The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Book I, Chapter 6

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August 2020

[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 1, pp. 54–59 (1792).]

CHAP. VI.

Concerning the Essence of Mathematical Genera and Species⁸.

It now remains, that we consider what subsistence or essence ought to be assigned to mathematical genera and species? Whether we must deduce their origin and subsistence from sensible objects, or from abstraction, or from a collection of such things as are dispersed by parts into one common definition; or must allow them an existence prior to that of sensibles, as Plato affirms, and as the progression of universal being demonstrates? First then, if we affirm that mathematical species are composed from sensibles; whilst the soul from material triangles or circles, forms in herself the trigonic, or circular species, by a kind of secondary generation; I would ask from whence is derived the great certainty and accuracy of definitions? For it must either proceed from sensibles, or from the soul herself. But from sensibles is impossible, for these, in a continual flow of generation and decay, do not for a moment retain an exact sameness of being; and consequently fall far short of the exactness contained in the definitions themselves. It must therefore proceed from the soul, which, by her immaterial nature, procures perfection from the

Every thing white, is an animal: Every bird is white: Therefore, Every bird is an animal.

⁸I would particularly recommend this chapter to modern mathematicians, most of whom, I am afraid, have never considered whether or not the subjects of their speculation have any real subsistence: though it is surely an enquiry worthy the earnest attention of every liberal mind. For if the objects of mathematical investigation are merely imaginary, I mean the point without parts, the line without breadth, &c. the science, founded on these false principles, must of course be entirely delusive. Indeed, an absolutely true conclusion, can never flow from an erroneous principle, as from its cause: as the stream must always participate of its source. I mean such a conclusion as is demonstrated by the proper cause, $\pi\lambda\dot{\gamma}\nu$ où $\delta\iota\delta\tau$, $\dot{\alpha}\lambda\lambda$ ' $\delta\tau$, says Aristotle, in his first Analytics; that is, a syllogism from false principles will not prove the *why*, but only simply *that it is*: indeed it can only simply prove *that it is*, to him who admits the false propositions; because he who allows the premises, cannot deny the conclusion, when the syllogium is properly constructed. Thus we may syllogize in the first figure,

And the conclusion will be true, though the major and minor terms are false; but then these terms are not the causes of the conclusion, and we have an inference without a proof. In like manner, if mathematical species are delusive and fictitious, the conclusions deduced from them as principles, are merely hypothetical, and not demonstrative.

imperfect, accurate subtility from that which is neither accurate nor subtle, and rekindles the light of ideas from the obscure and unreal objects of sense.

For where shall we find, amongst sensible objects, an indivisible nature, such as that of a point, or a line without the dimension of breadth, or a superficies without depth, or the ever constant proportion of sides, and exact rectitude of angles? For my part, I cannot see where, since all divisible natures are thus mixed and confused together, nothing sincere, nothing free from its contrary, but things every yielding to separation, as well such as are removed by distance of place, as those which are united together. How then shall we obtain this durable essence for these immoveable natures from the ever fluctuating forms of sense? For whatever derives its existence from moveable beings, must of necessity be mutable and frail. And how shall we gain this perfect accuracy for the stable species, from the inaccurate and imperfect? For whatever is the cause of a conception, always immutable, is itself much more stable than its effect. We must therefore admit the soul to be the generator of these mathematical species and reasons. But if she contains them in herself, as first exemplars, she gives them an essential being, so that the generations are nothing else than propagations of species, which had a prior subsistence in herself: and thus we shall speak agreeably to the sentiments of Plato, and discover the true essence of mathematical entities. But if the soul, though she neither possess nor received the mathematical reasons prior to the energies of sense, yet fabricates this admirable immaterial building, and generates this fair series of speculations; how can she discern whether her productions are stable and constant, or things which the winds may dissipate, and phantoms rather than realities? What standard can she apply as the measure of their truth? Or how, since she is destitute of their essence, can she generate such a variety of reasons? For from such an hypothesis, we make their subsistence fortuitous, not tending to any scientific bound. Mathematical species are therefore the genuine offspring of the soul: nor does she derive from sensible objects the definitions she frames, but rather the first are propagated from the second; they are the energies of soul, which, as it were, pregnant with forms, delivers her immaterial progeny into the dark and fluctuating regions of matter, as evidences of the permanent duration of her species.

Again, if we collect mathematical reasons from externals, why are not demonstrations composed from sensibles, better than the demonstrations of universal and simple species? For we say, in order to the investigation of any thing sought, that the principles and propositions, should be applied to the conclusions. If then, particulars are the causes of universals, and sensibles the sources of reasoning, why does the boundary of demonstration always refer to that which is more universal, and not to that which is partial and particular? And how can we prove that the essence of intelligibles is more allied to demonstration than the essence of sensibles? For thus they speak⁹: his knowledge is not legitimate, who demonstrates that the isosceles, the equilateral, or the scalene triangle, have angles equal to two right; but he possesses science, properly so called, who demonstrates this of every triangle simply, or of triangle itself. And again, that universals, for the purpose of demonstration, are superior to particulars; that demonstrations concern things more universal; but that the principles from which demonstrations are composed, have a priority of existence, and a precedency in nature to singulars, and are the causes of the propositions they prove. It is very remote, therefore, from the nature of Apodictical sciences, that from converse with things of posterior origin, and from the dark perceptions of sense, they should falsely collect their indubital propositions. I add farther, that they who affirm this, make the soul of a baser nature than the material species themselves. For if matter derives from nature beings essential, and participating a high degree of entity and evidence; but the soul by a posterior energy, receives these from sensible objects, and fashions in herself resemblances and images of posterior origin, contemplating vile essences, and abstracting from matter, the forms inseparable from its nature; do they not make the soul more obscure and indigent than matter itself? For matter is the receptacle of forms materialized, as the soul is of species immaterialized. But in this case, matter would be the place of primary beings, and the soul of such as are secondary and subordinate: matter and its forms obtaining the lead in being, and existing as the sources of the subsistence of immaterial forms. Lastly, the material forms would have an essential existence, the others only an intentional denomination. How then can the soul, which is the first participant of intellect, and an intellective essence, and which derives from thence consummate knowledge, and a plenitude of life, become the receptacle of the most obscure species, the lowest in the order of things, and participating the most imperfect existence. But this opinion, which has been sufficiently exploded by others, needs no farther confutation.

