The Commentaries of Proclus on the First Book of Euclid's Elements of Geometry
Translated by Thomas Taylor
(London, 1792)
Proposition 34

Transcribed by David R. Wilkins

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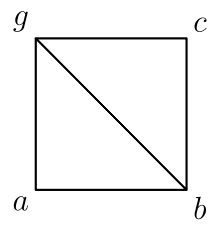
[Thomas Taylor, *The Philosophical and Mathematical Commentaries of Proclus*, Vol. 2, pp. 172–177 (1792).]

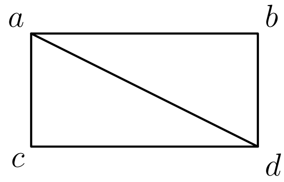
PROPOSITION XXXIV. Theorem XXIV.

> The opposite sides and angles of parallelogrammic spaces are equal to each other and they are bisected by the diameter.

As from the preceding theorem, he had assumed a parallelogram already constructed, he now contemplates its primarily inherent properties, and such things as express its peculiar constitution. But these are the following: *that the sides and angles which are opposite, are equal, and that the spaces themselves are bisected by the diameter.* For that part of the proposition relates to the spaces, which says: *and they are bisected by the diameter.* So that the area itself, is that whole which is bisected, and not the angles through which the diameter passes. These three properties then, are essentially inherent in parallelograms, the equality of the opposite sides and angles, and the bisection of the spaces by the diameter. And you may observe that the properties of parallelograms are investigated from all these, viz. from the sides, from the angles, and from the areas. But as there are four kinds of parallelograms, which Euclid defines in the hypotheses[178], viz. a quadrangle, oblong, rhombus, and rhomboides, it deserves to be remarked, that if we divide these four into rectangles, and non-rectangles, we shall find, that not only the diameters bisect these spaces, but that the diameters themselves, are, indeed, in rectangles equal, but in non-rectangles unequal, as was observed in the preceding theorem. But if we divide them into equilateral, and non-equilateral, we shall again find that in the equilateral figures, not only the spaces are bisected by the diameters, but likewise the angles through which they are drawn: but in non-equilaterals this is never the case. For in a quadrangle, and a rhombus, the diameters bisect the angles, and not the spaces only: but in an oblong, and a rhomboides, they alone bisect the spaces. For let there be a quadrangle, or a rhombus, $g\,c\,a\,b$, and a diameter $g\,b$. Because, therefore, the sides $g\,c$, $c\,b$, are equal to the sides $g\,a$, $a\,b$ (for they are equilateral), and the angles $g\,c\,b$, $g\,a\,b$, are equal (for they are opposite), and the basis also is common, hence, all are equal to all; and on this account the angles $c\,g\,a$, $a\,b\,c$, are bisected. Again, let there be an oblong, or rhomboides given. If, therefore, the angle $b\,a\,c$, and the angle $c\,d\,b$, is bisected by the diameter,

[178]In the definitions which are with great propriety called by the Platonist hypotheses, because their evidence is admitted without proof, which at the same time they are capable of receiving form [*sic.*] the first philosophy.





but the angle $c\,a\,d$, is equal to the angle $a\,d\,b$[179], the angle also $b\,a\,d$, will be equal to the angle $a\,d\,b$. Hence, the side also $a\,b$, will be equal to the side $b\,d$. But they are unequal; and consequently the angle $b\,a\,c$, is not bisected by the diameter, nor its equal the angle $c\,d\,b$. That I may therefore comprehend the whole in a few words, in a quadrangle the diameters are equal, on account of the rectitude of the angles, and the angles are bisected by the diameters, on account of the equality of the sides, and the areas are bisected by the diagonal, on account of the common property of parallelograms: but in an oblong, the diameters are indeed equal, because it is a rectangle, but the angles are not bisected by the diameters, because it is not equilateral, though the division of spaces into equal parts, is also inherent in this figure, so far as it is a parallelogram: but in a rhombus the diameters are unequal, because it is not a rectangle, but the spaces are not only bisected by these, because it is a parallelogram, but the angles also, because it is equilateral; and in the remaining figure, i. e. a rhomboides, the diameters are unequal, because it is not a rectangle, and the angles are cut by these into unequal parts, because it is not equilateral, and the spaces alone situated at each part of the diagonals, are equal, because it is a parallelogram. And thus much concerning observations of this kind, which exhibit the diversity found in the four divisions of parallelograms.

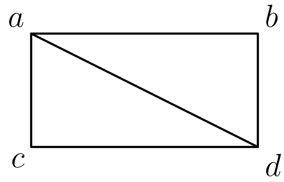
But we must not pass over in silence, the artificial consequence appearing in this theorem, that of theorems, some are universals, but others non-universals. But we shall speak concerning each of these, when we divide *the*

[179] [DRW—Printed This angle is printed as $a\,a\,b$ in the 1792 publication, but this is here corrected to $a\,d\,b$. The relevant passage of Proclus's Greek text, in the edition of Friedlein (1873), reads as follows (with α, β, γ and δ corresponding to a, b, c and d respectively: πάλιν ἔστω τὸ αὐτὸ ἑτερόμηκες ἢ ῥομβοειδές. εἰ οὖν δίχα τέμνεται ἡ ὑπὸ γαβ, ἀλλ᾽ ἡ ὑπὸ γαδ ἴση τῇ ὑπὸ αδβ, ἴση ἔσται καὶ ἡ ὑπὸ βαδ τῇ ὑπὸ αδβ, ὥστε καὶ ἡ αβ τῇ βδ. ἀλλ᾽ εἰσὶν ἄνισοι.]

