# The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 40

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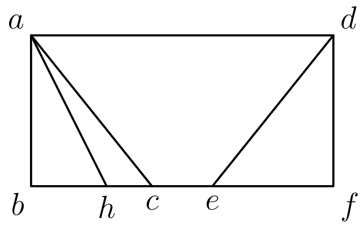
August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 190–191 (1792).]

PROPOSITION XL. Theorem XXX.

Equal triangles which are upon equal bases, and at the same parts, are between the same parallels.

There is the same mode of conversion too in the present theorem, and a similar demonstration; and that part of the deduction to an impossibility, which is omitted by the institutor of the Elements, is demonstrated after the same manner, and there is no occasion for repetition. But since these three conditions are in the aforesaid propositions, *situation upon equal, or on the same bases*; *position between the same parallels*; *and equality of triangles and parallelograms*, it is manifest that we may variously convert, by always connecting two, and leaving one. For we either suppose the basees the same, or equal, and the triangles and parallelograms between the same parallels, and thus we form four theorems; or we consider the triangles and parallelograms equal, and the bases the same, or equal, and thus we produce another four, two of which the elementary institutor omits, viz. those which respect parallelograms, but the other two relative to triangles, he exhibits; or lastly, when we have assumed them equal, and between the same parallels, we prove the remainder, that they are either upon the same, or upon equal bases, and produce another four, which the institutor of the Elements entirely neglects. For there is the same demonstration in these, except that two of these four are not essentially true. Thus, equal parallelograms or triangles, between the same parallels, are not necessarily upon the same base: but all this is true in these hypotheses, that they are upon the same or equal bases; but the other does not entirely follow the assumed hypotheses. Hence, as all these theorems are ten, the geometrician speaks of six, and neglects four, lest he should labour in vain, by repetition, since the demonstration is the same. For it may be shewn in triangles, that if they are equal, and between the same parallels, they will either be upon the same, or upon equal bases. For let it be denied, an if possible, let the triangles $a\,b\,c$, $d\,e\,f$, have these conditions, upon unequal bases $b\,c$, $e\,f$. Let too, $b\,c$, be the greater, and cut off $b\,h$, equal to $e\,f$, and connect $a\,h$. Because, therefore, the triangles $a\,b\,h$, $d\,e\,f$, are upon equal bases $b\,h$, $e\,f$, and between the same parallels, they are equal. But the triangles also, $a\,b\,c$, $d\,e\,f$, are supposed equal. Hence, the triangles $a\,b\,c$, $a\,b\,h$, are equal, which is impossible. The bases, therefore, of the triangles $a\,b\,c$, $a\,b\,h$, are not unequal. And the mode of demonstration




will be the same in parallelograms. Since, therefore, the ostensive method is the same, and the impossibility the same, viz. that the whole is equal to its part, it is not improperly omitted by the elementary institutor. And thus we have shewn, that there are necessarily ten theorems, and have enumerated what are omitted, and shewn the reason for their omission. But let us now pass to the following propositions.