The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 43

Transcribed by David R. Wilkins

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 196–198 (1792).]

PROPOSITION XLIII. Theorem XXXII.

The complements of parallelograms, situated about the diameter of every parallelogram, are equal to each other.

The beginning of this commentary is wanting.

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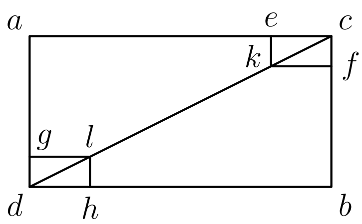
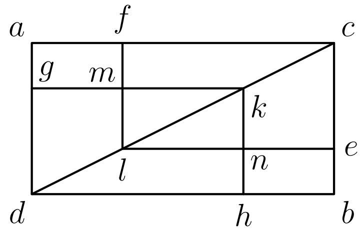
that parallelograms are not mutually conjoined according to one point, and that the complements are not quadrilateral; it is requisite that placing this also as a case, we should regard the same accident. For let there be a parallelogram $a\,b$, having the parallelograms $c\,k$, $d\,l$, about the same diameter, and let a certain right line $k\,l$, which is a part of the diameter intervene between them. Again, therefore, you may say the same, viz. that the triangle $a\,c\,d$

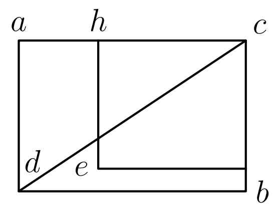


is equal to the triangle $b\,c\,d$, and the triangle $e\,c\,k$, to the triangle $k\,c\,f$; and likewise the triangle $d\,g\,l$, to the triangle $d\,h\,l$. The remaining figure, therefore $a\,g\,l\,k\,e$, of five sides, is equal to the remaining five-sided figure $b\,f\,k\,l\,h$. But these were the complements. Again, if the parallelograms are neither conjoined according to a point, nor distant from each other, but mutually cut each other, on this hypothesis also, the demonstration will be the same. For let there be a parallelogram $a\,b$, and a diameter $c\,d$, and let parallelograms be constructed about it, one of which is $e\,c\,f\,l$, but the other, by which this also is intersected, $d\,g\,k\,h$. I say that the complements $f\,g$, $e\,h$, are equal. For since the whole triangle $d\,g\,k$, is equal to the whole triangle $d\,h\,k$, but a part of it also, the triangle $k\,l\,m$, is equal to the triangle $k\,l\,n$; (since $l\,k$ is a parallelogram); hence the remaining trapezium $d\,l\,n\,h$, is equal to the remaining trapezium $d\,l\,m\,g$. But the triangle $a\,d\,c$, is equal to the triangle $b\,c\,d$, and the triangle $f\,c\,l$, in the parallelogram $e\,f$, to the triangle $e\,c\,l$, and



the trapezium $d\,g\,m\,l$, to the trapezium $d\,h\,n\,l$. The remaining quadrilateral figure, therefore $g\,f$, is not unequal to the remaining quadrilateral figure $e\,h$. And hence, the theorem is exhibited according to all its cases. But there are three only, and neither more nor less. For the parallelograms consisting about the same diameter, either cut each other or touch each other, according to a point, or are distant from each other by a certain part of the diameter.

But the institutor of the Elements assumes the appellation of *complements*, from the thing itself, so far as these also, besides two parallelograms fill up the whole: and on this account, it was not of itself thought worthy of being remembered in the definitions. For, indeed, variety is requisite to its declaration, such as the knowledge of a parallelogram, and what those parallelograms are, which are about the diameter of the whole parallelogram; since, when these are explained, this likewise becomes known. But those parallelograms are about the same diameter, which have a part of the whole diameter for their own: and those which have not this condition, are by no means about the same diameter. For when the diameter of the whole parallelogram is cut by the sides of an internal parallelogram, then this parallelogram is not about the same diameter with the whole parallelogram. As for example, in the parallelogram $a\,b$, let the diameter $c\,d$, cut the side $e\,h$, of the parallelogram $c\,e$. The parallelogram, therefore, $e\,c$, is not about the



same diameter with the parallelogram $c\,d$.