The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 44

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PROPOSITION XLIV. PROBLEM XII.

To a given right line, to apply a parallelogram equal to a given triangle, in an angle which is equal to a given right lined angle.

According to the Familiars of Eudemus, the inventions respecting the *application*, *excess*, and *defect* of spaces, is ancient, and belongs to the Pythagoric muse. But junior mathematicians receiving names from these, transferred them to the lines which are called conic, because one of these they denominate a parabola, but the other an hyperbola, and the third an ellipsis[191]; since, indeed these ancient and divine men, in the plane description of spaces on a terminated right line, regarded the things indicated by these appellations. For when a right line being proposed, you adapt a given space to the whole right line, then that space is said to be *applied*; but when you make the longitude of the space greater than that of the right line, then the space is said to *exceed*; but when less, so that some part of the right line is external to the described space, then the space is said to be *deficient*. And after this manner, Euclid, in the sixth book, mentions both excess and defect. But in the present problem he requires application, wishing to apply to a given right line a parallelogram equal to a given triangle, but also an application to a determinate right line. As for example, a triangle being given, having an area of twelve feet, but a right line being proposed, whose length is four feet, we may apply to the right line a parallelogram equal to the triangle, if when we assume the whole length of four feet, we find how many feet the breadth ought to contain, that the parallelogram may become equal to the triangle. When, therefore, we have discovered that the breadth is three feet, and have multiplied the length with the breadth, the proposed angle being right, we shall obtain the desired space. And such is the verb *to apply*, formerly delivered by the Pythagoreans. But there are three things given in the present problem; one, the right line to which it is to be so applied, that it may become the whole side of that space; but the other is the triangle to which that which is applied ought to be equal; and the third is the angle to which it is requisite that the angle of the space should be equal. And here it is a gain perspicuous, that when the angle is right, the space which is applied,

[191]See Sims. Sec. Con Lib. I. Prop. 13. and Lib. II. Prop. 23, and Lib. III. Prop. 48.

is either a quadrangle, or an oblong; but when it is either acute or obtuse, the space is either a rhombus, or rhomboides. Besides, this too is manifest, that the right line ought to be finite; since this cannot be accomplished on an infinite line. At the same time, therefore, as he says, *to apply to a given right line*, he indicates that the right line must be necessarily finite. But he uses in the construction of the present problem, the construction of a parallelogram equal to a given triangle; since, as we have observed, application is not the same with construction. For the latter, indeed, constructs both the whole space, and all the sides; but the former, when it has one side given, constitutes on this the space, because it is neither deficient, nor exceeds according to this extension, but uses this one side which comprehends the area. But you may perhaps say, why does he use theorems, when he shews triangles equal to triangles; but problems, when he shews triangles equal to parallelograms? We reply, it is because equality spontaneously arises in things of the same species; but requires orin, and fabrication, in things of a dissimilar species, on account of the mutation subsisting according to species, since it is by itself difficult of invention.