The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 37

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[Thomas Taylor, The Philosophical and Mathematical Commentaries of Proclus, Vol. 2, pp. 185–186 (1792).]

PROPOSITION XXXVII. THEOREM XXVII.

Triangles which are upon the same base, and between the same parallels, are equal to each other.

> The beginning of this Commentary is wanting. * * *

* for those being equal, the spaces are unequal; and when these are unequal, those are shewn to be equal. And this is the case with Chorographers, when they reason concerning the magnitudes of cities, from their ambits. But formerly, certain persons deceived their partners, in the distribution of their possessions, deluding them by an excess of ambit, so as to make them believe that they received a greater portion of land, when they received a greater ambit; and that they were gainers, but changing spaces into areas of less ambit.¹⁸⁶ Thus two isosceles triangles being proposed, one of which has each of its equal sides, containing five parts, but the base six: and the other has each of its equal sides five parts, but the base eight; and let these parts be, for instance, cubits, or digits, these triangles will very much deceive the ignorant in their choice. For the ambit of the one is eighteen, and of the other sixteen measures. But a geometrician is not ignorant that the spaces are equal, though the ambits are unequal; since the area of each is twelve measures. For if you draw a perpendicular from the vertex, you will bisect the bases, and cause the half of the one to be three but of the other four measures: but the perpendicular on the contrary, will be there equal to four, but here equal to three; since it is requisite that the square from the quinary, should be equal to the squares from the perpendicular, and the half of the base. But if the base of the one is equal to three, the perpendicular must be four; and if the base of the other is four, its perpendicular must be three. When, therefore, you have multiplied the half of the base with the perpendicular, you will have a space equal to the triangle: but this is the same in each, whether you multiply the quaternary with the ternary, or the ternary with the quaternary. And we have made these observations for the purpose of shewing that the equality of spaces is not to be entirely received from the ambits. Nor should we wonder, that though triangles upon the same base,

¹⁸⁶See Comment 8, of the third book, with its note.

may be infinitely increased between the same parallels, according to the remaining sides, yet the equality of the spaces immutably remains. But those triangles are said to be between the same parallels, which have their bases upon one of the parallel lines, and fix their vertices on the remainder; and whole vertices being connected, form one right line, parallel to the bases on the right line.