The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book II, Chapter 4

Transcribed by David R. Wilkins

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 1, pp. 98–101 (1792).]

C H A P. IV.

On the Origin of Geometry, and its Inventors.

But let us now explain the origin of geometry, as existing in the present age of the world. For the demoniacal Aristotle[42] observes, that the same opinions often subsist among men, according to certain orderly revolutions of the world: and that sciences did not receive their first constitution in our times, nor in those periods which are known to us from historical tradition, but have appeared and vanished again in other revolutions of the universe; nor is it possible to say how often this has happened in past ages, and will again take place in the future circulations of time. But, because the origin of arts and sciences is to be considered according to the present revolution of the universe, we must affirm, in comformity with the most general tradition, that geometry was first invented by the Egyptians, deriving its origin from the mensuration of their fields: since this, indeed, was necessary to them, on account of the inundation of the Nile washing away the boundaries of land belonging to each. Nor ought it to seem wonderful, that the invention of this as well as of other sciences, should receive its commencement from convenience and opportunity. Since whatever is carried in the circle of generation, proceeds from the imperfect to the perfect. A transition, therefore, is not undeservedly made from sense to consideration, and from this to the nobler energies of intellect[43]. Hence, as the certain knowledge of numbers received its origin among the Phœnicians, on account of merchandise and commerce, so geometry was found out among the Egyptians from the distribution of land. When Thales, therefore, first went into Egypt, he transferred this knowledge from thence into Greece: and he invented many things himself, and communicated to his successors the principles of many. Some of which were, indeed, more universal, but others extended to sensibles. After him Ameristus, the brother of Stesichorus the poet, is celebrated as one who touched upon, and tasted the study of geometry, and who is mentioned by Hippias the Elean,

[42]In lib. i. de Cælo, tex. 22. et lib. i. Metco. cap. 3. Aristotle was called demoniacal by the Platonic philosophers, in consequence of the encomium bestowed on him by his master, Plato, "That he was the dæmon of nature." Indeed his great knowledge in things subject to the dominion of nature, well deserved this encomium; and the epithet *divine*, has been universally ascribed to Plato, from his profound knowledge of the intelligible world.

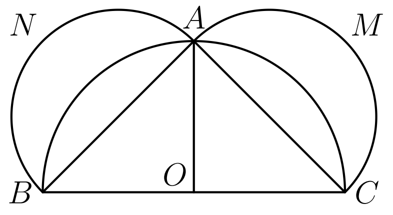
[43]Εἲς νοῦν, is wanting in the original, but is supplied by the excellent translation of Barocius.

as restoring the glory of geometry. But after these, Pythagoras changed that philosophy, which is conversant about geometry itself, into the form of a liberal doctrine, considering its principles in a more exalted manner; and investigating its theorems immaterially and intellectually; who likewise invented a treatise of such things as cannot be explained[44] in geometry, and discovered the constitution of the mundane figures. After him, Anaxagoras the Clazomenian succeeded, who undertook many things pertaining to geometry. And Oenipides the Chian, was somewhat junior to Anaxagoras, and whom Plato mentions in his Rivals, as one who obtained mathematical glory. To these, succeeded Hippocrates, the Chian, who invented the quadrature of the lunula[45], and Theodorus the Cyrenean, both of them eminent in geometrical knowledge. For the first of these, Hippocrates composed geometrical

[44] Ἀλόγων, in the printed Greek, which Fabricius, in his Bibliotheca Græca, vol. i. page 385, is of opinion, should be read ἀναλόγων; but I have rendered the word according to the translation of Barocius, who is likely to have obtained the true reading, from the variety of manuscripts which he consulted.

[DRW—The following comment, relevant to this translation, appears in a footnote to a review by Augustus De Morgan of Peyrard's edition of Euclid's *Elements of Geometry.* De Morgan understood the sense of Proclus to be that "Pythagoras wrote on *incommensurables*," and set out his reasons in the accompanying footnote as follows: "Ἀλόγων is the Greek word, which always meant incommensurables. But Barocius, whose Latin is highly esteemed, translated it, *quæ explicari non possunt*, and the late Thomas Taylor, the Platonist, who translated Proclus with the love of a disciple, follows Barocius, and cites Fabricius, who thought the words should be ἀναλόγων, proportionals. But surely 'incommensurables' makes perfect sense, and we know that some rather acute ideas of incommensurables must have preceded Euclid's theory of proportion. The words of Proclus are, τὴν τῶν ἀλόγων πραγματείαν καὶ τῶν κοσμικῶν σχημάτων σύστασιν ἀνεῦρε."]

