The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book II, Chapter 8

Transcribed by David R. Wilkins

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CHAP. VIII.

Concerning the Order of Geometrical Discourses.

BUT let us now explain the universal order of the discourses contained in geometry. Because then, we assert that this science consists from hypothesis[56], and demonstrates its consequent propositions from definite principles (for one science only, I mean the first philosophy, is without supposition, but all the rest assume their principles from this) it is necessary that he who constructs the geometrical institution of elements, should separately deliver the principles of the science, and separately the conclusions which flow from those principles; and that he should render no reason concerning the nature or truth of the principles, but should confirm by reasons, the things consequent to these geometric principles. For no science demonstrates its own principles, nor discourses concerning them; but procures to itself a belief of their reality, and they becomd more evident to the particular science to which they belong than the things derived from them as their source. And these, indeed, science knows by themselves; but their consequents, through the medium of these. For thus, also, the natural philosopher propagates his reasons from a definite principle, supposing the existence of motion. Thus too, the physician, and he who is skilled in any of the other sciences and arts. For if any one mingles principles, and things flowing from principles into one and the same, he disturbs the whole order of knowledge, and conglutinates things which can never mutually agree; since a principle, and its emanating consequent, are naturally distinct from each other. In the first place, therefore (as I have said), principles in the geometric institution are to be distinguished from their consequents, which is performed by Euclid in each of his books; who, before every treatise, exhibits the common principles of this science; and afterwards divides these common principles into hypothesis, petitions, and axioms. For all these mutually differ; nor is an axiom, petition, and hypothesis the same, according to the demoniacal Aristotle; but when that which is assumed in the order of a principle, is indeed known to the learner, and credible by itself, it is an axiom: such as, that things equal to the same, are mutually equal to each other. But when any one, hearing

[56]The reader will please to observe, that the definitions are, indeed, hypotheses, according to the doctrine of Plato, as may be seen in the note to chap. i. book I. of this work.

another speak concerning that of which he has no self-evident knowledge, gives his assent to its assumption, this is hypothesis. For that a circle is a figure of such a particular kind, we presume (not according to any common conception) without any preceding doctrine. But when, again, that which is asserted was neither known, nor admitted by the learner, yet is assumed, then (says he) we call it petition; as the assumption that all right angles are equal. But the truth of this is evinced by those who study to treat of some petition, as of that which cannot by itself be admitted by any one. And thus, according to the doctrine of Aristotle[57], are axiom, petition, and supposition distinguished. But oftentimes, some denominate all these hypotheses, in the same manner as the Stoics call every simple enunciation an axiom. So that, according to their opinion, hypotheses also will be axioms; but, according to the opinion of others, axioms will be called suppositions. Again, such things as flow from principles are divided into problems and theorems. The first, indeed, containing the origin, sections, ablations, or additions of figures, and all the affections with which they are conversant; but the other exhibiting the accidents essential to each figure. For, as things effective of science, participate of contemplation, in the same manner things contemplative previously assume problems in the place of operations. But formerly some of the ancient mathematicians thought that all geometrical propositions should be called theorems, as the followers of Speusippus and Amphinomus, believing, that to contemplative sciences, the appellation of theorems is more proper than that of problems; especially since they discourse concerning eternal and immoveable objects. For origin does not subsist among things eternal: on which account, problems cannot have any place in these sciences; since they enunciate origin, and the production of that which formerly had no existence, as the construction of an equilateral, or the description of a square on a given right line, or the position of a right line at a given point. It is better, therefore (say they), to assert that all propositions are of the speculative kind: but that we perceive their origin, not by production, but by knowledge, receiving things eternal as if they were generated; and on this account we ought to conceive all those theorematically, but not problematically. But others, on the contrary, think that all should be called problems: as those mathematicians who have followed Menæchmus. But that the office of problems is two-fold, sometimes, indeed, to procure the thing sought; but at other times when they have received the determinate object of enquiry, to see, either what it is, or of what kind it is, or what affectation it possesses, or what its relation is to another. And, indeed, the assertions of each are right; for the followers of Speusippus well perceive. Since the problems of geometry are not of the same

