The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book II, Chapter 1

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B O O K II.

CHAP. I.

What Part Geometry is of Mathematics, and what the Matter is of which it consists.

In the preceding discourses we have considered those common properties which respect the whole of the mathematical science; and this we have done agreeable to the doctrine of Plato; at the same time collecting such particulars as pertain to our present design. But consequent to this it is requisite that we should discourse on geometry itself, and on the proposed institution of the elements, for the sake of which we have undertaken the whole of the present work. That geometry then, is a part of the whole of mathematics, and that it obtains the second place after arithmetic, since it is perfected and bounded by this, (for whatever in geometry may be expressed and known, is determined by arithmetical reasons) has been asserted by the ancients, and requires no long discussion in the present enquiry. But we also may be able to relate our opinion on this particular, if we consider what place, and what essence its subject matter[29] is allotted among the universality of things. For

[29]The design of the present chapter is to prove that the figures which are the subjects of geometric speculation, do not subsist in external and sensible matter, but in the receptacle of imagination, or the matter of the phantasy. And this our philosopher proves with his usual elegance, subtilty, and depth. Indeed, it must be evident to every attentive observer, that sensible figures fall far short of that accuracy and perfection which are required in geometrical definitions: for there is no sensible circle perfectly round, since the point from which it is described is not without parts; and, as Vossius well observes, (de Mathem. p. 4.) there is not any sphere in the nature of things, that only touches in a point, for with some part of its superficies it always touches the subjected plane in a line, as Aristotle shews Protagoras to have objected against the geometricians. Nor must we say, with that great mathematician Dr. Barrow, in his Mathematical Lectures, page 76, "that all imaginable geometrical figures, are really inherent in every particle of matter, in its utmost perfection, though not apparent to sense; just as the effigies of Cæsar lies hid in the unhewn marble, and is no new thing made by the statuary, but only is discovered and brought to sight by his workmanship, i. e. by removing the parts of mattter by which it is overshadowed and involved. Which made Michael Angelus, the most famous carver, say, *that sculpture was nothing but a purgation from things superfluous. For take all that is superfluous*, (says he) *from the wood or stone, and the rest will be the figure you intend.* So, if the hand of an angel (at least the power of God) should think fit to polish any

from a proper survey of this, the power of the science which knows this subject matter, the utility arising from it, and the good acquired by its learners, will immediately appear. Indeed, some one may doubt in what genus of things he ought to place geometrical matter, so as not to deviate from the truth it contains. For if the figures concerning which geometry discourses, exist in sensible natures, and cannot be separated from the dark receptacle of matter; how can we assert that geometry frees us from sensible objects, that it brings us to an incorporeal essence, that it accustoms us to an inspection of intelligibles, and prepares us for intellectual energy? Where shall we every survey among sensible objects a point without parts, or a line destitute of breadth, or a superficies without profundity, or the equality of lines from the centre to the circumference; or the multangles, and all the figures of many bases, concerning which geometry informs us? Lastly, after what manner can the reasons of such a science remain free from all possible confutation; since, indeed, sensible forms and figures are susceptive of the more and the less, are all moveable and mutable, and are full of material variety; among which equality subsits mixt and confused with its contrary inequality, and into which things without parts have proceeded into partition, and interval, darkened by the shades of matter, and lost in its infinite folds? But if the subjects of geometry are removed from matter, are pure forms, and are separated from sensible objects: they will be all of them, without doubt, void of parts, incorporeal, and destitute of magnitude. For extension, tumor, and interval, approach to forms, on account of the material receptacle in which they are involved, and which receives things destitute of parts, distributed into parts; things void of dimension, extended into dimension; and immove-