If then, mathematical species do not subsist by material abstraction, nor by a collection of those common properties inherent in individuals; nor are at all, in their origin, posterior to sensibles, nor derived in any manner from them: it is necessary that the soul should either deduce them from herself, or from intellect; or lastly, from herself and intellect united. But if from herself alone, Whence do the images of intellectual species arise; whence do they

⁹Aristotle, in his last Analytics. The reader will please to observe, that the whole force of this nervous, accurate, and elegant reasoning, is directed against Aristotle; who seems unfortunately to have considered, with the moderns, that mathematical species subsist in the soul, by an abstraction from sensibles. See the preceding Dissertation.

derive their middle nature, linking, as it were, the divisible and indivisible essence together, if they do not participate the fullness of entity from primary essences? Lastly, how, upon this hypothesis, are the first exemplars, paradigms, or ideas, which subsist in intellect, the principles of universals? But if they are derived from intellect alone into the soul, how can the soul remain self-operative, and self-motive, if her inherent reasons flow from an external source, and are regulated by its operations? And in what respect does the soul differ from matter, which is all things in mere dormant capacity, but generates nothing appertaining to material species? It remains, therefore, that the soul deduces these species from herself, and intellect; and that she is the absolute consummation of the forms which originate from intellectual exemplars, but which are allotted from themselves a transition to permanent being. The soul, therefore, is by no means to be compared to a smooth tablet, void of all reasons; but she is an ever-written tablet, herself inscribing the characters in herself, of which she derives an eternal plenitude from intellect. For soul is a certain subordinate intellect, revolving round an intellect prior to herself, formed to its image, and participating its divine irradiations. If then, this superior intellect is all things intellectualy, soul will all things animally; if the first exists as the exemplar, soul will be as its image; if as contracted and united in itself, soul as divisible and expanded. And this is what Plato understood, when in his Timæus, he composes the soul of the world from all things, dividing her according to harmonical reasons, and analogies¹⁰ assigning to her the first principles effective of figures, I mean the right and circular line, and giving an intellectual motion to her inherent circles. All mathematical species, therefore, have a primary subsistence in the soul: so that, before sensible numbers, there are to be found in her inmost recesses, self-moving numbers; vital figures, prior to the apparent, ideal proportions of harmony previous to concordant sounds; and invisible orbs, prior to the bodies which revolve in a circle. So that soul is the prolific abundance of all these, and is another ornament producing herself, and produced from a proper principle, filling herself with life, and at the same time filled from the demiurgus of the universe, in an incorporeal and indistant manner. When, therefore, she produces and unfolds her latent reasons, she then detects every science and virtue. The essence of soul then consists in these species, nor must we suppose her inherent numbers to be a multitude of units, nor her

¹⁰[DRW—perhaps, here, harmonic ratios and proportions. The sentence in Friedlein's edition of Proclus's Commentaries (Friedlein, p. 16, 16–22) reads as follows: δ δὴ καὶ ὁ Πλάτων εἰδὼς ἐκ πάντων ὑφίστησι τῶν μαθματικῶν εἰδῶν τὴν ψυχὴν καὶ κατ' ἀριθμοὺς αὐτὴν διαιραῖ καὶ συνδεῖ ταῖς ἀναλογίαις καὶ τοῖς ἁρμονικοῖς λόγοις, καὶ τὰς πρωτουργοὺς ἀρχὰς τῶν σχημάτων ἐν αὐτῆ καταβάλλεται, τό τε εὐθὺ καὶ τό περιφερές, καὶ κινεῖ τοὺς ἐν αὐτῆ κύκλους νοερῶς.]

archytipal ideas of divisible forms to be corporeal: but we must conceive all these as subsisting ever vitally, and intellectually, as the exemplars of apparent numbers, figures, reasons and motions. And here we must follow the doctrine of Timæus, who derives the origin, and consummates the fabric of the soul, from mathematical forms, and reposes in her nature the causes of every thing which exists. For the seven bounding $terms^{11}$, comprehending the principles of all numbers, lines, planes and solids, pre-exist in soul according to cause. And again, the principles of figures are placed in her essence, according to a demiurgical power. And lastly, the first of all motions, which embraces every other motion in its comprehensive ambit, is co-existent with soul. For the principle of every thing which is moved is a circle, and the circular motion. The mathematical reasons, therefore, which fully consummate the soul, are essential, and self-moving: and the soul, by her cogitative power, diffusing, propagating, and evolving these, from her profound recesses, constitutes all the fair variety of mathematical sciences. Nor will she ever cease to generate, and waken into energy, succeeding species, which she divests her indivisible reasons of their intellectual simplicity. For she previously received all things, after a primary manner; and according to her infinite power, from pre-existent principles, deduces a beautiful series of various speculations.

 $^{^{11}\}mathrm{Viz.}$ 1, 2, 4, 8, 3, 9, 27. Concerning which, see lib. iii. of Proclus's excellent Commentary on the Timæus.