[57]In his last Analytics. See the preceding Dissertation.

kind, with such as are mechanical. For these are sensibles, and are endued with origin, and mutation of every kind. And, on the other hand, those who follow Menæchmus do not dissent from truth; since the invention of theorems cannot by any means take place without an approach into matter; I mean intelligible matter. Reasons, therefore, proceeding into this, and giving form to its formless nature, are not undeservedly said to be assimilated to generations. For we say that the motion of our cogitation, and the production of its inherent reasons, is the origin of the figures situated in the phantasy, and of the affections with which they are conversant: for there constructions and sections, positions and applications, additions and ablations, exist: but every thing resident in cogitation, subsists without origin and mutation. There are, therefore, both geometrical problems and theorems. But, because contemplation abounds in geometry, as production in mechanics, all problems participate of contemplation; but every thing contemplative is not problematical. For demonstrations are entirely the work of contemplation; but every thing in geometry posterior to the principles, is assumed by demonstration. Hence, a theorem is more common: but all theorems do not require problems; for there are some which possess from themselves the demonstration of the thing sought. But others, distinguishing a theorem from a problem, say, that indeed every problem receives whatever is predicated of its matter, together with its own opposite; but that every theorem receives, indeed, its symptom predicate, but not its opposite. But I call the matter of these, that genus which is the subject of enquiry; as for instance, a triangle, quadrangle, or a circle: but the symptom predicate, that which is denominated an essential accident, as equality, or section, or position, or some other affection of this kind. When, therefore, any one proposes to inscribe an equilateral triangle in a circle, he proposes a problem; for it is possible to inscribe one that is not equilateral. But when any one asserts that the angles at the base of an isosceles triangle are equal, we must affirm that he proposes a theorem; for it is not possible that the angles at the base of an isosceles triangle should be unequal to each other. On which account, if any one forming problematically, should say that he wishes to inscribe a right angle in a semi-circle, he must be considered as ignorant of geometry; since every angle in a semicircle is necessarily a right one. Hence, propositions which have an universal symptom, attending the whole matter, must be called theorems; but those in which the symptom is not universal, and does not attend its subject, must be considered as problems. As to bisect a given terminated right line, or to cut it into equal parts: for it is possible to cut it into unequal parts. To bisect every rectilinear angle, or divide it into equal parts; for a division may be given into unequal parts. On a given right line to describe a quadrangle; for a figure that is not quadrangular may be described. And, in short, all of this kind belong to the problematic order. But the followers of Zenodotus, who was familiar with the doctrine of Oenopides, but the disciple of Andron, distinguish a theorem from a problem, so far as a theorem enquires what the symptom is which is predicated of the matter it contains; but a problem enquires what that is, the existence of which is granted. From whence the followers of Possidonius define a theorem a proposition, by which it is enquired whether a thing exists or not; but a problem, a proposition, in which it is enquired what the thing is, or the manner of its existence. And they say that we ought to form the contemplating proposition by enunciating, as that every triangle has two sides greater than the remaining one, and that the angles at the base of every isosceles triangle are equal: but we must form the problematical proposition, as if enquiring whether a triangle is to be constructed upon this right line. For there is a difference, say they, absolutely and indefinitely, to enquire whether the thing proposed is from a given point to erect a right line at right angles to a given line, and to behold what the perpendicular is. And thus, from what has been said, it is manifest there is some difference between a problem and a theorem. But that the elementary institution of Euclid, also, consists partly of problems, and partly of theorems, will be manifest from considering the several propositions. Since, in the conclusion of his demonstrations, he sometimes adds (which was to be shewn) sometimes (which was to be done) the latter sentence being the mark or symbol of problems, and the former of theorems. For although, as we have said, demonstration takes place in problems, yet it is often for the sake of generation; for we assume demonstration in order to shew, that what was commanded is accomplished: but sometimes it is worthy by itself, since the nature of the thing sought after may be brought into the midst. But you will find Euclid sometimes combining theorems with problems, and using them alternately, as in the first book; but sometimes abounding with the one and not the other. For the fourth book is wholly problematical; but the fifth is entirely composed from theorems. And thus much concerning the order of geometrical propositions.