The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Introduction to Book IV

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 145–146 (1792).]

B O O K IV.

WHATEVER can be said in an elementary institution, concerning the origin and equality of triangles, we may learn from the preceding discourse. But after this, the narration of Euclid is concerning quadrilateral figures, and he particularly teaches us concerning parallelograms[169] together with the contemplation of these delivering the doctrine of trapeziums. For a quadrilateral figure, (as we have formerly observed in our discourse on hypotheses,) is divided into parallelogram and trapezium; and a parallelogram into other certain species, and in like manner a trapezium. But because a parallelogram, on account of its participation of equality, possesses disposition and order, but a trapezium has neither the same, nor a similar order; Euclid's principal discourse, is with propriety, concerning parallelograms, but he also contemplates together with these a trapezium. For from the section of parallelograms, the origin of trapeziums will appear, as will be manifest as we proceed. But because again, it is not possible that any thing can be said of the construction or equality of parallelograms, without the consideration of parallels, (for as it is manifest from the very name, that is, a parallelogram, which is circumscribed by parallel right lines in an opposite position,) hence, he necessarily assumes from parallels the beginning of his doctrine, but have advanced a little from these, he enters on the doctrine of parallelograms, employing one middle theorem, between the elementary institution of each, because he appears to contemplate a certain symptom inherent in parallels: but he delivers the first origin of a parallelogram. For such is the proposition, which says, that *right lines which join equal and parallel right lines towards the same parts, are themselves equal and parallel.* For in this theorem, indeed, a certain accident to equal and parallel right lines is considered: but from the connection a parallelogram appears, having its sides opposite and parallel. And from hence it is manifest that the discourse concerning parallels, was necessarily pre-assumed. But three things are to be assumed, essentially inherent in parallels, which they essentially express, and are converted with them, not only the three together, but every one separately assumed from the rest. Of these, one is, *that when a right line cuts parallel lines, the alternate*

[169] [DRW—The first six occurrences of the word *parallelogram* in this passage are spelled 'paralellogram', but the translator, Thomas Taylor, thereafter usually, but not invariably, employs the spelling 'parallelogram'. In this transcription, instances of the former spelling have been replaced by the latter.]

angles are equal; but the second, *that when a right line cuts parallel lines, the internal angles are equal to two right*; and the third, *that in consequence of a right line cutting parallel lines, the external is equal to the internal and opposite angle*. For when any one of these *symptoms* is demonstrated, we have sufficient authority to affirm that the right lines are parallel. But other mathematicians, also, have been accustomed to discourse after this manner concerning lines, delivering the symptoms of every species. For Apollonius, in each of the conical lines, shews what a *symptom* is, as also Nicomedes in his Treatise on Conchoids, and Hippias in his Quadratics, and Perseus in his Spirals. Since after their origin, that which is essentially inherent in these lines, and according to what it is inherent, being assumed, distinguishes a constructed form from all others. After the same manner, therefore, the institutor of the Elements, first of all investigates, the symptoms of parallels.