The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book II, Chapter 2

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CHAP. II.

What kind of Science Geometry is.

BUT let us now speak of that science which possesses a power of contemplating the universal forms participated by imaginative matter. Geometry, therefore, is endued with the knowledge of magnitudes and figures, and of the terms and reasons subsisting in these; together with the passions, various positions and motions which are contingent about these. For it proceeds, indeed, from an impartible point, but descends even to solids, and finds out their multiform diversities. And again, runs back from things more composite, to things more simple, and to the principles of these: since it uses compositions and resolutions, always beginning from suppositions, and assuming its principles from a previous science: but employing all the dialectic ways. In principles, by the divisions of forms from their genera, and by defining its orations. But in things posterior to principles, by demonstrations and resolutions. As likewise, it exhibits things more various, proceeding from such as are more simple, and returning to them again. Besides this, it separately discourses of its subjects; separately of its axioms; from which it rises to demonstrations; and separately of essential accidents, which it shews likewise are resident in its subjects. For every science has, indeed, a genus, about which it is conversant, whose passions it proposes to consider: and besides this, principles, which it uses in demonstrations; and essential accidents. Axioms, indeed, are common to all sciences (though each employs them in its peculiar subject matter), but genus and essential accident vary according to the sciental variety. The subjects of geometry are therefore, indeed, triangles, quadrangles, circles, and universally figures and magnitudes, and the boundaries of these. But its essential accidents are divisions, ratios, contacts, equalities, applications, excesses, defects, and the like. But its petitions and axioms, by which it demonstrates every particular are, this, to draw a right line from any point to any point; and that, if from equals you take away equals, the remainders will be equal; together with the petitions and axioms consequent to these. Hence, not every problem nor thing sought is geometrical, but such only as flow from geometric principles. And he who is reproved and convicted from these, is convinced as a geometrician. But whoever is convinced from principles different from these, is not a geometrician, but is foreign from the geometric contemplation. But the objects of the non-geometric investigation, are of two kinds. For the thing sought for, is either from entirely different principles, as we say that a musical enquiry is foreign from geometry, because it emanates from other suppositions, and not from the principles of geometry: or it is such as uses, indeed, geometrical principles, but at the same time perversely, as if any one should say, that parallels coincide. And on this account, goemetry also exhibits to us instruments of judging, by which we may know what things are consequent to its principles, and what those are which fall from the truth of its principles: for some things attend geometrical, but others arithmetical principles. And why should we speak of others, since they are far distant from these? For one science is more certain than another (as Aristotle says[34]) that, indeed, which emanates from more simple suppositions, than that which uses more various principles; and that which tells the *why*, than that which knows only the simple existence of a thing; and that which is conversant about intelligibles, than that which touches and is employed about sensibles. And according to these definitions of certainty, arithmetic is, indeed, more certain than geometry, since its principles excel by their simplicity. For unity is void of position, with which a point is endued. And a point, indeed, when it receives position, is the principle of geometry: but unity, of arithmetic. But geometry is more certain than spherics; and arithmetic, than music. For these render universally the causes of those theorems, which are contained under them. Again, geometry is more certain than mechanics, optics, and catoptrics. Because these discourse only on sensible objects. The principles, therefore, of geometry and arithmetic, differ, indeed, from the principles of other sciences; but the hypotheses of these two, alternately differ and agree according to the difference we have already described. Hence, also, with respect to the theorems which are demonstrated in these sciences, some are, indeed, common to them, but others peculiar. For the theorem which says, *every proportion may be expressed*, alone belongs to arithmetic; but by no means to geometry: since this last science contains things which cannot be expressed[35]. That theorem also, which affirms, that *the gnomons of quadrangles are terminated according to the least*[36], is the property of arithmetic: for in geometry, a minimum cannot be given. But those things are peculiar to geometry, which are conversant about positions; for numbers have no position: which respect

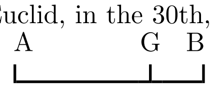
[34]In his first Analytics, t. 42.

[35]Such as the proportion of the diagonal of a square to its side; and that of the diameter of a circle, to the periphery.