object of investigation, which has, indeed, one part universal, but the other non universal. For though every theorem may seem to be universal, and every thing exhibited by the elementary institutor may appear to be of this kind (as in the the present he may not only seem to assert, that in all parallelograms universally, the opposite sides and angles are equal, but likewise that each is bisected by the diameter), yet we must say that some things are universally exhibited, but other not universally. For it is customary to call the *universal* which affirms the truth concerning every thing of which it is predicated, differently from *that universal*, comprehending all things in which the same symptom is inherent. Thus it is universal, that every isosceles triangle has three angles equal to two right, because it is true of all isosceles triangles: and it is universal that every triangle has three angles equal to two right, because it comprehends all things, in which this is essentially inherent. On which account we affirm that the possession of three angles equal to two right, is to be primarily manifested of a triangle. According to this signification, therefore, of theorems, calling some universal, but others non-universal, we must affirm that the present theorem, has, indeed, one of its objects of investigation universal, but the other non-universal. For the possession of opposite sides and angles that are equal, is a universal, since it is alone inherent in parallelograms: but that the diameter bisects the space, is not universal, because it does not comprehend all things in which this symptom is beheld; for this is inherent in a circle and ellipsis. And it appears, indeed, that primary conceptions of such like concerns, are more particular, but that in their progress they comprehend the whole. For when the ancients had contemplated that a diameter bisects an ellipsis, circle, and parallelogram, they afterwards surveyed that which was common in these. But we are deceived (says Aristotle[180]) when a non-universal is exhibited as universal, because that common something in which the symptom is primarily inherent, is nameless. For we cannot say what that is, which is common to numbers and magnitudes, motions and sounds; and it is likewise difficult to express what is common to an ellipsis, circle, and parallelogram. For one of these figures is right-lined, but the other circular, and the third mixt; and on this account we conceive that he exhibits universally, who demonstrates that a diameter bisects every parallelogram, because we do not at the same time perceive that common something, on account of which, this is true. This then in parallelograms, is not an universal of this kind, on account of the aforesaid cause; but the proposition is universal, which asserts, that every parallelogram has its opposite sides and angles equal. For if any figure is supposed, having its opposite sides and angles equal, it may be shewn to

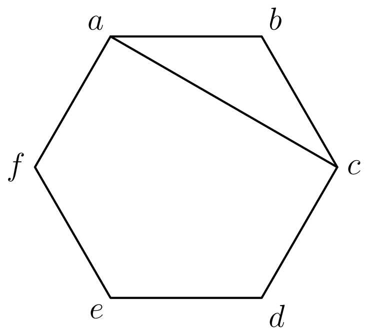
[180]In his last Analytics. See page 49, of the Dissertation, Vol. I. of this work.

be a parallelogram. Thus let such a figure be $a\,b\,c\,d$, and its diameter $a\,d$. Because, therefore, the sides $a\,b$, $b\,d$, are equal to the sides $a\,c$, $c\,d$, and the



angles comprehended by them are equal, and the base common, all will be equal to all. The angle, therefore $b\,a\,d$, is equal to the angle $a\,d\,c$, and the angle $a\,d\,b$, to the angle $c\,a\,d$. Hence, $a\,b$, is parallel to $c\,d$, and $a\,c$ to $b\,d$. And on this account the figure $a\,b\,c\,d$, is a parallelogram. And thus much may suffice for observations of this kind.

But the institutor of the Elements seems to have composed the name of parallelograms, by taking an occasion from the preceding theorem. For when he had shewn that right lines, which conjoin equal and parallel right lines at the same parts, were themselves also equal and parallel, it is evident that he pronounces as well the opposite sides which conjoin, as those which are conjoined, to be parallel: but that he very properly calls the figure which is contained by parallels, a parallelogram, in the same manner as he denominates that which is comprehended by right lines rectilineal. And it is evident that the institutor of the Elements places a parallelogram among quadrilateral figures. But it is worthy our observation and enquiry, whether every right-lined figure, which is composed from equal sides, since it is equilateral and equiangular, is to be called a parallelogram. For a figure of this kind also, has its opposite sides equal and parallel, as likewise the opposite angles equal. As for example, a sexangle, and an octangle, and a decangle. Thus, if you conceive a sexangle $a\,b\,c\,d\,e\,f$, and connect a right line $a\,c$, you may shew that $a\,f$ is parallel to $c\,d$. For the angle at the point b, is one right, and the



third part of a right angle; and this is true of every angle of a sexangle, since it is equiangular. Besides the side $a\,b$, is equal to the side $b\,c$, for it is placed equilateral. Each of the angles, therefore, $b\,a\,c$, $b\,c\,a$, is a third part of a right angle. Hence, the angles $f\,a\,c$, $a\,c\,d$, are right angles. And on this account $a\,f$, is parallel to $c\,d$. In like manner we may shew that the other opposite sides are parallel, and the same may be evinced in an octangle, and in the remaining figures of this kind. If, therefore, that is a parallelogram which is comprehended by parallels oppositely situated, a parallelogram will likewise subsists among non-quadrilateral figures. But it appears that with the institutor of the Elements a parallelogram is quadrilateral. And this is particularly perspicuous in that theorem, in which he says, *that a parallelogram which has the same base with a triangle, and is between the same parallels, is double of the triangle*: for this is alone true in quadrilateral figures.