particle of matter, without vacuity, a spherical superficies would appear to the eyes, of a figure exactly round; not a created anew, but as unveiled and laid open from the disguises and covers of its circumjacent matter." For this would be giving a perfection to sensible matter, which it is naturally incapable of receiving: since external body is essentially full of pores and irregularities, which must eternally prevent its receiving the accuracy of geometrical body, though polished by the hand of an angel. Besides, what polishing would ever produce a point without parts, and a line without breadth? For though body may be reduced to the greatest exility, it will not by this means ever pass into an incorporeal nature, and desert its triple dimension. Since external matter, therefore, is by no means the receptacle of geometrical figures, they must necessarily reside in the catoptric matter of the phantasy, where they subsist with an accuracy sufficient for the energies of this science. It is true, indeed, that even in the purer matter of imagination, the point does not appear perfectly impartible, nor the line without latitude: but then the magnitude of the point, and the breadth of the line is indefinite, and they are, at the same time, unattended with the qualities of body, and exhibit to the eye of thought, magnitude alone. Hence, the figures in the phantasy, are the proper recipients of that universal, which is the object of geometrical speculation, and represent, as in a mirror, the participated subsistence of those vital and immaterial forms which essentially reside in the soul.

able natures accompanied with motion. How then, if this is the case, shall we cut a right line, triangle, and circle? How can we speak of the diversities of angles, and the increments and decrements of triangular and quadrangular figures? Or how exhibit the contacts of circles or right lines? For all these evince that the geometric matter consists of parts, and does not reside among indivisible reasons. Such then are the doubts concerning the matter of geometry, to which we may add, that Plato considers the forms of geometry as placed in cogitation; and grants, that we advance from sensibles to forms of this kind, and that we rise from sensibles to intellect, though (as we have previously observed) the reasons subsisting in cogitation are indivisible, are separated by no interval, and subsist according to the peculiarity of the soul. But if reasons are to be rendered agreeable to things themselves, and to the doctrine of Plato, the following division must be adopted. [30]Every

[30]This division is elegantly explained by Ammonius, (in Porphyr. p. 12.) as follows. "Conceive a seal-ring, which has the image of some particular person, for instance, of Achilles, engraved in its seal, and let there be many portions of wax, which are impressed by the ring. Afterwards conceive that some one approaches, and perceives all the portions of wax, stamped with the impression of this one ring, and keeps the impressions of the ring in his mind: the seal engraved in the ring, represents the universal, prior to the many: the impression in the portions of wax, the universal in the many: but that which remains in the intelligence of the beholder, may be called the universal, after and posterior to the many. The same must we conceive in genera and species. For that best and most excellent artificer of the world, possesses within himself the forms and exemplars of all things: so that in the fabrication of man, he looks back upon the form of man resident in his essence, and fashions all the rest according to its exemplar. But if any one should oppose this doctrine, and assert that the forms of things do not reside with their artificer, let him attend to the following arguments. The artificer either knows, or is ignorant of that which he produces: but he who is ignorant will never produce any thing. For who will attempt to do that, which he is ignorant how to perform? since he cannot act from an irrational power like nature, whose operations are not attended with animadversion. But if he produces any thing by a certain reason, he must possess a knowledge of every thing which he produces. If, therefore, it is not impious to assert, that the operations of the Deity, like those of men, are attended with knowledge, it is evident that the forms of things must reside in his essence: but forms are in the demiurgus, like a seal in the ring; and these forms are said to be prior to the many, and separated from matter. But the species man, is contained in each particular man, like the impression of the seal in the wax, and is said to subsist in the many, without a separation from matter. And when we behold particular men, and perceive the same form and effigy in each, that form seated in our soul, is said to be after the many, and to have a posterior generation: just as we observed in him, who beheld many seals impressed in the wax from one and the same ring. And this one, posterior to the many, may be separated from body, when it is conceived as not inherent in body, but in the soul: but is incapable of a real separation from its subject." We must here, however, observe, that when Ammonius speaks of the knowledge of the Deity, it must be conceived as far superior to ours. For he possesses a nature more true than all essence, and a perception clearer than all knowledge. And as he produced all things by his unity, so by an ineffable unity of apprehension, he knows the universality