[45] The quadrature of the Lunula is as follows. Let ABC be a right-angled triangle, and BAC a semi-circle on the diameter BC: BNA a semi-circle described on the diameter AB; AMC a semi-circle described on the diameter AC.



Then the semi-circle BAC is equal to the semi-circles BNA, and AMC together: (because circles are to each other as the squares of their diameters, 31, 6.) If, therefore, you take away the two spaces BA, AC common on both sides, there will remain the two lunulas BNA, AMC, bounded on both sides with circular lines, equal to the right-angled triangle BAC. And if the line BA, be equal to the line AC, and you let fall a perpendicular to the hypothenuse BC, the triangle BAO will be equal to the lunular space BNA, and the triangle COA will be equal to the lunula CMA. Those who are curious, may see a long account of an attempt of Hippocrates to square the circle, by the invention of the lunulas, in Simplicius on Aristotle's Physics, lib. i.

elements: but Plato, who was posterior to these, caused as well geometry itself, as the other mathematical disciplines, to receive a remarkable addition, on account of the great study he bestowed in their investigation. This he himself manifests, and his books, replete with mathematical discourses, evince: to which we may add, that he everywhere excites whatever in them is wonderful, and extends to philosophy. But in his time also lived Leodamas the Thasian, Architas the Tarentine and Theætetus the Athenian; by whom theorems were increased, and advanced to a more skilful constitution. But Neoclides was junior to Leodamas, and his disciple was Leon; who added many things to those thought of by former geometricians. So that Leon also constructed elements more accurate, both on account of their multitude, and on account of the use which they exhibit: and besides this, he discovered a method of determining when a problem, whose investigation is sought for, is possible, and when it is impossible. But Eudoxus the Cnidian, who was somewhat junior to Leon, and the companion of Plato, first of all rendered the multitude of those theorems which are called universals more abundant; and to three proportions added three others; and things relative to a section, which received their commencement from Plato, he diffused into a richer multitude, employing also resolutions in the prosecution of these. Again, Amyclas the Heracleotean, one of Plato's familiars, and Menæchmus, the disciple, indeed, of Eudoxus, but conversant with Plato, and his brother Dinostratus, rendered the whole of geometry as yet more perfect. But Theudius, the Magnian, appears to have excelled, as well in mathematical disciplines, as in the rest of philosophy. For he constructed elements egregiously, and rendered many particulars more universal. Besides, Cyzicinus the Athenian, flourished at the same period, and became illustrious in other mathematical disciplines, but especially in geometry. These, therefore, resorted by turns to the Academy, and employed themselves in proposing common questions. But Hermotimus, the Colophonian, rendered more abundant what was formerly published by Eudoxus and Theætetus, and invented a multitude of elements, and wrote concerning some geometrical places. But Philippus the Mendæan[46], a disciple of Plato, and by him inflamed in the mathematical disciplines, both composed questions, according to the institutions of Plato, and proposed as the object of his enquiry whatever he thought conduced to the Platonic philosophy. And thus far historians produce the perfection of this science. But Euclid was not much junior to these, who collected elements, and constructed many of those things which were invented by Eudoxus; and perfected many which were discovered by Theætetus. Besides, he reduced to invincible demonstrations, such things as were exhibited by others with a

[46]So Barocius reads, but Fabricias Μεομαῖος.

weaker arm. But he lived in the times of the first Ptolemy: for Archimedes mentions Euclid, in his first book, and also in others. Besides, they relate that Euclid was asked by Ptolomy [*sic.*], whether there was any shorter way to the attainment of geometry than by his elementary institution, and that he answered, there was no other royal path which led to geometry. Euclid, therefore, was junior to the familiars of Plato, but more ancient than Eratosthenes and Archimedes (for these lived at one and the same time, according to the tradition of Eratosthenes) but he was of the Platonic sect, and familiar with its philosophy: and from hence he appointed the constitution of those figures which are called Platonic[47], as the end of his elementary institutions.

[47] The five regular bodies, the pyramid, cube, octaedron, dodecaedron and icosaedron; concerning which, and their application to the theory of the universe, see Kepler's admirable work, De Harmonia Mundi.