The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 47

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 204–206 (1792).]

PROPOSITION XLVII. Theorem XXXIII.

> In right angled triangles, the quadrangle, which is described from the side subtending the right angle, is equal to the quadrangles which are described from the sides comprehending the right angle.

If we attend to the historians of antiquity, we shall find them referring the present theorem to Pythagoras, and asserting that he sacrificed an ox for its invention. For my own part, I admire those who first investigated the truth of this theorem: but I possess a greater admiration for the elementary institutor, not only because he establishes its truth by evident demonstration, but likewise, because he persuades us by scientific reasons, which cannot be confuted of a theorem more universal than this in his sixth book[194]. For in that he shews universally, *that in right-angled triangles, the figure described from the side subtending the right angle, is equal to the figures described from the sides comprehending the right angle, when they are similar to the former figure, and are similarly described.* For every quadrangle is similar to every quadrangle; but all right-lined figures similar to each other, are not quadrangles; since in triangles, and other multangles, similitude is inherent. Hence, the reason which demonstrates that the figure described from the side subtending the right angle, whether it is quadrangular, or of some other form, is equal to the figures subsisting about the right angle, similar to the former, and similarly described; exhibits something more universal, and which possesses a greater power of producing science, than the reason exhibits, affirming a quadrangle alone, equal to quadrangles. For in the former case, it becomes manifest by a universal ostension, that the rectitude of the angle affords to the figure described from its subtending side, equality, to all the figures, subsisting about its comprehending sides, similar to the former, and similarly described: just as obtuseness is the cause of excess; but acuteness of diminution. But how this theorem is evinced, will be perspicuous, when we comment on it in the sixth book.

But let us now consider the truth of the present theorem, only adding this, that *universal* ought not to be shewn here, by him who has taught nothing concerning the similitude of right lined figures, and the doctrine of

[194] In the 31st proposition.

proportion: for many things which are here exhibited more particularly, are in that theorem shewn more universally by the same method. The institutor of the elements, therefore, shews the thing proposed in the present, from the common contemplation of parallelograms. But since right-angled triangles are two-fold, i. e. either isosceles, or scalene; in isosceles triangles, we shall never find numbers corresponding with the sides: for there is no quadrangular number, exactly double of another quadrangular number; since the square from the septenary is double of the square from the quinary, by a deficience of unity. But in scalene triangles it is possible, that numbers may be assumed, so as evidently to evince, that the square from the side subtending the right angle, is equal to the squares from the sides subsisting about the right angle. And of this kind is the triangle in the republic, whose right angle is contained by the ternary, and quaternary, but is subtended by the quinary. The quadrangle, therefore, from the quinary, is equal to the quadrangles from the other numbers: for this is twenty-five: but the quadrangle from the ternary is nine, and from the quaternary sixteen. And thus what we have asserted is perspicuous in numbers.

But there are delivered certain methods of inventing triangles of this kind, one of which they refer to Plato, but the other to Pythagoras, as originating from odd numbers. For Pythagoras places a given odd number, as the least of the sides about the right angle, and when he has received the quadrangle produced from this number, and diminished it by unity, he places the half of the remainder, as the greatest of the sides about the right angle; and when he has added unity to this, he produces the remaining side which subtends the right angle. Thus for example, when he has assumed the ternary, and has produced from it a quadrangular number, and from this number nine, has taken unity, he assumes the half of eight, that is four, and to this again he adds unity, and makes five; and thus discovers a right-angled triangle, having one of its sides of three, but the other of four, and the other of five units. But the Platonic method originates from even numbers. For when he has assumed a given even number, he places it as one of the sides about the right angle, and when he has divided this into half, and has produced a quadrangular number from the half, when he has added unity to this quadrangle, he forms the subtending side, but when he has taken unity from the quadrangle, he forms the remaining side about the right angle. Thus for example, when he has assumed the number four, and has multiplied the half of this into itself, and produced four, when he takes away unity he forms the number three, but when he adds unity, he produces the number five; and thus he has the same triangle effected, as by the Pythagoric method. For the square from the number five, is equal to the squares from the numbers three, and four. And thus much for the digression of the present narration. But as the demonstration of the elementary institutor is perspicuous, I do not think, that any thing should be added, because it would be superfluous; but we should be content with what is written. For those who have added any thing more, as the familiars of Hero and Pappus, have been obliged to assume in an affair of no difficulty, some of the propositions of the sixth book; and the cause which regards this affair. We shall therefore pass on to the following theorem.