# The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Introduction to Book III

Transcribed by David R. Wilkins

August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 1–6 (1792).]

# B O O K III.

## *Concerning Petitions and Axioms.*

SINCE[127] the principles of geometry are triply divided into Hypotheses, Petitions, and Axioms, the difference between these we have explained in the preceding books. But we now intend to discourse more accurately of petition and axiom, as especially necessary to our present design. For hypotheses, which are also called definitions, we have already explained. It is common, therefore, as well to axioms as to petitions, to require no demonstration, and no geometrical faith: but to be received as manifest, and to become the principles of the rest. But they differ mutually from each other, in the same manner in which we have distinguished theorems from problems. For as in theorems we propose to perceive and know that which follows a subject; but in problems we are ordered to compare and do something: in the same manner also in axioms, we must receive whatever is manifest of

[127]In the two preceding books of this work, our author has displayed an uncommon degree of philosophic elegance and depth; and in the present two, he no less manifests the greatest geometrical accuracy and skill. In the former he elevates us from participated truth to truth itself; and from the glimmering light of universals reflected in the catoptric bosom of the phantasy, to the bright refulgence of ideas. In the latter he combines geometry and philosophy, occasionally cloathes the rigid accuracy of demonstration with the enchanting imagery of divine imaginations, and unites the graces of diction with the precision and sanctity of truth. Yet his genius, though rapid as a torrent, never passes beyond the bounds of propriety; and though his thoughts are vehement and vast, they are at the same time orderly and majestic. For my own part I confess myself enamoured with the grandeur of his diction, astonished with the magnificence of his conceptions, and enlightened by the irradiations of his powerful genius. And I desire nothing so much as that others may experience similar effects from this admirable work. I only add, that the study of this second part is absolutely necessary to a perfect comprehension of Euclid's method and meaning; and to the understanding geometry completely and philosophically. It is easy indeed to learn a science in a manner sufficient for mechanical purposes; for this is accomplished by the *many*: but it is arduous to learn it with a view to the perception of truth; for this is alone the province of a *few*. It is easy to be knowing in effects, for these are obvious and common; but it is difficult to investigate causes, for these are occult and rare. In short, a general and confused apprehension of a science may be readily obtained, without much labour and toil; but a particular and accurate knowledge requires liberal application, and patient endurance. For the one is like the distant prospect of a country, in which the larger parts are alone conspicuous to the observer's eye; but the other resembles a near and distinct view, in which every thing is recognized essential to the perfection of the whole.

itself, and easily apprehended by our untaught conceptions; but in petitions we must receive whatever is easy to be done and compared, (since in admitting these, thought is not fatigued) and whatever requires no variety, and no kind of construction. Hence evident and indemonstrable cognition, and unconstructed assumption, distinguish petitions from axioms. Just as demonstrative cognition, and an assumption of things sought, together with construction, separates theorems from problems. For it is every where requisite, that principles in simplicity, indemonstrability, and self-evidence, should excel things posterior to principles. For universally (says Speusippus) of the things which cogitation pursues, some of its energies it produces without a various progression, prepares them for future enquiry, and has a more evident apprehension of these than of visible objects: but others which it is not able immediately to follow, by a transition proceeding from their nature, these it endeavours by *consequence* to pursue. Thus for example, *to draw a right line from one point to another*, it receives as evident, and easy to be done. For since in this case the line is composed from the indeclinable flux of a point, and at the same time advances in an orderly progression, because it no where more or less declines, it necessarily falls in another point. Again, if one extremity of a right line abiding, the other is moved about it, it will describe a circle without any labour. But if any one wishes to describe a helix of one revolution, it requires a more various operation. For it is generated by various motions. Likewise if any one wishes to construct an equilateral triangle, he will require a certain method for its construction. For the geometrical intellect says, when I understand a right line, which abides according to one of its extremities, but is moved about it according to the other, and at the same time conceive a point, which is moved in the line from the abiding extreme, I have described a helix of one revolution. For when at the same time both the extremity of the right line, which describes the circle, and the point which is moved in the right line, arrive at the same point, and coincide, they produce for me such a helix. And again, when I describe equal circles, and draw right lines from the common section to the centre of the circles, and a right line from one centre to the other, I shall have an equilateral triangle. The production of these, therefore, is very remote from a simple apprehension, and primary notion. For we are content to pursue the progressions of their origin. Hence it happens that these are compared with greater ease or difficulty, and are exhibited with many or fewer mediums, according to the habit of those who enter on this undertaking: but that they require demonstration and construction, on account of the property of the things sought, which wants the evidence of petitions and axioms.

