The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 38

Transcribed by David R. Wilkins

August 2020

[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 186–187 (1792).]

PROPOSITION XXXVIII. Theorem XXVIII.[187]

Triangles which are on equal bases, and between the same parallels, are equal to each other.

The present theorem also is local, because it corresponds in proportion with parallelograms, and supposes the situation of triangles upon equal bases. But Euclid seems, to me, to have delivered one demonstration by the first proposition of the sixth book of these four theorems, two of which are exhibited in parallelograms, and two in triangles: and two of which are on the same base, and the other two on equal bases. But that Euclid has performed this is unknown to the vulgar. For after he had shewn that triangles and parallelograms, which are under the same altitude, have the same proportion to each other as their bases, nothing demonstrates all these four theorems more universally, from proportion, than this theorem: since to possess the same altitude, is nothing else than being constituted between the same parallels. For all figures between the same parallels, are under the same altitude, and the contrary: since the altitude is the perpendicular, which extends itself from one parallel to the rest. In that proposition, therefore, it is shewn by proportion, that triangles and parallelograms, under the same altitude, that is, situated between the same parallels, are to each other as their bases, and so when the bases are equal, the spaces are equal; and when those are double, these will be double; and when the bases have any other proportion, the spaces also will have to each other the same proportion. But for the present, because it is not proper that he should use proportion, who has not yet explained its nature, he is content with equality and identity alone: for the identity of bases is collected from equality. Hence, these four theorems are comprehended in that one; not only because he shews by one demonstration, whatever are contained in these four, but likewise, because he adds what was wanting to their perfection, viz. identity of proportion, though the bases are unequal. But that this theorem, also, has many cases, and that it is possible that the bases of the triangles may be assumed, either having the same part as in parallelograms; or possessing no common part, but touching each other according to one point; or entirely separate, so that a line may intervene between them, is manifest, even to such as are endued with slender

[187] [DRW—Printed XXVII. in Thomas Taylor's 1792 translation.]

capacities. And this too is evident, that according to all cases, however the bases or vertices may be situated, the same method of proceeding must be adopted as in parallelograms; viz. parallels to the sides must be drawn, and produced both ways, and the equality of the triangles exhibited.