[36]The gnomons, from which square numbers are produced, are odd numbers in a natural series from unity, i. e. 1, 3, 5, 7, 9, 11, &c. for these, added to each other continually, produce square numbers *ad infinitum*. But these gnomons continually decrease from the highest, and are at length terminated by indivisible unity.

contacts; for contact is found in continued quantities: and which are conversant about ineffable proportions: for where division proceeds to infinity, there also that which is ineffable is found[37]. But things common to both these sciences, are such as respect divisions, which Euclid treats of in the second book; except that proposition which divides a right line into extreme and mean proportion[38]. Again, of these common theorems, some, indeed, are transferred from geometry into arithmetic; but others, on the contrary, from arithmetic into geometry: and others similarly accord with both, which are derived into them from the whole mathematical science. For the permutation, indeed, conversions, compositions, and divisions of ratios are, after this manner, common to both. But such things as are commensurable, arithmetic first beholds; but afterwards geometry, imitating arithmetic. From whence, also, it determines such things to be commensurables of this kind, which have the same mutual ratio to one another, as number to number; because commensurability principally subsists in numbers. For where number is, there also that which is commensurable is found; and where commensurable is, there also number. Lastly, geometry first inspects triangles and quadrangles: but arithmetic, receiving these from geometry, considers them according to proportion. For in numbers, figures reside in a causal manner. being excited, therefore, from effects, we pass to their causes, which are contained in numbers. And at one time, we indifferently behold the same accidents, as when every polygon is resolved by us into triangles[39]: but at another time, we are

[37]This doctrine of ineffable quantities, or such whose proportion cannot be expressed, is largely and accurately discussed by Euclid, in the tenth book of his Elements: but its study is neglected by modern mathematicians, *because it is of no use*, that is, because it contributes to nothing mechanical.

[38]This proposition is the 11th of the second book: at least, the method of dividing a line into extreme and mean proportion, is immediately deduced from it; which is done by Euclid, in the 30th, of the sixth book. Thus, Euclid shews (II. 2.) how to divide the line (

) A B, so that the rectangle under the whole A B, and the segment G B, may be equal to the square made from A G: for when this is done, it follows, that as A B is to A G, so is A G to G B; as is well known. But the proposition, as Dr. Barrow observes, cannot be explained by numbers; because there is not any number which can be so divided, that the product from the whole into one part, may be equal to the square from the other part. [DRW—The proposition proving this result is in fact Euclid, *Elements*, II. 11, not II. 2.]

[39]All polygonous figures, may, it is well known, be resolved into triangles; and this is no less true of polygonous numbers, as the following observations evince. All number originates from indivisible unity, which corresponds to a point: and it is either linear, corresponding to a line; or superficial, which corresponds to a superficies; or solid, which imitates a geometrical solid. After unity, therefore, the first of linear numbers is the duad; just as every finite line is allotted two extremities. The triad is the first of the superficial numbers; as the triangle of geometrical figures. And the tetrad, is the first of solids;

content with what is nearest to the truth, as when we find in geometry one quadrangle the double of another, but not finding this in numbers, we say that one square is double of another, except by a deficience of unity. As for instance, the square from 7, is double the square from 5, wanting one. But we have produced our discussion to this length, for the purpose of evincing the communion and difference in the principles of these two sciences. Since it belongs to a geometrician to survey from what common principles common theorems are divided; and from what principles such as are peculiar proceed; and thus to distinguish between the geometrical, and non-geometrical, referring each of them to different sciences.

because a triangular pyramid, is the first among solid numbers, as well as among solid figures. As, therefore, the monad is assimilated to the point, so the duad to the line, the triad to the superficies, and the tetrad to the solid. Now, of superficial numbers, some are triangles, others squares, others pentagons, hexagons, heptagons, &c. Triangular numbers are generated from the continual addition of numbers in a natural series, beginning with unity. Thus if the numbers 1, 2, 3, 4, 5, &c. be added to each other continually, they will produce the triangular numbers 1, 3, 6, 10, 15, &c. and if every triangular number be added to its preceding number, it will produce a square number. Thus 3 added to 1 makes 4; 6 added to 3 is equal to 9; 10 added to 6 is equal to 16; and so of the rest. Pentagons, are produced from the junction of triangular and square numbers, as follows. Let there be a series of triangular numbers 1, 3, 6, 10, 15, &c.

And of squares 1, 4, 9, 16, 25, &c.

Then the second square number, added to the first triangle, will produce the first pentagon from unity, i. e. 5. The third square added to the second triangle, will produce the second pentagon, i. e. 12; and so of the rest, by a similar addition. In like manner, the second pentagon, added to the first triangle, will form the first hexagon from unity; the third pentagon and the second triangle, will form the second hexagon, &c. And by a similar proceeding, all the other polygons may be obtained.