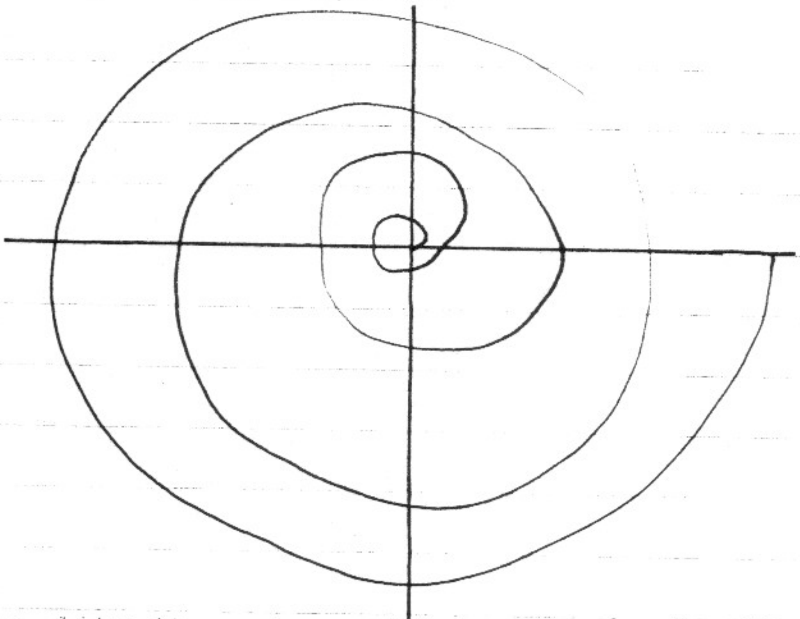
So the point shown is (4cm , 45).

In both systems equations represent different shapes.

In normal co-ordinate geometry (rectilinear) y = 3x + 4 is a line and looks a bit like this.



In polar form r = 3 + 4 is a spiral.



Looking vaguely like the above. Some shapes such as circles , spirals and other similar figures are more easily written in polar form. More information on polar form can be found in most books on complex numbers , co-ordinate geometry or trigonometry.