

The Commentaries of Proclus on the First
Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 39

Transcribed by David R. Wilkins

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[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 187–189 (1792).]

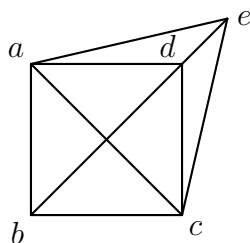
PROPOSITION XXXIX. THEOREM XXIX.¹⁸⁸

Equal triangles, which are upon the same base, and at the same parts, are between the same parallels.

When it was proposed to exhibit equality to us, then it was requisite to make four theorems, receiving two in parallelograms, but the other two in triangles, situated either upon the same, or upon equal bases. But now by conversion, we neglect the theorems which are converse in parallelograms, and esteem such as are converse in triangles worthy of relation. And the reason of this is, because the mode of demonstration in parallelograms, is the same indifferently, by a deduction to an impossibility, and the construction is similar. But we are content when we have exhibited the way in more simple figures, I mean triangles, to leave to the more curious the same mode of reasoning in the rest: since it is easy, at the same time, to perceive that there is the same method in these. For when we assume equal parallelograms, upon the same base, or upon equal bases, we must say that they are also between the same parallels. For if they are not, either one of them falls within, when the parallels which are in the other are produced; or without. But which ever case is assumed, when we receive it and its parallels, we may exhibit the same consequences as in triangles, I mean that the whole will be equal to its part: but this is impossible. It is however manifest, that the institutor of the Elements very properly adds the particle, *and at the same parts*. For it is possible that equal triangles, may be assumed upon the same base, one, indeed, at these parts, but the other at different parts, and yet these will not be entirely between the same parallels: for neither will they be contained under the same altitude. And on this account he added the particle.

But since a parallel may be drawn in a two-fold respect, according to an absurd hypothesis, i.e. either within or without, Euclid draws it within: but we can exhibit the same consequences, by drawing it without. For let the equal triangles abc , dbc , be upon base, and at the same parts. I say that they are between the same parallels, and that the right line connected at their vertices, is parallel to the base. Let the right line ad be connected. But if this is not parallel, let the line, external to this, i. e. ae be parallel, and let bd be produced to the point e , and connect ec . The triangle, therefore, abc ,

¹⁸⁸[DRW—Printed XXXIX. in Thomas Taylor's 1792 translation.]



is equal to the triangle ebc , the whole to the part. But this is impossible; and hence, the parallel line does not fall external to ad . But it is shewn by the institutor of the Elements, that neither does it fall within: and hence ad , is parallel to bc . Hence too, equal triangles, which are at the same parts, and upon the same base are parallel to each other. And thus the remaining part of the deduction to an impossibility is demonstrated. But it is worthy of observation, that since the conversion of theorems is triple (for either the whole is converted to the whole, as we have noticed, in the eighteenth and nineteenth theorems; or the whole to the part, as the sixth and fifth; or the part to the part, as the eighth and the fourth; for the whole is not a *datum*, in the one, and *an object of investigation* in the other, nor is *the object of investigation*, a *datum*, but a part) these triangular theorems appear to be of this kind. For, that the triangles are equal, is an object of investigation in the preceding; but this is not a datum alone in these, because it assumes, besides this, a part of that which was hypothesis in those. For to stand upon the same, or upon equal bases, is a datum in these, as well as in those, except that in these hypotheses he adds something which was neither an object of investigation, nor a datum in these; since the particle *at the same parts*, is over and above extrinsically assumed.