The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 46

Transcribed by David R. Wilkins

August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 201–203 (1792).]

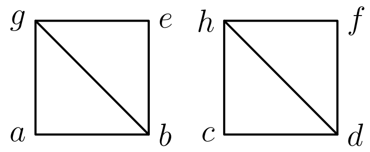
PROPOSITION XLVI. Problem XIV.

To describe a quadrangle from a given right line.

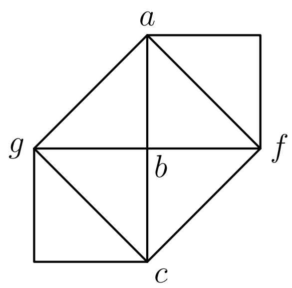
Euclid requires this problem, most particularly, in the construction of the following theorem. But he appears to have been desirous to deliver the origin of the two best rectilineal figures, viz. the equilateral triangle, and the quadrangle; because these right lined figures are required in the constitution of the mundane figures, and particularly of those four, to which origin and dissolution belong. For the icosaedron, and the octaedron, and the pyramid, are composed from equilateral triangles; but the cube from quadrangles. And on this account, as it appears to me, he has principally *constructed* the former, but *described* the latter. For he has discovered appellations adapted to these figures: since the equilateral triangle, so far as its composition is various, requires *construction*; but the quadrangle, so far as it originates from one side, requires *description.* For we cannot produce a triangle in the same manner as a quadrangle, by multiplying the number of a given right line into itself; but when we have conjoined right lines produced by other means, with the extremities of the given right line, we construct from these one equilateral triangle; and the description of circles, profits in discovering that point from which it is requisite to connect right lines, to the extremes of the proposed right line. But these observations are indeed perspicuous.

It may, however, be shewn, that the right lines, from which quadrangles are described, being equal, the quadrangles also shall be equal. For let the right lines $a\,b$, $c\,d$, be equal, and from $a\,b$, describe the quadrangle $a\,b\,e\,g$,[193], but from $c\,d$, the quadrangle $c\,d\,h\,f$, and connect the right lines $g\,b$, $h\,d$. Because, therefore, the right lines $a\,b$, $c\,d$, are equal, $a\,g$, $h\,c$, are also equal; and they comprehend equal angles, and the base $g\,b$, is equal to the base $h\,d$, and the triangle $a\,b\,g$, to the triangle $c\,d\,h$; and the doubles of these

[193][DRW—In the 1792 publication of Thomas Taylor's translation of Proclus's *Commentaries*, the vertex at the top right corner of the first square is labelled as c, and is referenced as such in the accompanying text, notwithstanding that the bottom left vertex of the second square is also labelled as c. In Friedlein's edition of the Greek text of Proclus's *Commentaries* (p. 424, 9–18), the vertex at the top right corner of the first square is labelled ε. Indeed, in that edition, the vertices of the first square, in anticlockwise order from the bottom left corner are labelled α, β, ε, η, whilst the corresponding vertices of the second square are labelled γ, δ, ζ, θ. Accordingly, in this transcription, e has been used in place of c to denote the top right corner of the first square.]



are equal. Hence the quadrangle $a\,e$, is not unequal to the quadrangle $c\,f$. But the converse of this also is true. For if the quadrangles are equal, the right lines, also, from which they are described, will be equal. Thus let the quadrangles $a\,f$, $c\,g$ be equal, and let them be so placed, that the side $a\,b$, may be in a right line with the side $b\,c$. Since therefore, the angles are right,



the right line $f\,b$, will be in a direct position, with the right line $b\,g$. Let the right lines $f\,c$, $a\,g$, $a\,f$, $c\,g$, be connected. Because, therefore, the quadrangle $a\,f$, is equal to the quadrangle $c\,g$; the triangle, also $a\,f\,b$, is equal to the triangle $c\,b\,g$. Let the common triangle $b\,c\,f$, be added. The whole triangle, therefore, $a\,c\,f$, is equal to the whole triangle $c\,f\,g$. Hence, $a\,g$ is parallel to $f\,c$. Again, because, as well $a\,f\,g$, as the angle $c\,g\,b$, is the half of a right angle $a\,f$, is parallel to $c\,g$. The right line, therefore, $a\,f$, is equal to the right line $c\,g$, since they are the opposite sides of a parallelogram. Because, therefore, there are two triangles $a\,b\,f$, $b\,c\,g$, which have the alternate angles equal, since $a\,f$, $c\,g$, are parallel; likewise one side $a\,f$, equal to the side $c\,g$, the side, also, $a\,b$, shall be equal to the side $b\,c$, and the side $b\,f$, to the side $b\,g$. And thus it is shewn, that the quadrangles $a\,f$, $c\,g$, being equal, the sides, also, from which they are described are equal.