The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry Translated by Thomas Taylor (London, 1792) Proposition 36

Transcribed by David R. Wilkins

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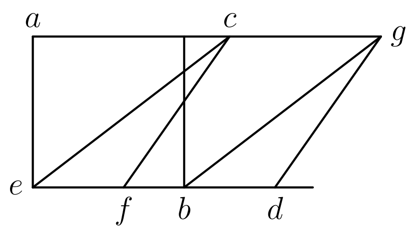
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 183–185 (1792).]

PROPOSITION XXXVI. Theorem XXVI.

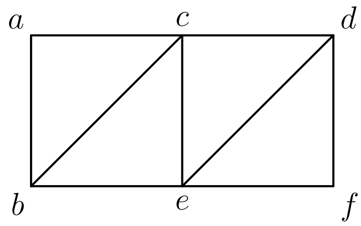
Parallelograms which are upon equal bases, and between the same parallels, are equal to each other.

The preceding theorem assumed, indeed, the same bases, but this receives them equal, and different from each other. But it is common to both, to suppose the parallelograms between the same parallels. It is requisite, therefore, that they should neither fall within, nor without their subject parallel lines. For parallelograms are said to be between the same parallels, when their bases and opposite sides are adapted to the same parallels. As to the rest, the institutor of the Elements, as he had assumed the bases entirely separate, exhibits the theorem. But nothing hinders our receiving them with this hypothesis, so that they may have a common part. For let $a\,b$, $c\,d$, be parallelograms upon equal bases $e\,b$, $f\,d$, having a common part, and constructed between the same parallels. I say that they are equal. Let the lines $e\,c$, $b\,g$ be connected. Because, therefore, $e\,f$, is equal to $b\,d$ (for the base $e\,b$ was



supposed equal to the base $f\,d$), but the side $c\,f$, is equal to the side $d\,g$, and the angle $c\,f\,e$, is equal to the angle $g\,d\,b$, and hence, $c\,e$ is equal to $b\,g$. But it is also parallel to it. Hence, $c\,b$ is a parallelogram, and has the same base with each of the parallelograms $a\,b$, $c\,d$, and is between the same parallels. The parallelogram, therefore, $a\,b$, is equal to the parallelogram $c\,d$.

But if any one should suppose that the bases of the parallelograms have neither a common part, nor are separate from each other, but (which is the only remaining hypothesis) that they touch each other in one point, as in the parallelograms $a\,e$, $e\,d$, we must say that the base $b\,e$ is equal to the base $e\,f$, and to the side $c\,d$. Hence, also, the right line $c\,b$, is equal to the right line $d\,e$, and is parallel to it. For the lines which join equal and parallel lines, are themselves also equal and parallel. Hence, $b\,d$ is a parallelogram, and is upon



the same base, and between the same parallels, with the parallelograms $c\,b$, $d\,e$. The parallelograms, therefore, $c\,b$, $d\,e$, are equal. But according to the first conception of a theorem, we may divide the constructions by asserting that the bases have either a common part, or touch each other, or are distant from each other. It is however possible, that though they may touch each other, as $b\,e$, $e\,f$, yet the whole parallelogram $d\,e$ may be supposed external to the side $c\,e$; or one side of the parallelogram $c\,f$, may be the diameter of the parallelogram $a\,e$; or the side $c\,e$, may cut the side $a\,c$; or the side $a\,c$, being produced beyond a, the side $c\,e$, may fall as the diameter of the parallelogram increased towards a, when the side $d\,f$, becomes the same as a line drawn from a to f; or the side $c\,e$, may cut the side $a\,c$, produced beyond a; or the side $a\,c$, may be still farther produced beyond a, so that the side $c\,e$ may fall beyond the point, to which $a\,c$ was extended in the preceding case, and the side $d\,f$, may cute the line produced beyond a[185]. * * *.

[185]The present Commentary is imperfect, both in the Greek, and the translation of Barocius; who observes that the conclusion is wanting in all the copies which he had an opportunity of perusing. Those who are curious may consult his scholium, in which he has endeavoured to complete it.