universal, and one thing containing many, is either naturally disposed to be thought of in particulars, or to appear such, because it possesses its existence in these; is inseparable from them; is disposed and distributed in them; and together with these is either moved, or firmly and immoveably abides. Or it is adapted to subsist prior to many, and to possess a power of generating multitude, affording to many things images from itself, being furnished with a nature destitute of parts, from the essences which it participates, and raising various participations to secondary natures: or it is disposed to be formed by thought, from the many, to possess a generating existence, and to reside in the last place in the many. For, according to these modes of subsistence, we shall find, I think, that some subsist before the many, others in the many, and others from the relation and predication which they possess to these. But, that I may absolve all in one word, universal forms being threefold, we shall consider the differences of that form which many participate, which exists in many, and fills particular natures according to its subject matter. Besides this, establishing a twofold order of participants, one subsisting in sensible objects, but the other in the phantasy, (since matter is twofold; one indeed, of things united with sense, but the other of such as fall under the inspection of phantasy, as Aristotle asserts, in a certain place[31] we must allow that the universal, which is distributed in the many, is likewise twofold. The one, indeed, sensible, as being that which sensible objects participate; but the other imaginative, as that which subsists in the *many* of the phantasy. For the phantasy, on account of its forming motion, and because it subsists with, and in body, always receives impressions which are both divided and figured. So that whatever is known by it, is allotted a correspondent existence: on which account, Aristotle[32] does not hesitate to call it passive intellect. But if it is intellect, which is it not impassive, and destitute of matter? And if it operates with passion, how can it with propriety be called intellect? For impassivity, indeed, properly belongs to intellect and intelligent nature: but passivity is very remote from such an essence. But (unless I am deceived) Aristotle being willing to explain the middle nature between cognitions the most primary, and such as are the last, calls it at the same time intellect, because similar to primary cognitions, and passive from that alliance which it possesses with such as are posterior. For first cognitions are indeed destitute of figures and forms; comprehending in themselves, intelligible natures, energizing about themselves, united with the objects of knowledge, and free from all extrinsical impression and passion. But last cog-

of things.

[31]In lib. vii. Metaphys. 35 & 39.

[32]In lib. iii. de Anima, tex. 20.

nitions exercise themselves through the medium of instruments, are rather passions than energies, admit extrinsical knowledge, and move themselves together with their various subjects. For such (says Plato) are the sensations which arise from violent passions. But the phantasy, obtaining a middle centre in the order of cognitions, is excited, indeed, by itself, and produces that which falls under cogitation: but because it is not separate from body, it deduces into partition, interval, and figure, the objects of its knowledge, from the indivisibility of an intellectual life. Hence, whatever it knows, is a certain impression and form of intelligence. For it understands the circle, together with its interval, void, indeed, of external matter, but possessing intelligible matter. On this account, like sensible matter, it does not contain one circle only: for we behold in its receptacle, distance, together with the more and the less, and a multitude of circles and triangles. If then an universal nature is distributed in sensible circles, since each of these completes a circular figure, and they are all mutually similar, subsisting in one reason, but differing in magnitude or subjects: in like manner, there is a common something in the circles, which subsist in the receptacle of the phantasy, of which all its circles participate, and according to which they all possess the same form; but in the phantasy they possess but one difference only, that of magnitude. For when you imagine many circles about the same centre, they all of them exist in one immaterial subject and life, which is inseparable from a simple body, which, by the possession of interval, exceeds an essence destitute of parts; but they differ in magnitude and parvitude, and because they are contained and contain. Hence, that universal is two-fold, which is understood as subsisting in the many; one, indeed, in sensible forms; but the other in such as are imaginative. And the reason of a circular and triangular figure, and of figure universal, is twofold. The one subsisting in intelligible, but the other in sensible matter. But prior to these is the reason which resides in cogitation, and that which is seated in nature herself. The former being the author of imaginative circles, and of the one form which they contain; but the other, of such as are sensible. For there are circles existing in the heavens, and universally those produced by nature, the reason does not fall under a cogitative distribution. For in incorporeal causes, things possessing interval, are distinguished by no intervals: such as are endued with parts, subsist without parts: and magnitudes without the diffusion of magnitude, as on the contrary in corporeal causes, things without parts subsist divisibly, and such as are void of magnitude with the extension of magnitude. Hence, the circle resident in cogitation, is one, simple and free from interval: and magnitude itself is there destitute of magnitude; and figure expressed by no figure: for such are reasons separate from matter. But the circle subsisting in the phantasy, is divisible, figured, endued with interval, not one only, but