Petition, therefore, and axiom, are simple and easy to be apprehended. But petition, indeed, commands us to fabricate, and provide a certain matter, in order to the assignation of the *symptom*, which possesses an easy and simple apprehension: but axiom pronounces a certain essential accident, of itself known to the hearers. As that *fire is hot*, of any other of those manifest truths, he who doubts of which, we consider as either wanting sense or punishment. Hence, petition and axiom are of the same genus; but they differ in the above-mentioned manner. For each is an indemonstrable principle, but this after one mode, and that after another, as we have already observed. But some think that all these should be called petitions, in the same manner as all problems, *things sought*. For Archimedes beginning his book of *Equiponderants*, *we desire it may be granted* (says he) *that things equally heavy, from equal lengths, will equally ponderate*; though some rather chuse to call this an axiom. But others call all these axioms, in the same manner as they denominate every thing a theorem, which requires demonstration. For, according to the same proportion, as it seems they pass from proper names to such as are common. Nevertheless, as a problem differs from a theorem, so petition from axiom: though both these last are indemonstrable, and the former require demonstration. And the one, indeed, is assumed as easy to be done, but the other is granted as easy to be known by the common consent of all men. After this manner, therefore, Geminus distinguishes petitions from axioms.

But others will perhaps say, that petitions are indeed proper to the geometrical matter: but that axioms are common to the universal theory, which is conversant about the *how much*, and *the how many*. For the geometrician knows *that* which requires *that all right angles are equal*, and *that every finite straight line may be produced straight forwards*: but *that* which says, *things equal to one and the same are equal to each other*, is a common conception, which not only the arithmetician employs, but every one endued with science, accommodating that which is common to his own particular matter. But Aristotle (as we have before observed[128]) says, that petition, since it is demonstrable, is not granted by the hearer, yet is received as a principle: but that axiom is of itself indemonstrable, and that this is confessed by all, according to habit, though some, for the sake of disputation, have doubted its evidence. Since then, there are these three differences, according to the first, which by operating, and knowle[d]ge only distinguishes petition from axiom, it is manifest that that which says *all right angles are mutually equal, is not a petition*. Nor the fifth, which says, *if a right line falling on two right lines makes the internal angles towards the same parts less than two right, those right lines infinitely produced, shall coincide towards the parts in which the angles less than two right subsist*. For these are neither assumed in

[128]See the second section of the Dissertation, Vol. I.

construction, nor do they command any thing to be done: but they exhibit a certain *symptom*, inherent in right angles, and in right lines, departing from angles less than two right. But, according to the second difference, that will not be an axiom which says, that *two right lines cannot comprehend space*, which some at present consider as an axiom. For this is proper to the geometric matter, as likewise that which affirms *that all right angles are equal.* But according to the the third difference, which is Aristotelic, all those which produce their credibility by a certain demonstration, are petitions; but whatever is indemonstrable, are axioms. Apollonius, therefore, in vain endeavours to deliver the demonstrations of axioms: for Geminus very properly observes, that some have attempted demonstrations of indemonstrables, and have endeavoured from more unknown mediums to prove things manifest to all, into which error Apollonius has fallen, who wishes to prove the axiom true, which says, *that things equal to one, and the same, are equal to each other*: but that others assume in the place of indemonstrables, things requiring demonstration. As is the case with Euclid himself, in the fourth and fifth petition. For some say, that this last, as ambiguous, requires demonstration. Indeed, is it not ridiculous, that theorems should be assigned as indemonstrable, the converse of which are demonstrable? For that the internal angles of coincident right lines are less than two right, Euclid himself shews in the theorem, which says, *that two angles of every triangle, however taken, are less than two right*: besides, it may be perspicuously shewn, that not every thing equal to a right angle is a right angle. Hence, says Geminus, the converse of these are not to be granted indemonstrable. It seems therefore, according to the ordination of this man, that there are, indeed, three petitions: but that the other two, and the converse of these, require demonstrating science: and that in the axioms, the one which says, that two right lines cannot comprehend space, is superfluously added, since its credibility must be derived from demonstration. And thus much concerning the difference of petitions and axioms. Again, of axioms, some are proper to arithmetic, but others to geometry; and others are common to both: for that which says *every number is measured by unity*, is an arithmetical axiom. But that which says *equal right lines agree amongst themselves*, as also this which affirms *that every magnitude is divisible in infinitum*, are geometrical axioms: but the one which says *that things equal to the same, are mutually equal*, and all of this kind are common to both. However, it must be observed, that each science uses such as the last, according to its proper subject; as geometry in magnitudes, but arithmetic in numbers. In like manner of petitions, some are peculiar to particular sciences, but others are common to all. For you must call the petition which requires to be granted, *that a number may be divided into the least parts*, peculiar to arithmetic: but this, *that every finite* *straight line may be produced straight forwards*, peculiar to geometry; and the one which desires us to grant, *that quantity may be infinitely increased*, common to both; for this passion is equally found to reside in number and magnitude.