The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book II, Chapter 7

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 1, pp. 104–106 (1792).]

CHAP. VII.

From whence the Name of Elementary Institution originated, and why Euclid is called the Institutor of Elements.

But what gave rise to the name of elementary institution, and of element itself, from which elementary institution was derived? To this we shall reply, by observing, that of theorems some are usually called elements, but others elementary, and others again are determined beyond the power of these. Hence, an element is that whose consideration passes to the science of other things, and from which we derive a solution of the doubts incident to the particular science we investigate. For as there are certain first principles of speech, most simple and indivisible, which we denominate elements, and from which all discourse is composed; so there are certain principal theorems of the whole of geometry, denominated elements, which have the respect of principles to the following theorems; which regard all the subsequent propositions, and afford the demonstrations of many accidents essential to the subjects of geometric speculation. But things elementary are such as extend themselves to a multitude of propositions, and possess a certain simplicity and sweetness, yet are not of the same dignity with elements; because their contemplation is not common to all the science to which they belong, as is the case in the following theorem, that in triangles, perpendiculars, drawn from their angles to their sides, coincide in one point[53]. Lastly, whatever neither possesses a knowledge extended into multitude, nor exhibits any thing skilful and elegant, falls beyond the elementary power. Again, an element, as Menæchmus says, may have a twofold definition. For that which confirms, is an element of that which is confirmed; as the first proposition of Euclid with respect to the second, and the fourth with regard to the fifth. And thus, indeed, many things may be mutually called elements one of another; for they are mutually confirmed. Thus, because the external angles of right-lined figures, are equal to four right angles, the multitude of internal ones equal to right angles; and, on the contrary, that from this is exhibited[54] Besides, an element is otherwise

[53]Because this is true only in isosceles and equilateral triangles

[54]This follows from the 32d proposition of the first book of Euclid, and is demonstrated by Dr. Barrow, in his scholium to that proposition.

[DRW—See the conclusion of Proclus's commentary on Proposition 32 of Book I of Euclid's *Elements*. Now the sum of the internal and external angles of a convex n-sided called that into which, because it is more simple, a composite is dissolved. But it must be observed, that every element cannot be called the element of every thing: but such as are more principal are the elements of such as are constituted in the reason of the thing effected; as petitions are the elements of theorems. And, according to this signification of an element, Euclid's elements are constructed. Some, indeed, of that geometry which is conversant about planes; but others of stereometry. In the same manner, likewise, in arithmetic and astronomy, many have composed elementary institutions. But it is difficult, in each science, to chuse and conveniently ordain elements, from which all the peculiarities of that science originate, and into which they may be resolved. And among those who have undertaken this employment, some have been able to collect more, but others fewer elements. And some, indeed, have used shorter demonstrations: but others have extended their treatise to an infinite length. And some have omitted the method by an impossibility; but others that by proportion; and others, again, have attempted preparations against arguments destroying principles. So that many methods of elementary institution have been invented by particular writers on this subject. But it is requisite that this treatise should entirely remove every thing superfluous, because it is an impediment to science. But every thing should be chosen, which contains and concludes the thing proposed; for this is most convenient and useful in science. The greatest care, likewise, should be paid to clearness and brevity; for the contraries to these, disturb our cogitation. Lastly, it should vindicate to itself, the universal comprehension of theorems, in their proper bounds: for such things as divide learning into particular fragments, produce an incomprehensible knowledge. But in all these modes, any one may easily find, that the elementary institution of Euclid excels the institutions of others. For its utility, indeed, especially confers to the contemplation of primary figures: but the transition from things more simple to such as are more various, and also that perception, which from axioms possesses the beginning of knowledge, produces clearness, and an orderly tradition: and the migration from first and principal theorems to the objects of enquiry effects the universality of demonstration. For whatever he seems to omit, may either be known by the same ways, as the construction of a scalene and isosceles triangle[55]: or because they are difficult, and

polygon, taken together, is $2n$ right angles. Thus, if it is shown that the sum of the external angles is four right angles, then the sum of the internal angles must be $2n-4$ right angles, and vice versa. The wording of Thomas Taylor's translation here seems obscure. The Greek text in Friedlein's edition of Proclus's *Commentaries* (p. 73, 1–4) reads as follows: δείκνυται γὰρ καὶ ἐκ τοῦ τέτρασιν ὀρθαῖς εἶναι ἴσας τὰς ἔξω τῶν εὐθυγράμμων γωνίας τὸ πλῆθος τῶν ἐντὸς ὀρθαῖς ἴσων καὶ ἀνάπαλιν ἐκ τούτου ἐκεῖνο.]

[55]The method of constructing these is shewn by our philosopher, in his comment on the

capable of infinite variety, they are far more remote from the election of elements, such as the doctrine of perturbate proportions, which Apollonius has copiously handled: or, lastly, because they may easily constructed from the things delivered, as from causes, such as many species of angles and lines. For these, indeed, were omitted by Euclid, and are largely discoursed of by others, and are known from simple propositions. And thus much concerning the universal elementary institution of geometry.

first proposition, as will appear in the second volume of this work.