The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 48

Transcribed by David R. Wilkins

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[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 2, pp. 206–209 (1792).]

PROPOSITION XLVIII. THEOREM XXXIV.

If the quadrangle described from one side of a triangle, is equal to the quadrangles described from the other two sides of the triangle: then the angle comprehended by the remaining two sides of the triangle is right.

This theorem is the converse of the preceding, and the whole is converted to the whole. For if the triangle is rectanglar, the quadrangle which is described, from the side subtending the right angle, is equal to the quadrangles described from the other sides: and if the square from this, is equal to the squares from the other sides, the triangle is rectangular, because it has the angle right, which is comprehended by the remaining sides. And the demonstration of the Elementary institutor is indeed conspicuous. But when there is a triangle a b c, having the quadrangle, which is described from the side a c, equal to the quadrangles from the sides a b, b c, since in the triangle, a right line from the point b, is raised at right angles to the side b c, if it should be said, that the right line must be raised at right angles, to other parts, and not at those to which the elementary institutor raises it, we assert that this is an impossibility. For it can neither fall within, nor without the triangle; and can be no other than a b. For if possible, let it fall as b e. Because, therefore,



the angle e b c, is right, the angle c f b, is doubtless acute; and hence, the remaining angle a f b, will be obtuse. The side, therefore, a b, is greater than the side b f. Let a line b e, be placed equal to a b, and connect e c. Because, therefore, the angle e b c, is right, the quadrangle described from the side e c, is equal to the quadrangles from the sides e b, b c. But e b is equal to b a. The quadrangle, therefore, from the side e c, is equal to the quadrangles from the sides a b, b c. But the quadrangle from the side a c was also equal to the same. Hence the quadrangle from the side e c, is equal to that which is described from the side a c; and so e c is equal to a c. Two right lines, therefore, b e, ec, are equal to the two ba, ac, each to each, and are constructed upon the right line bc, which is impossible. And hence, the line raised at right angles, does not fall within the right line ab.

But neither can it fall without, towards other parts of the right line ab. For if possible, let it fall as bg, and let bg be equal to ab, and connect cg. Because, therefore, the angle gbc is right, the quadrangle described



on the side gc, is equal to the quadrangles from the sides bg, bc. But the quadrangle also, from the side a c, was equal to the quadrangles from the sides ab, bc, but ab is equal to qb; and so qc is equal to ac. But the right line gb, also, is equal to the right line ba, upon one right line bc, which is impossible. Hence, the right line which is raised from the point b, at right angles to bc, neither falls within, nor without the side ab; and therefore falls upon it. And so the objection is dissolved. But the institutor of the Elements, thus far completes his first book, in which he has delivered many species of conversions; (for he often converts the whole of theorems to the whole, and wholes to parts, and parts to parts) and has invented a great variety of problems; (for he has delivered the sections, positions, constructions, and applications of lines and angles. He likewise touches upon that mathematical place which is called admirable; and sufficiently brings local theorems into our remembrance. Besides, he unfolds the elementary institution of universal and particular theorems, and indicates the difference of indeterminate, and determinate problems; all which, attending him in his progress, we have orderly explained. Lastly, he refers the whole book to one purpose. I mean the elementary institution, of the contemplation respecting the more simple rectilineal figures; and finally, he investigates their constructions, and considers their essential properties. But we, indeed, shall give thanks to the gods, should we be able to comment on the other books, in a similar manner. In the mean time, if other cares should prevent the execution of our design, it is my opinion, that such as are studious of these contemplations, ought to expound the other books, after the same mode; by investigating that which is every where difficult, and pertinent to the subject, and capable of an easy division. For, indeed, the commentaries which are circulated at the present period, are replete with great and various confusion, because, at the same time, the neither infer any assignation of cause, nor dialectic judgment, nor philosophic contemplation.

END OF THE COMMENTARIES.