one and many, nor form alone, but distributed with form. And the circle, in sensible objects, is composite, distant with magnitude, diminished by a certain reason, full of ineptitude, and very remote from the purity of immaterial natures. We must therefore say, that geometry, when it asserts any thing of circle and diameter, and of the passions and affections which regard the circle; as of contacts, divisions, and the like: neither teaches nor discourses concerning sensible forms, (since it endeavours to separate us from these), not yet concerning the form resident in cogitation, (for here the circle is one, but geometry discourses of many, proposing something of each, and contemplating the same of all: and here it is indivisible, but the geometric circle is divisible); but we must confess, that it considers itself universal itself; yet as distributed in imaginative circles. And that it beholds, indeed, one circle[33]: and by the medium of another, contemplates the circle resident in the depths of cogitation: but by another, different from the preceding, fabricates the fair variety of its demonstrations. For since cogitation is endued with reasons, but cannot behold them contractedly, separated from material figure; it distributes and removes them, and draws them forth seated in the shadowy bosom of the phantasy, and placed in the vestibules of primary forms; revolving in it, or together with it, the knowledge of these: loving, indeed, a separation from sensibles, but finding imaginative matter proper for the reception of its universal forms. Hence, its intellection does not subsist without the phantasy. And the compositions and diversions of figures are imaginative; and their knowledge is the way which leads us to that essence pursued by cogitation: but cogitation itself, does not yet arrive at this stable essence, while it looks abroad to externals, contemplates its internal forms according to these, uses the impressions of reasons, and is moved from itself to external and material forms. But if it should ever be able to return to itself, when it has contracted intervals and impressions, and beholds multitude without impression, and subsisting uniformly; then it will excellently perceive geometrical reasons, void of division and interval, essential and vital, of which there is a copious variety. And this energy will be the best end of the geometric study; and truly the employment of a Mercurial gift, bringing it back as from a certain Calypso, and her detaining charms, to a more intellectual knowledge; and freeing it from those *forming* apprehensions with which the mirror of the phantasy is replete. Indeed, it is requisite that a true geometrician should be employed in this meditation, and should establish, as his proper end, the excitation and transition from the phantasy to cogitation

[33]That is, geometry first speculates the circle delineated on paper, or in the dust: but by the medium of the circular figure in the phantasy, contemplates the circle resident in cogitation; and by that universal, or circular reason, participated in the circle of the phantasy, frames its demonstrations.

alone; and that he should accomplish this by separating himself from intervals, and the passive intellect to that energy which cogitation contains. For by this means he will perceive all things without an interval, the circle and diameter without a part, the polygons in the circle, all in all, and yet every one separate and a-part. Since, on this account, we exhibit also in the phantasy, both circles inscribed in polygons, and polygons in circles; imitating the alternate exhibition of reasons destitute of parts. Hence, therefore, we describe the constitutions, the origin, divisions, positions, and applications of figures: because we use the phantasy, and distances of this kind proceeding from its material nature; since form itself is immoveable, without generation, indivisible, and free from every subject. But whatever form contains occultly, and in an indistant manner, is produced into the phantasy subsisting with intervals, divisibly and expanded. And that which, indeed, produces the forms of geometric speculation, is cogitation; but that form which they are produced, is the form resident in cogitation: and that in which the produced figure resides is what is called the passive intellect. Which folds itself about the impartibility of true intellect, separates from itself the power of pure intelligence free from interval, conforms itself according to all formless species, and becomes perfectly every thing from which cogitation itself, and our indivisible reason consists. And thus much concerning the geometric matter, as we are not ignorant of whatever Porphyry the Philosopher has observed in his miscellanies, and whatever many of the Platonists describe. But we think that the present discussions are more agreeable to geometric dissertations, and to Plato himself, who subjects to geometry the objects of cogitation. For these mutually agree among themselves; because the causes, indeed, of geometrical forms, by which cogitation produces demonstrations, pre-exist in demonstration itself: but the particular figures which are divided and compounded, are situated in the receptacle of the phantasy.